EE-2025 Fall-99

Lecture 5

Harmonics & Time-Varying Sinusoids

10-Sept-99

Web-CT Info

- <u>Check the Bulletin Board for msgs</u>
- Old Quizzes & Problems are linked
  - Quiz #1 on 20-Sept (Monday)
- Prob Set #2 due Today
- On-Line Homework due every Friday

Lab Info

- Lab #2 Report
  - Turn in during your lab time
  - Write-up sections 4 and 5
  - Include INSTRUCTOR VERIFICATION
- Lab #3 will be posted today
  - Music Notation will be needed

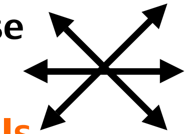
  - <u>ERRORS ? ALWAYS Check Bulletin Board</u>

READING ASSIGNMENTS

- This Lecture:
  - Chapter 3, pp. 57-68
  - Chapter 3, pp. 68-77
- Next Lecture: Chapter 4
  - Sampling: A-to-D and D-to-A

Problem Solving Skills

- Math Formula
  - Sum of Cosines
  - Amp, Freq, Phase
- Recorded Signals
  - Speech
  - Music
  - No simple formula



- Plot & Sketches
  - S(t) versus t
  - Spectrum
- MATLAB
  - Numerical
  - Computation
  - Plotting list of numbers

LECTURE OBJECTIVES

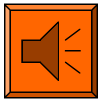
- Signals with HARMONIC Frequencies
  - Add Sinusoids with $f_k = kf_0$

$$x(t) = A_0 + \sum_{k=1}^{N} A_k \cos(2\pi k f_0 t + \varphi_k)$$

- FREQUENCY can change vs. TIME
  - Chirps: $x(t) = \cos(\alpha t^2)$
  - Introduce Spectrogram Visualization (`specgram.m`)

HISTORY

- Jean Baptiste Joseph Fourier
  - 1807 thesis (memoir)
  - http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians/Fourier.html
- "All signals x(t) can be SYNTHESIZED as sum of sinusoids—maybe an infinite number are needed"



  - EVEN THIS ONE:

Joseph Fourier

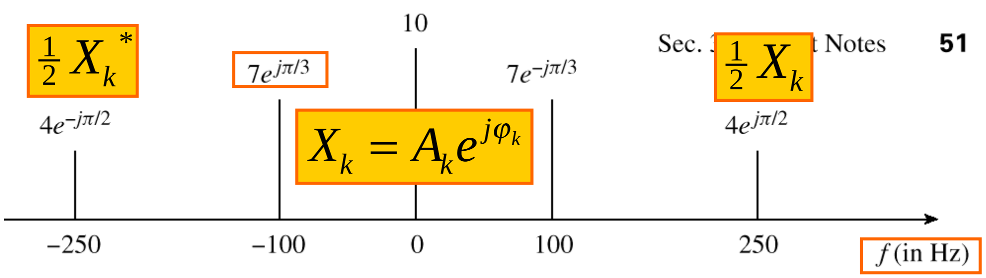
lived from 1768 to 1830

Fourier studied the mathematical theory of heat conduction. He established the partial differential equation governing heat diffusion and solved it by using infinite series of trigonometric functions.

Find out more at:
http://www-history.mcs.st-andrews.ac.uk/history/Mathematicians/Fourier.html

FREQUENCY DIAGRAM

- Recall Complex Amplitude vs. Freq



$$x(t) = 10 + 14\cos(200\pi t - \pi/3) + 8\cos(500\pi t + \pi/2)$$

Summary: GENERAL FORM

$$x(t) = A_0 + \sum_{k=1}^{N} A_k \cos(2\pi f_k t + \phi_k)$$

$$X_0 = A_0 e^{j0}$$

$$x(t) = X_0 + \sum_{k=1}^{N} \Re e\left\{X_k e^{j2\pi f_k t}\right\}$$

$$X_k = A_k e^{j\phi_k}$$

frequency is f_k.

$$x(t) = X_0 + \sum_{k=1}^{N} \left\{\frac{X_k}{2} e^{j2\pi f_k t} + \frac{X_k^*}{2} e^{-j2\pi f_k t}\right\}$$

PERIODIC SIGNALS

- Repeat every T secs
  - Definition

$$x(t) = x(t+T)$$

  - Example:

$$x(t) = \cos^2(3t)$$

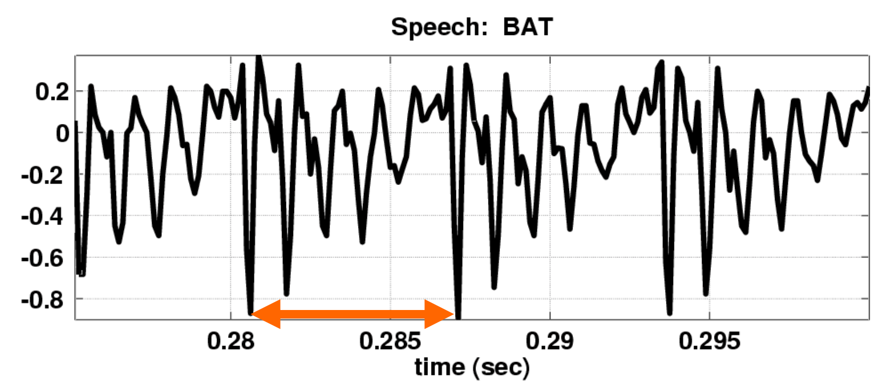
$T = ?$

$T = \frac{\pi}{3}$

  - Speech can be “quasi-periodic”

Speech Signal: BAT

- Nearly Periodic in the Vowel Region
  - Period is (Approximately) T = 0.0065 sec



Harmonic Signal Spectrum

$$x(t) = A_0 + \sum_{k=1}^{N} A_k \cos(2\pi k f_0 t + \varphi_k)$$

$$X_k = A_k e^{j\varphi_k}$$

$$x(t) = X_0 + \sum_{k=1}^{N} \tfrac{1}{2} X_k e^{j2\pi k f_0 t} + \sum_{k=1}^{N} \tfrac{1}{2} X_k^* e^{-j2\pi k f_0 t}$$

Period of Complex Exponential

$$x(t) = e^{j\omega t}$$

$$x(t+T) = x(t)\ ?$$

Definition: Period is T

$$e^{j\omega(t+T)} = e^{j\omega t}$$

$$\Rightarrow e^{j\omega T} = 1 \quad \Rightarrow \omega T = 2\pi k$$

$$e^{j2\pi k} = 1$$

$$\omega = \frac{2\pi k}{T} = \left(\frac{2\pi}{T}\right)k = \omega_0 k$$

k = integer

HARMONIC SIGNAL

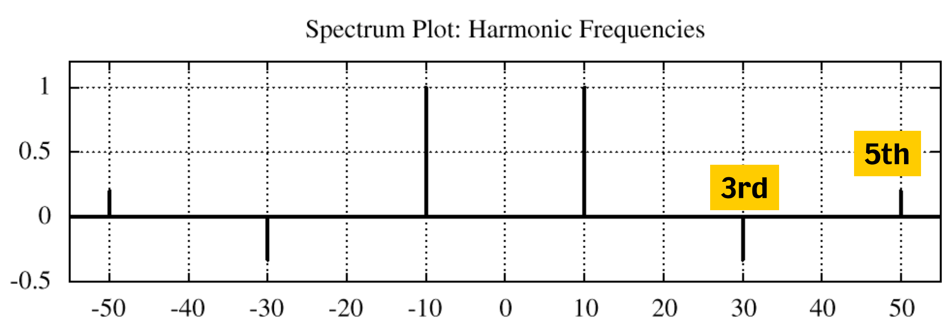
$$x(t) = A_0 + \sum_{k=1}^{N} A_k \cos(2\pi k f_0 t + \varphi_k)$$

$$f_k = k\, f_0 \qquad (\omega_0 = 2\pi\, f_0)$$

f_0 = fundamental frequency

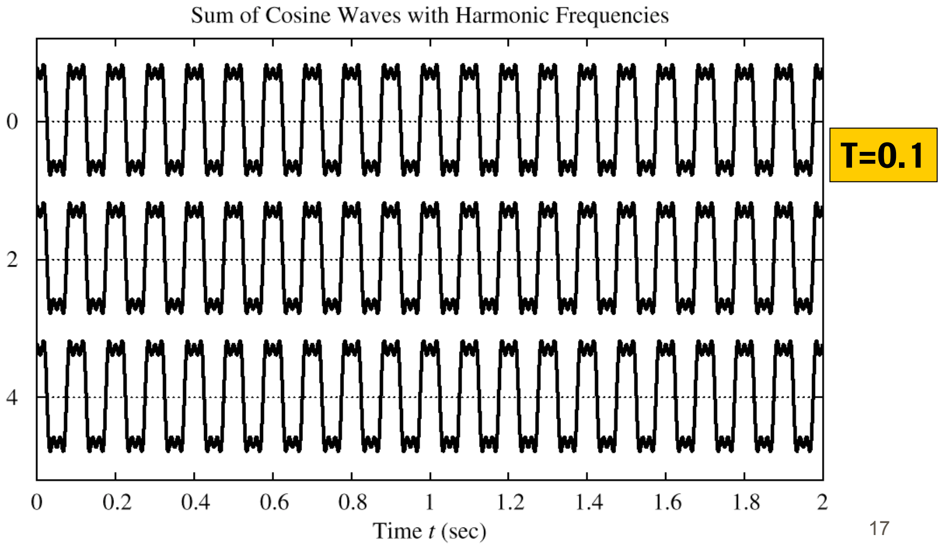
T_0 = fundamental Period $\quad f_0 = \frac{1}{T_0}$

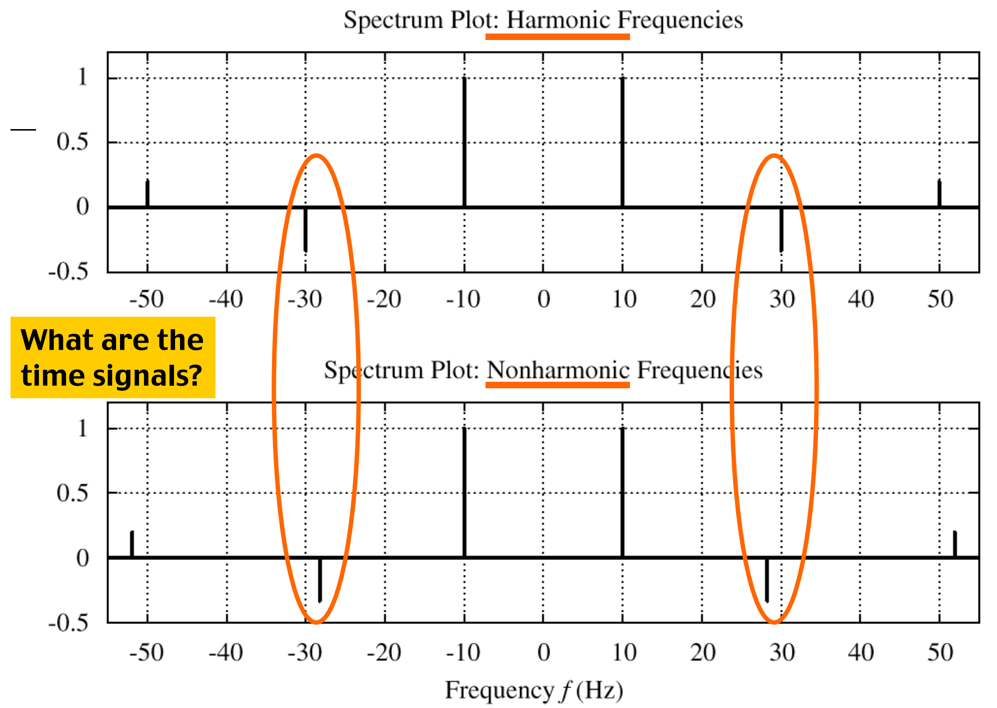
Harmonic Signal (3 Freqs)



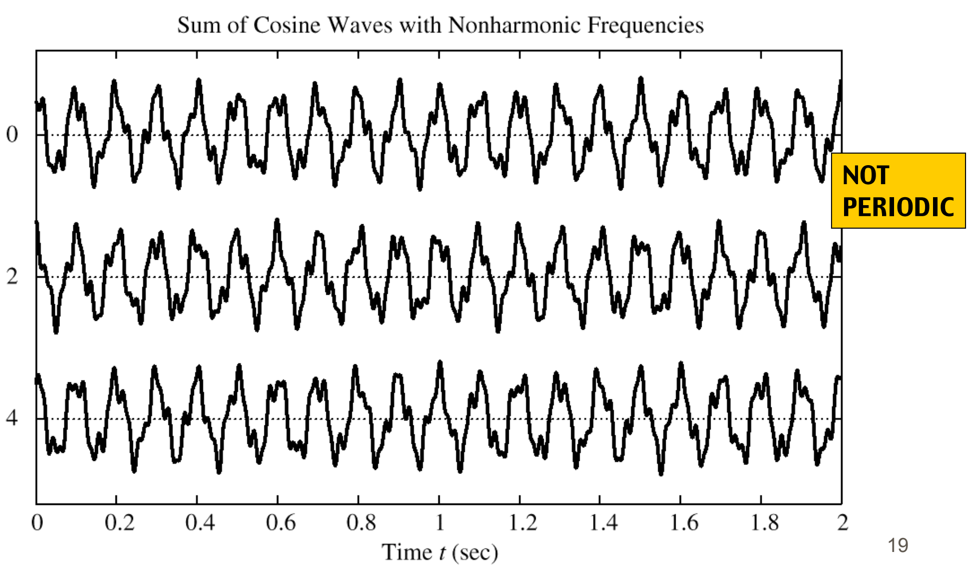
What is the fundamental frequency?

Harmonic Signal (3 Freqs)





NON-Harmonic Signal



Time-Varying FREQUENCIES Diagram

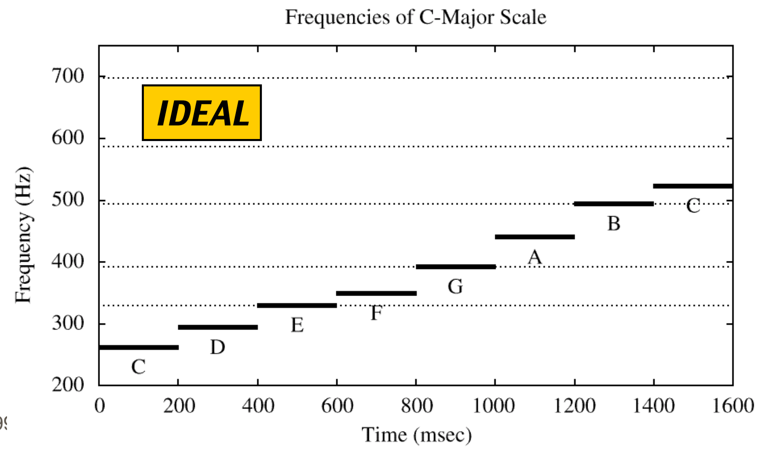
Frequency is the vertical axis

Figure 3.18 Sheet-music notation is a time–frequency diagram.

Time is the horizontal axis

STEPPED FREQUENCIES

- C–major SCALE: successive sinusoids
  - Frequency is constant for each note



FREQUENCY ANALYSIS

- Given a recording of a song, have the computer write the music
- Can a machine extract frequencies?
  - COMPUTE the spectrum for x(t)
    - During short intervals

Time-Varying FREQUENCIES Diagram

Frequency is the vertical axis

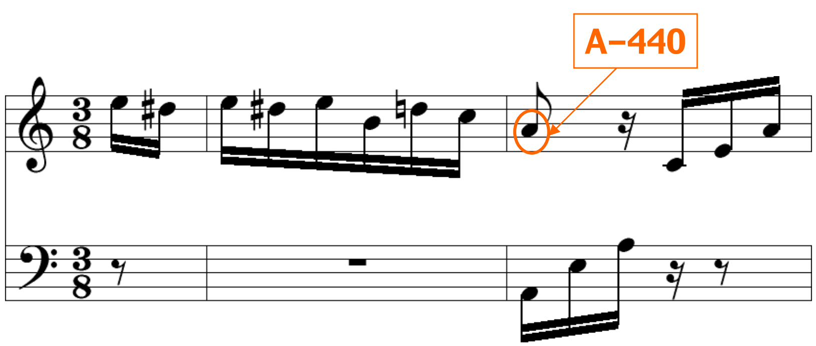


Figure 3.18 Sheet-music notation is a time–frequency diagram.

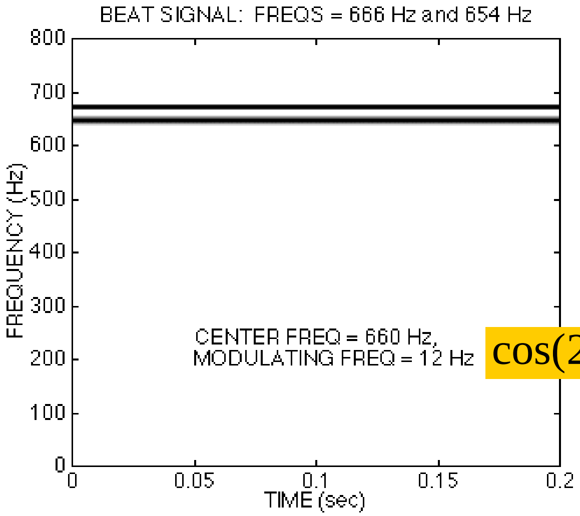
Time is the horizontal axis

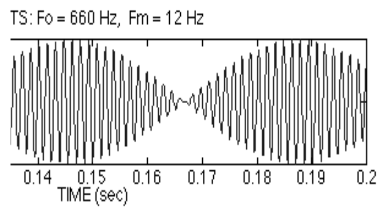
R-rated: ADULTS ONLY

- SPECTROGRAM Tool
  - MATLAB function is specgram.m
  - DSP First has spectgr.m (no plotting)
- ANALYSIS program
  - Takes x(t) as input
  - Produces spectrum values X_k
  - Breaks x(t) into SHORT TIME SEGMENTS
    - Then uses the FFT (Fast Fourier Transform)

SPECTROGRAM EXAMPLE

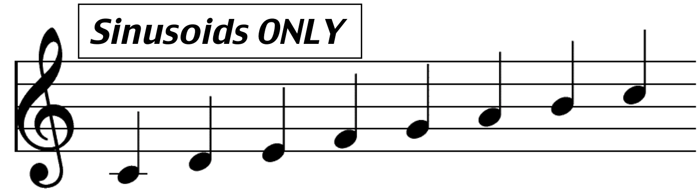
- Two Constant Frequencies: Beats

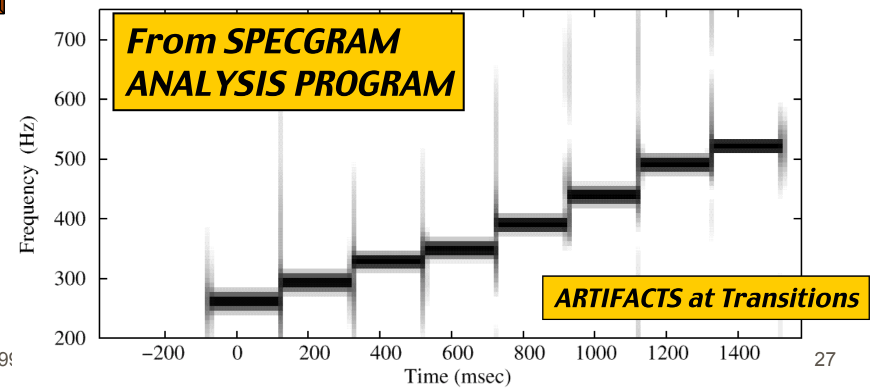




$\cos(2\pi(660)t)\cos(2\pi(12)t)$

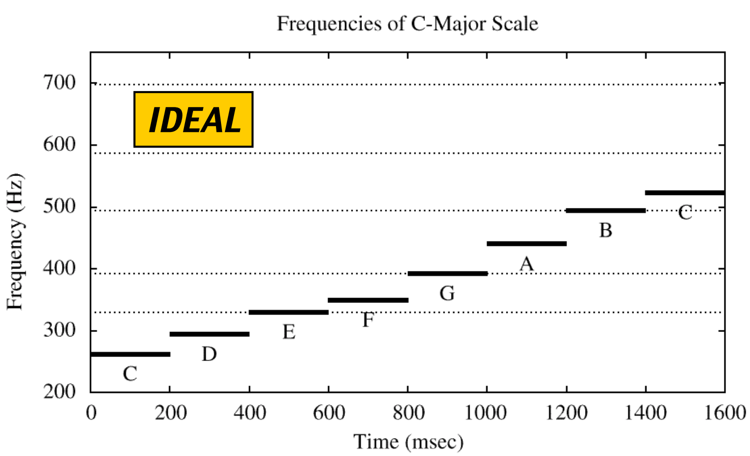
SPECTROGRAM of C-Scale



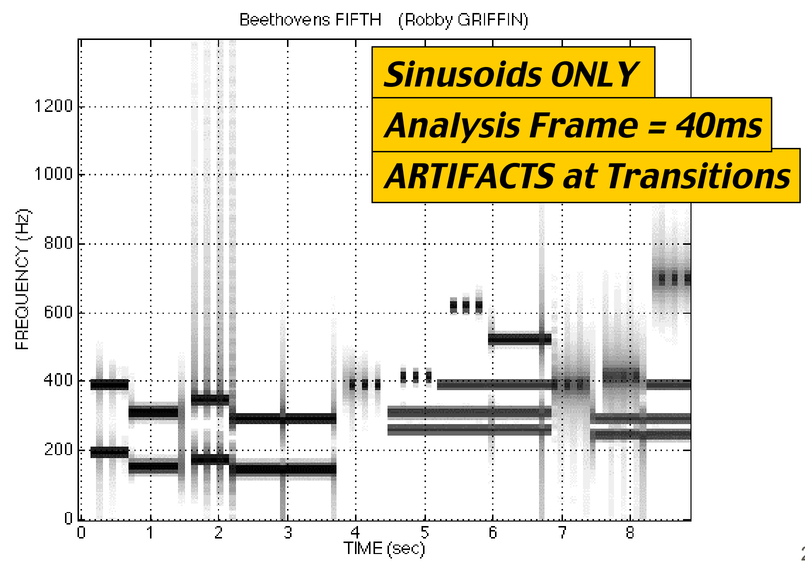


STEPPED FREQUENCIES

- C–major SCALE: successive sinusoids
  - Frequency is constant for each note



Spectrogram of LAB SONG



Time-Varying Frequency

- Frequency can change vs. time
  - Continuously, not stepped
- FREQUENCY MODULATION (FM)

$$x(t) = \cos(2\pi f_c t + v(t))$$

VOICE

- CHIRP SIGNALS
  - Linear Frequency Modulation (LFM)

New Signal: Linear FM

- Called Chirp Signals (LFM)
  - Quadratic phase

QUADRATIC

$$x(t) = A\cos(\alpha t^2 + 2\pi f_0 t + \varphi)$$

- Freq will change LINEARLY vs. time
  - Example of Frequency Modulation (FM)
  - Define “instantaneous frequency”

INSTANTANEOUS FREQ

- Definition

$$x(t) = A\cos(\psi(t))$$

$$\Rightarrow \omega_i(t) = \tfrac{d}{dt}\psi(t)$$

Derivative of the Phase

- For Sinusoid:

$$\psi(t) = 2\pi f_0 t + \varphi$$

$$\Rightarrow \omega_i(t) = \tfrac{d}{dt}\psi(t) = 2\pi f_0$$

Makes sense

INSTANTANEOUS FREQ of the Chirp

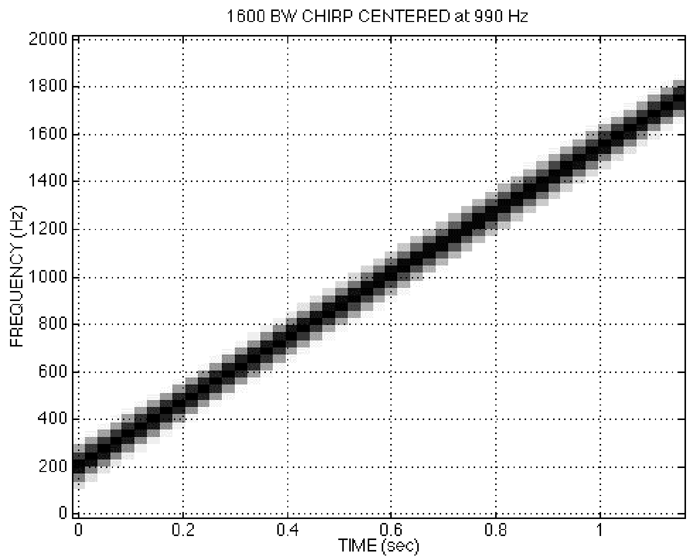
- Chirp Signals have Quadratic phase
- Freq will change LINEARLY vs. time

$$x(t) = A\cos(\alpha t^2 + \beta t + \varphi)$$

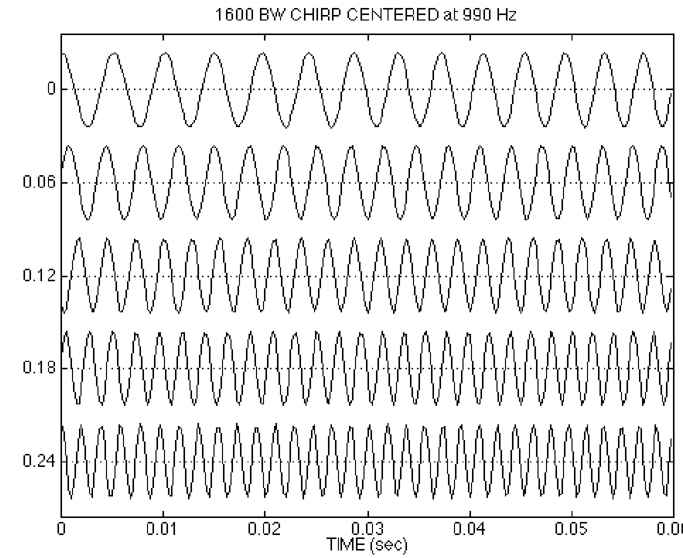
$$\Rightarrow \psi(t) = \alpha t^2 + \beta t + \varphi$$

$$\Rightarrow \omega_i(t) = \tfrac{d}{dt}\psi(t) = 2\alpha t + \beta$$

CHIRP SPECTROGRAM



CHIRP WAVEFORM



OTHER CHIRPS

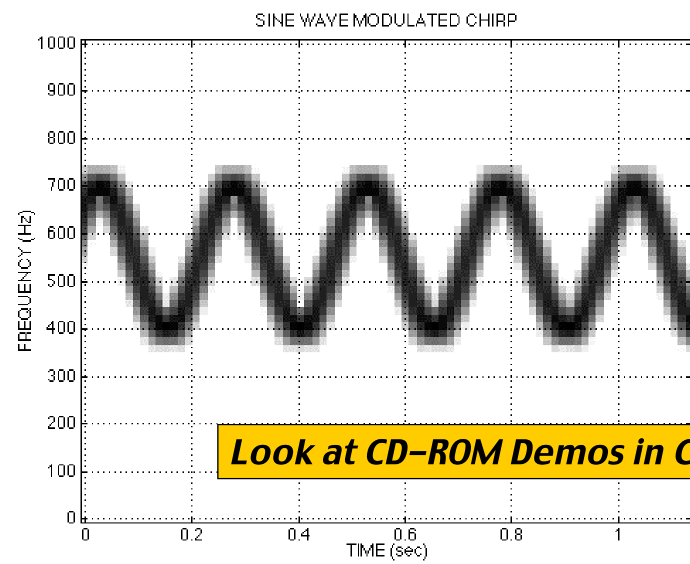
- $\psi(t)$ can be anything:

$$x(t) = A\cos(\alpha\cos(\beta t) + \varphi)$$

$$\Rightarrow \omega_i(t) = \tfrac{d}{dt}\psi(t) = -\alpha\beta\sin(\beta t)$$

- $\psi(t)$ could be speech or music:
  - FM radio broadcast

SINE-WAVE FREQUENCY MODULATION (FM)



Look at CD-ROM Demos in Ch 3