EE-2025 Fall-99

Lecture 26

Spectrum Analysis and Frequency Modulation (FM)

10-Dec-99

Info: Web-CT, Lab, HW

- Calendar: Final Exams
  - 11am section–Period 9, Weds, 12/15
  - 12pm section–Period 13, Fri, 12/17
  - NO switching allowed
- Reading Assignment:
  - You should have read Chapters 2–8 of DSP First and all chapters of the notes.
- Prob Set #14 – not handed in;
  - However, will be covered on FINAL EXAM

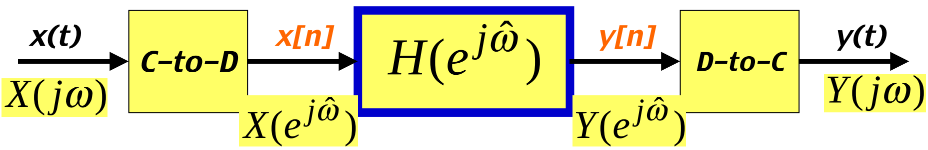
LECTURE OBJECTIVES

- Review Discrete–Time Filtering of Continuous–Time Signals
  - Basic Configuration
    - CT Input -> A/D -> DT System -> D/A -> CT Output
- Spectrum analysis
  - Spectrogram
- Frequency Modulation (FM)

FINAL EXAM

- FORMULA PAGES ?
  - Students bring TWO pages (2–sided)
- COVERAGE / EMPHASIS?
  - Fourier Transform
    - Sampling & Spectrum
  - Digital Filters: IIR & FIR & H(z)
  - Hard problems from Quiz #2, #3.
  - Homework & Old Quizzes

DT Filtering of CT Signals

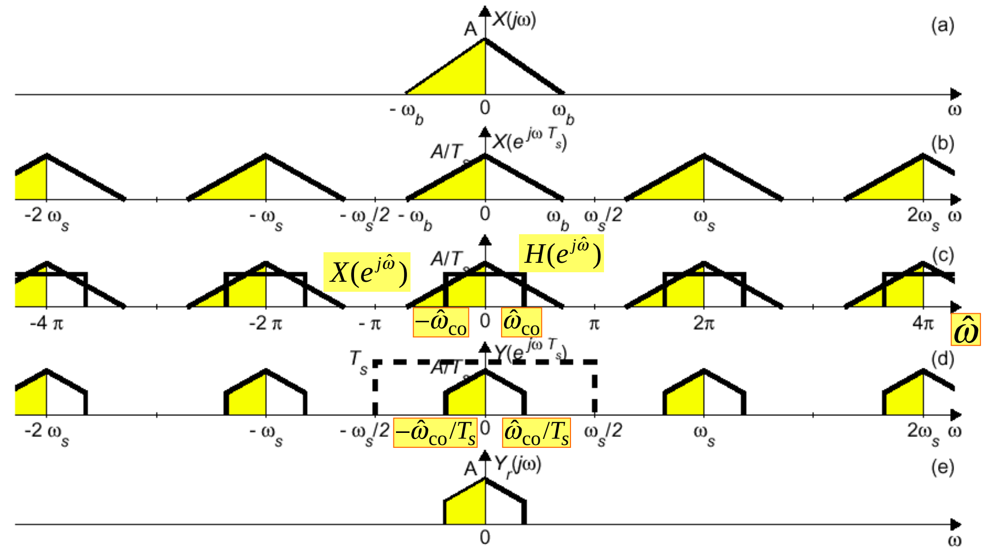


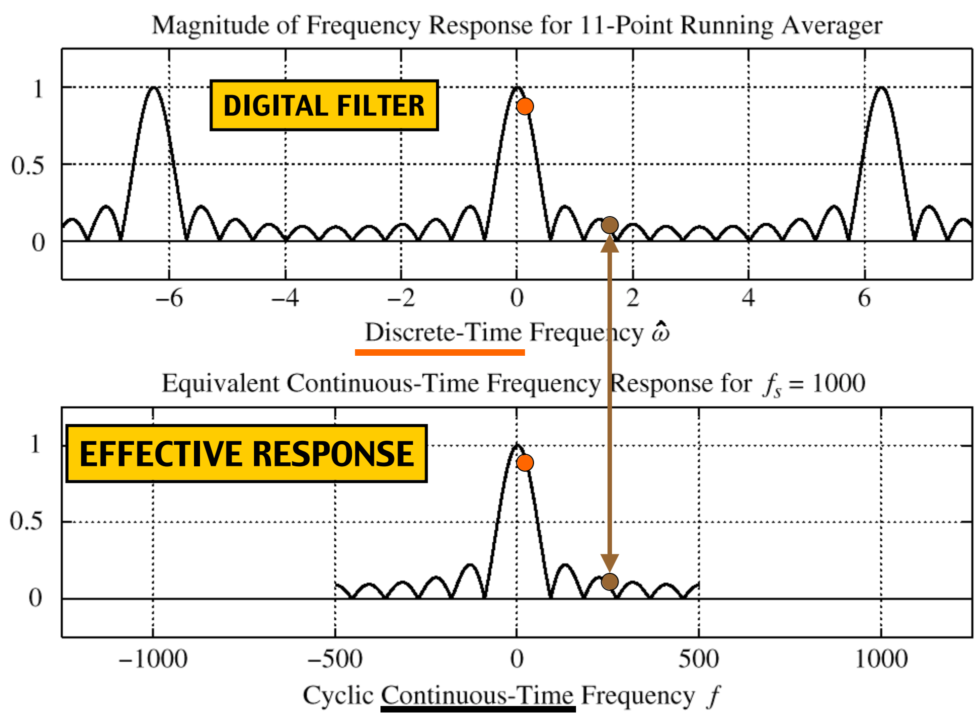
If no aliasing occurs in sampling x(t), then it follows that

$$Y(j\omega) = H_{\text{eff}}(j\omega)X(j\omega)$$

$$H_{\text{eff}}(j\omega) = \begin{cases} H(e^{j\omega T_s}) & |\omega| < \frac{1}{2}\omega_s \\ 0 & |\omega| > \frac{1}{2}\omega_s \end{cases}$$

Illustration of DT Filtering of a CT Signal





EFFECTIVE Freq. Response

- **Assume NO Aliasing, then**
  - **ANALOG FREQ <--> DIGITAL FREQ**

$$\hat{\omega} = \omega T_s = \frac{\omega}{f_s}$$

- **So, we can plot:**
- **Scaled Freq. Axis**

DIGITAL FILTER

$$H(e^{j\omega T_s}) \text{ vs. } \omega$$

ANALOG FREQUENCY

Education

- Ambrose Bierce:
- Education, noun: "That which discloses to the wise and disguises from the foolish their lack of understanding"

Time-Varying Frequency

- Frequency can change vs. time
  - Continuously, not stepped
- FREQUENCY MODULATION (FM)

$$x(t) = \cos(2\pi f_c t + v(t))$$

VOICE

- CHIRP SIGNALS
  - Linear Frequency Modulation (LFM)

Sinusoidal Modulation

$$y(t) = A(t)\cos[\theta(t)]$$

- Double sideband AM (DSBAM)

$$y(t) = x(t)\cos(\omega_c t)$$

- DSBAM with transmitted carrier

$$y(t) = [A + x(t)]\cos(\omega_c t)$$

- Phase modulation

$$y(t) = \cos[\omega_c t + x(t)]$$

- Frequency modulation

$$y(t) = \cos[\omega_c t + \int^t x(\tau)d\tau]$$

Frequency Modulation (FM)

Definition

$$y(t) = \cos[\omega_c t + \int_{-\infty}^{t} x(\tau)d\tau]$$

Angle of the cosine

$$\theta(t) = \omega_c t + \int_{-\infty}^{t} x(\tau)d\tau$$

Instantaneous frequency

$$\omega_i(t) = \frac{d\theta(t)}{dt} = \omega_c + x(t)$$

Example

- Input to modulator

$$\int^{t} x(\tau)d\tau = 50\sin(80\pi t) \qquad \omega_m = 80\pi$$

- Output of FM modulator

$$y(t) = \cos(8000\pi t + 50\sin(80\pi t))$$

- Instantaneous frequency

$$\omega_i(t) = \frac{d\theta(t)}{dt} = 8000\pi + 4000\pi\cos(80\pi t)$$

$$\omega_c = 8000\pi \qquad \Delta\omega = 4000\pi$$

R-rated: ADULTS ONLY

- SPECTROGRAM Tool
  - MATLAB function is specgram.m
  - DSP First has spectgr.m (no plotting)
- ANALYSIS program
  - Takes x(t) as input
  - Produces spectrum values X_k
  - Breaks x(t) into SHORT TIME SEGMENTS
    - Then uses the FFT (Fast Fourier Transform)

Computing the Fourier Transform

- Our analysis of sampling gave this:

$$X(e^{j\omega T_s}) = \sum_{n=-\infty}^{\infty} x(nT_s)e^{-j\omega n T_s} = \frac{1}{T_s}\sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s))$$

- If $X(j\omega)=0$ for $\omega > \omega_b$ and $\omega_s > 2\omega_b$,

$$\sum_{n=-\infty}^{\infty} x(nT_s)e^{-j\omega n T_s} = \frac{1}{T_s}X(j\omega) \quad \text{for } |\omega| < \omega_s/2$$

- This is something that we can compute!

Fast Fourier Transform (FFT)

- Assume x[n]=0 for n<0 and n>N−1 and consider frequencies

$$\omega_k = \frac{2\pi k}{NT_s} \qquad k = 0,1,\ldots,N$$

- Then

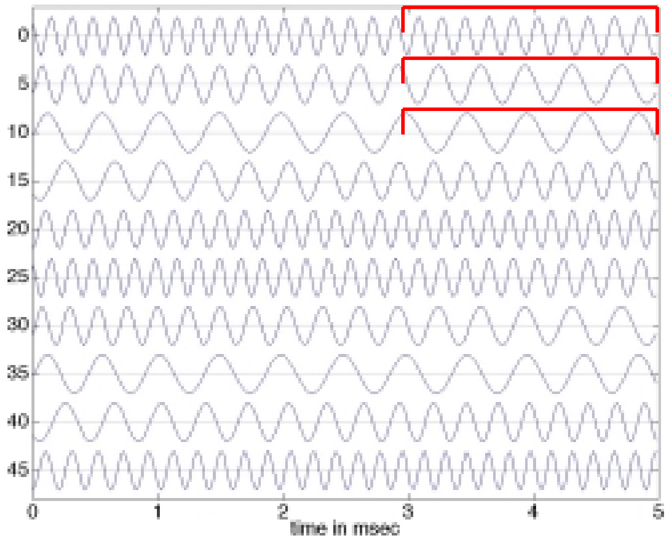
$$X\left(j\frac{2\pi k}{NT_s}\right) = T_s\left[\sum_{n=0}^{N-1} x(nT_s)e^{-j(2\pi k/N)n}\right]$$

$$k = 0,1,\ldots,N$$

can be evaluated with a fast algorithm.

$$y(t) = \cos(8000\pi t + 50\sin(80\pi t))$$

$$\omega_i(t) = \frac{d\theta(t)}{dt} = 8000\pi + 4000\pi\cos(80\pi t)$$



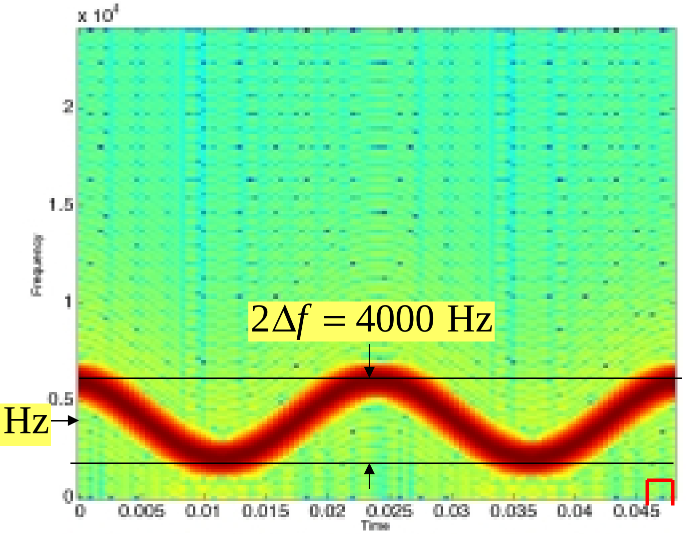
Spectrogram computes FT of short segments of a signal.

$$\omega_c = 8000\pi$$

$$\Delta\omega = 4000\pi$$

$$\omega_m = 80\pi$$

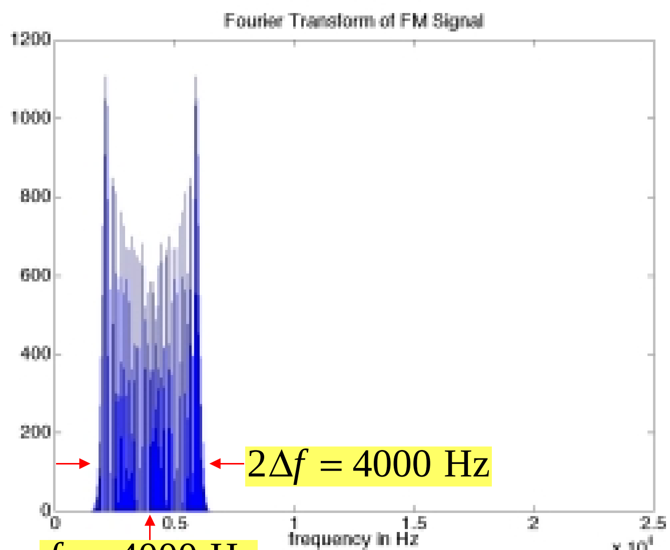
Spectrogram of FM Signal



$2\Delta f = 4000$ Hz

$f_c = 4000$ Hz

Fourier Transform of a Very Long segment of an FM Signal



$2\Delta f = 4000$ Hz

$f_c = 4000$ Hz

Carson's Rule

- **An empirical formula summarizes FM bandwidth**

$$\mathrm{BW}_y = 2\Delta\omega + \mathrm{BW}_x$$

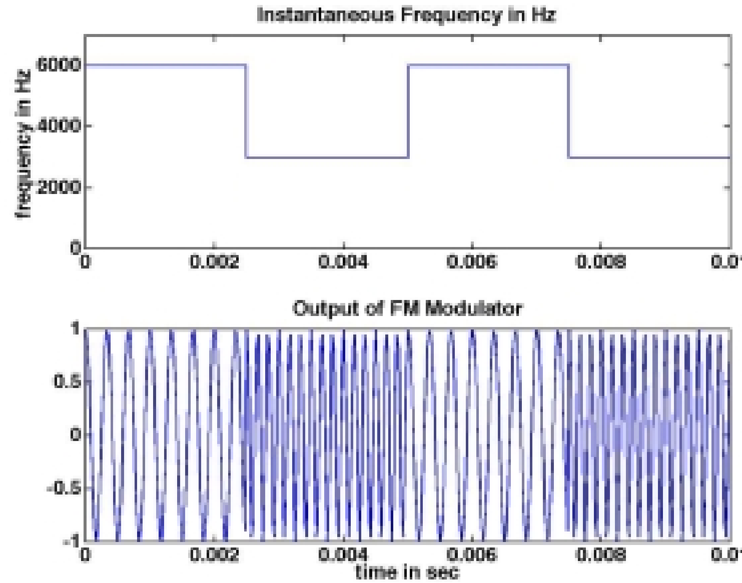
$\mathrm{BW}_x =$ Bandwidth of input signal

$\Delta\omega =$ Maximum frequency deviation

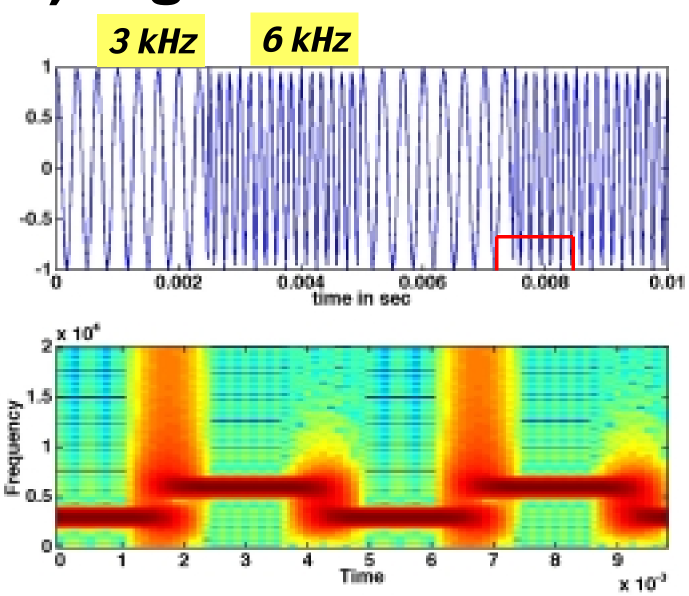
Sending Data Bits with FM

- Frequency Shift Keying
  - 0 = transmit frequency $f_c - f_1$
  - 1 = transmit frequency $f_c + f_1$
- Pulse Width
  - Duration of "0" or "1"
  - BW of pulse is INVERSELY proportional to pulse DURATION

Frequency-Shift Keying (FSK)



Frequency-Shift Keyed (FSK) Signal



Fourier Transform of a Very Long Segment of the Signal