EE-2025 Fall-99

Lecture 6

Sampling & Aliasing

13-Sept-99

Information

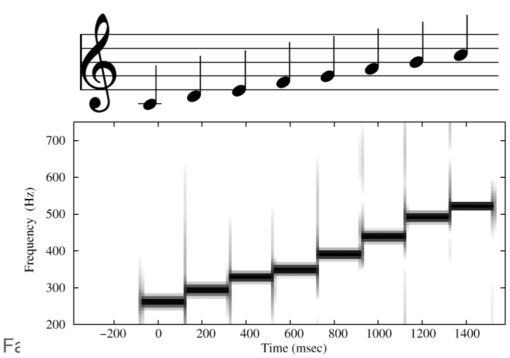
- Check the Bulletin Board for msgs
- Problem Set #3 due Friday
- Quiz #1 on 20-Sept (Monday)
  - Calculator; One page <u>hand-written</u> notes
  - Will include Periodic Signals
  - Quiz Review/Help on Sunday
    - PROBABLY 7PM in ECE Auditorium

READING ASSIGNMENTS

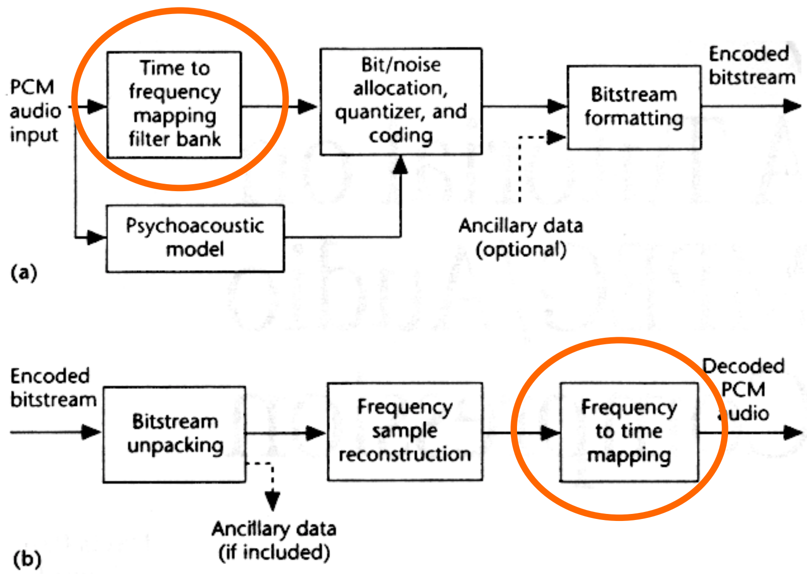
- This Lecture:
  - Chapter 4, pp. 83–94
- Other Reading:
  - Recitation: Chapter 4, pp. 90–100
    - Strobe Demo
  - Next Lecture: Chap. 4, pp. 100–111

CD-ROM DEMOS

- USE THE DEMOS
- Chapter 3: Spectrum
  - DEMOS of SPECTROGRAM
  - BEAT NOTES/AM
  - SPEECH
  - MUSIC



MP-3 Block Diagram



LECTURE OBJECTIVES

- SAMPLING can cause ALIASING
  - Sampling Theorem
  - Sampling Rate > 2(Highest Frequency)
- Spectrum for digital signals, x[n]
  - Normalized Frequency

$$\hat{\omega} = \omega T_s = \frac{2\pi f}{f_s} + 2\pi\ell$$

ALIASING

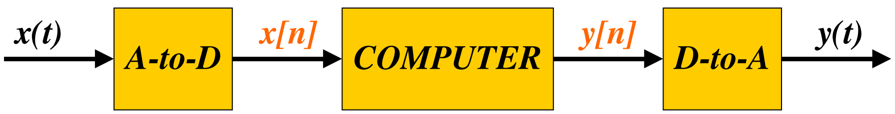
SYSTEMS Process Signals

- PROCESSING GOALS:
  - Change x(t) into y(t)
    - For example, more BASS
  - Improve x(t), e.g., image deblurring
  - Extract Information from x(t)

System IMPLEMENTATION

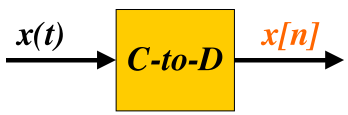
- ANALOG/ELECTRONIC:
  - Circuits: resistors, capacitors, op-amps

- DIGITAL/MICROPROCESSOR
  - Convert x(t) to numbers stored in memory



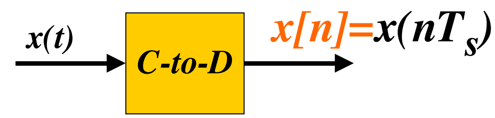
SAMPLING x(t)

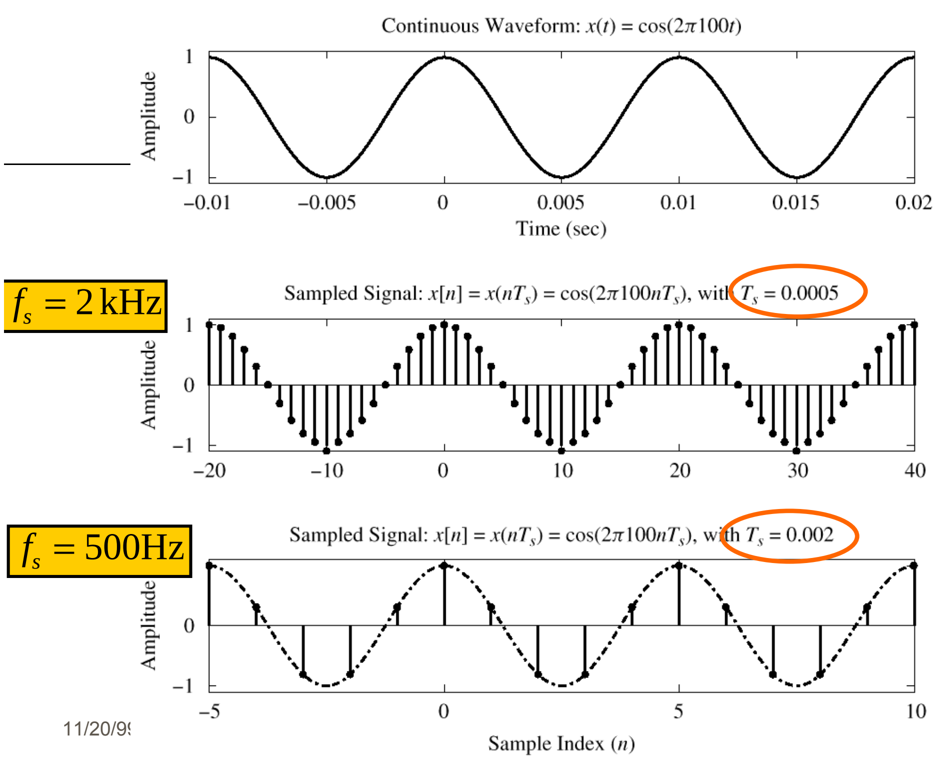
- SAMPLING PROCESS
  - Convert x(t) to numbers x[n]
  - "n" is an integer; x[n] is a sequence
  - "n" is the storage address in memory
- UNIFORM SAMPLING at $t = nT_s$
  - IDEAL: $x[n] = x(nT_s)$



SAMPLING RATE

- UNIFORM SAMPLING at $t = nT_s = n/f_s$
  - IDEAL: $x[n] = x(nT_s)=x(n/f_s)$
- SAMPLING RATE (f_s)
  - $1/T_s$ = NUMBER of SAMPLES PER SECOND
  - T_s = 125 microsec --> f_s = 8000 samples/sec
    - UNITS ARE HERTZ: 8000 Hz





SAMPLING THEOREM

- HOW OFTEN ?
  - DEPENDS on FREQUENCY of SINUSOID
  - ANSWERED by SHANNON/NYQUIST
  - ALSO DEPENDS on "<u>RECONSTRUCTION</u>"

> *Shannon Sampling Theorem*
> A continuous-time signal $x(t)$ with frequencies no higher than f_{max} can be reconstructed exactly from its samples $x[n] = x(nT_s)$, if the samples are taken at a rate $f_s = 1/T_s$ that is greater than $2f_{max}$.

STORING DIGITAL SOUND

- $x[n]$ is a SAMPLED SINUSOID
  - A list of numbers stored in memory
- CD rate is 44,100 samples per second
  - 16-bit samples
  - Stereo uses 2 channels
- Number of bytes for 1 minute is
  - 2 X (16/8) X 60 X 44100 = 10.584 Mbytes

DISCRETE-TIME SINUSOID

- Change x(t) into x[n] DERIVATION

$$x(t) = A\cos(\omega t + \varphi)$$

$$x[n] = x(nT_s) = A\cos(\omega n T_s + \varphi)$$

$$x[n] = A\cos((\omega T_s)n + \varphi)$$

$$x[n] = A\cos(\hat{\omega} n + \varphi)$$

$$\hat{\omega} = \omega T_s$$

DEFINE DIGITAL FREQUENCY

DIGITAL FREQUENCY $\hat{\omega}$

- DIGITAL FREQUENCY is NORMALIZED
- UNITS are radians, not rad/sec
- $\hat{\omega}$ VARIES from 0 to 2π, as f varies from 0 to the sampling frequency

$$\hat{\omega} = \omega T_s = \frac{2\pi f}{f_s}$$

ALIASING DERIVATION

- Other Frequencies give the same $\hat{\omega}$

If $x(t) = A\cos(2\pi(f + \ell f_s)t + \varphi)$ $\quad t \leftarrow \frac{n}{f_s}$

and we want : $x[n] = A\cos(\hat{\omega} n + \varphi)$

then : $$\hat{\omega} = \frac{2\pi(f + \ell f_s)}{f_s} = \frac{2\pi f}{f_s} + \frac{2\pi \ell f_s}{f_s}$$

$$\hat{\omega} = \omega T_s = \frac{2\pi f}{f_s} + 2\pi\ell$$

ALIASING CONCLUSIONS

- ADDING f_s or $2f_s$ or $-f_s$ TO THE FREQ of x(t) gives exactly the same x[n]
- $x[n] = x(n/f_s)$ HAS THE SAME VALUES
- GIVEN x[n], WE CAN'T DISTINGUISH f_o FROM $(f_o + f_s)$ or $(f_o + 2f_s)$
- CALLED ALIASING

NORMALIZED FREQUENCY

- DIGITAL FREQUENCY

Normalized Radian Frequency

$$\hat{\omega} = \omega T_s = \frac{2\pi f}{f_s} + 2\pi\ell$$

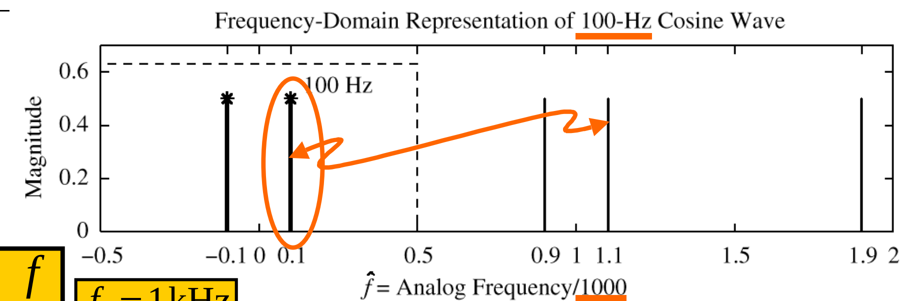
Normalized Cyclic Frequency

$$\hat{f} = \hat{\omega}/(2\pi) = f T_s = f/f_s$$

SPECTRUM for x[n]

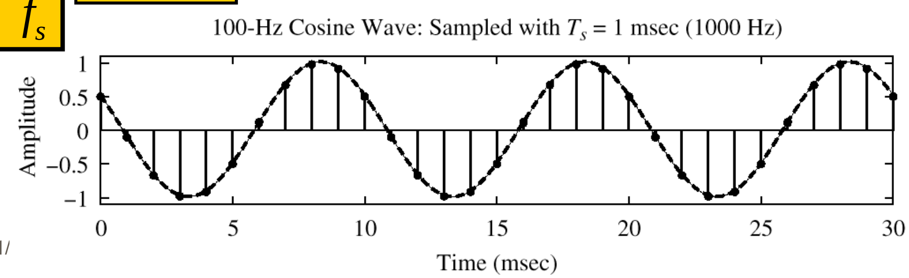
- PLOT versus NORMALIZED FREQUENCY
- INCLUDE ALL SPECTRUM LINES
  - ALIASES
    - ADD MULTIPLES of 2π
    - SUBTRACT MULTIPLES of 2π
  - FOLDED ALIASES
    - (to be discussed later)
    - ALIASES of NEGATIVE FREQS

SPECTRUM (DIGITAL)



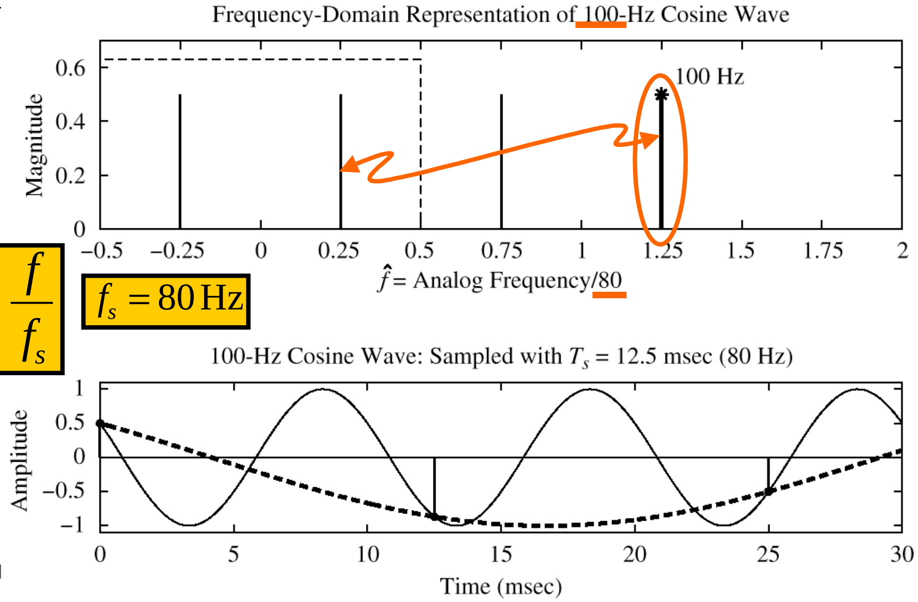
$$\hat{f} = \frac{f}{f_s}$$

$$f_s = 1\,\text{kHz}$$

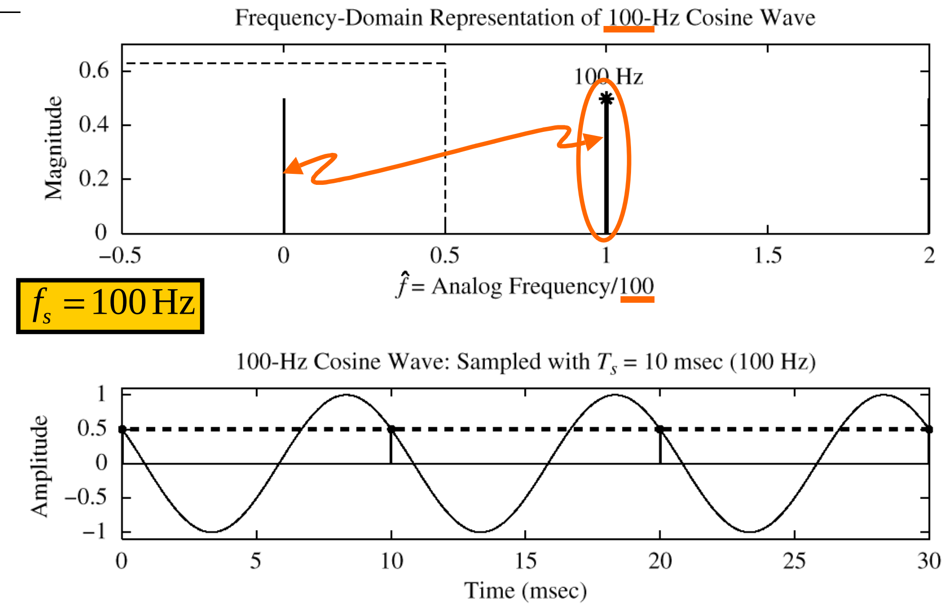


SPECTRUM of x[n]
ALIASING CASE

$$\hat{f} = \frac{f}{f_s}$$



SPECTRUM of x[n]
ALIASING to ZERO FREQ



SAMPLING DEMO (Chap. 4)