# EE-2025 Fall-99

**LECTURE #2**

**Complex Exponentials**

**27-Aug-99**

## INFORMATION

- MATLAB: Mon,T,Wed in VL–456 (6, 7?)
- LABS start next week
  - Attend correct section (in CoC–309)
  - Computer acct: gtxx, password: SSN
  - Verification must be signed during Lab
- RECITATIONS
  - Attend your assigned time



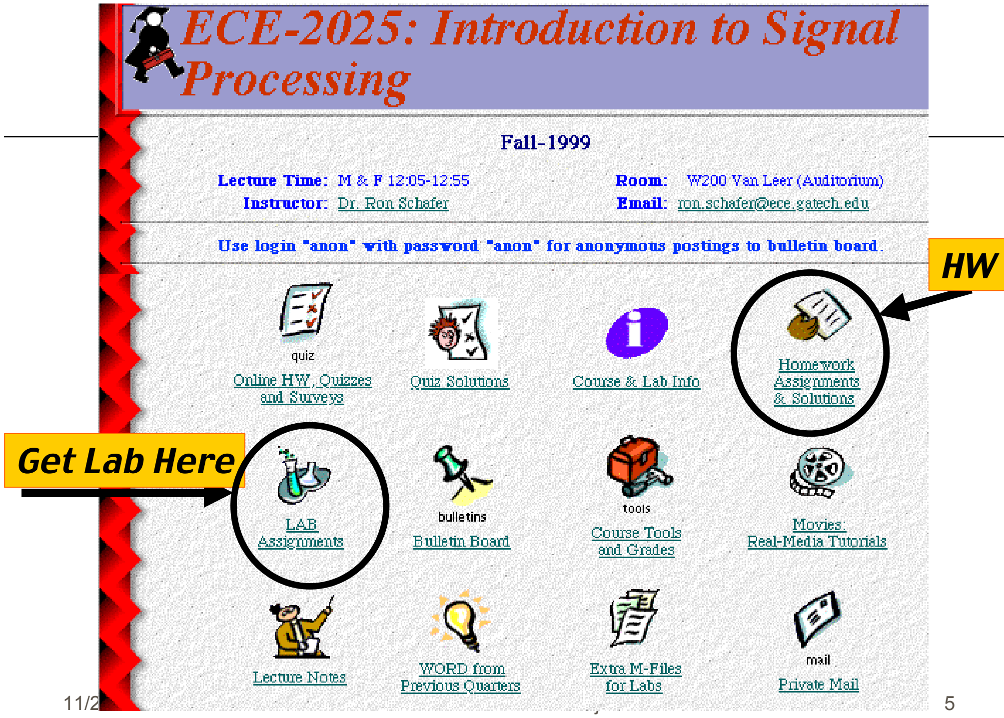
## HOMEWORK #1

- Written Part–conventional
  - *Hand in STAPLED papers—unfolded*
- On–Line in Web–CT
  - Under the "On–Line HW, Quizzes.." link
- BEFORE 11AM on Friday
- Several Easy Problems (ie, Drill)
  - Take it up to 5 times
  - Last score counts



## REMINDERS

- Web–CT Password:
  - SSN(4:8), 4th thru 8th digits of SSN
- Hard copy of Instructor Verification Sheet
  - Get PDF file of Lab#1 from WebCT
  - Lab #1 is different from the book
- HW #1 is due next Friday (in Lecture)
  - Get PDF file from WebCT






## READING ASSIGNMENTS

- This Lecture:
  - Chapter 2, pp. 17–32
- Appendix A: Complex Numbers
- Appendix B: MATLAB
- Next Lecture: finish Chap. 2, pp. 31–43



## LECTURE OBJECTIVES

- Define Sinusoid from a plot
- Relate TIME-SHIFT to PHASE
- Introduce an ABSTRACTION:
  - Complex Numbers represent Sinusoids
  - Complex Exponential Signal

$$z(t) = Ze^{j\omega t}$$



## SINUSOIDAL SIGNAL

$$A\cos(\omega t + \varphi)$$

- FREQUENCY $\omega$
  - Radians/sec
  - Hertz (cycles/sec)

$$\omega = (2\pi) f$$

- PERIOD (in sec)

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

- AMPLITUDE $A$
  - Magnitude
- PHASE $\varphi$

## PLOT a COSINE SIGNAL

- Given the Formula

$$5\cos(0.3\pi t + 1.2\pi)$$

- Make a plot
- Formula defines A, ω, and ϕ

$$A = 5$$
$$\omega = 0.3\pi$$
$$\varphi = 1.2\pi$$



## PLOTTING COSINE SIGNAL from the FORMULA

$$5\cos(0.3\pi t + 1.2\pi)$$

- Determine **period**:

$$T = 2\pi / \omega = 2\pi / 0.3\pi = 20/3$$

- Determine a **peak** location by solving

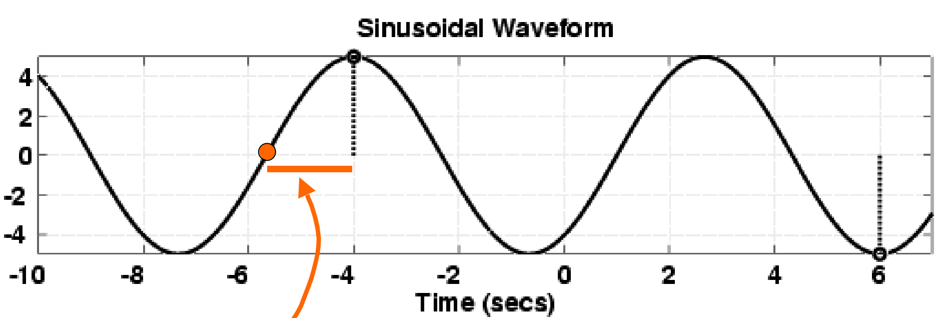
$$(\omega t + \varphi) = 0$$

- **Peak at t = −4**
- **Zero** crossing is T/4 before or after



## ANSWER for the PLOT

$$5\cos(0.3\pi t + 1.2\pi)$$




$$T/4 = (20/3)/4 = 5/3$$



## TIME-SHIFT

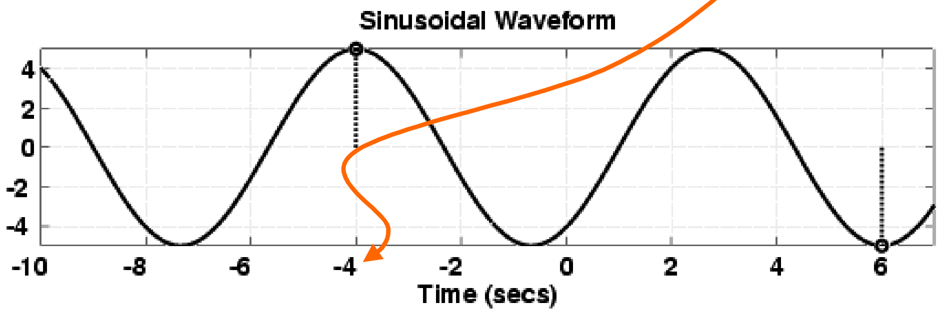
- In a mathematical formula replace t with $t - t_1$
- For example, $x(t-t_1) = \cos(\omega(t-t_1))$
- **Then the t=0 point moves to $t = t_1$**

$$x(t - t_1) = A\cos(\omega(t - t_1))$$

- Peak value of $\cos(\omega(t-t_1))$ is at $t = t_1$

# TIME-SHIFTED SINUSOID

$x(t) = 5\cos(0.3\pi(t+4)) = 5\cos(0.3\pi(t-(-4)))$






# PHASE <--> TIME-SHIFT

- Equate the formulas:

$$A\cos(\omega(t-t_1)) = A\cos(\omega t+\varphi)$$

- and we obtain: $-\omega t_1 = \varphi$
- or,

$$t_1 = \frac{-\varphi}{\omega}$$



# EX: Time-Shift from Phase

- Frequency : $\omega = 30\pi$
- Phase: $\phi = -0.2\pi$
- What is the time shift?
  - Also called the "time delay"
  - $t_1 = -\phi/\omega = -(-0.2\pi)/30\pi$
  - $t_1 = 1/150$ sec.
  - Note: T = 1/15 sec. (period)



# PHASE is AMBIGUOUS

- The cosine signal is periodic
  - Period is $2\pi$

$$A\cos(\omega t+\varphi+2\pi) = A\cos(\omega t+\varphi)$$

  - Thus adding any multiple of $2\pi$ leaves x(t) unchanged
  - if $t_1 = \frac{-\varphi}{\omega}$, then

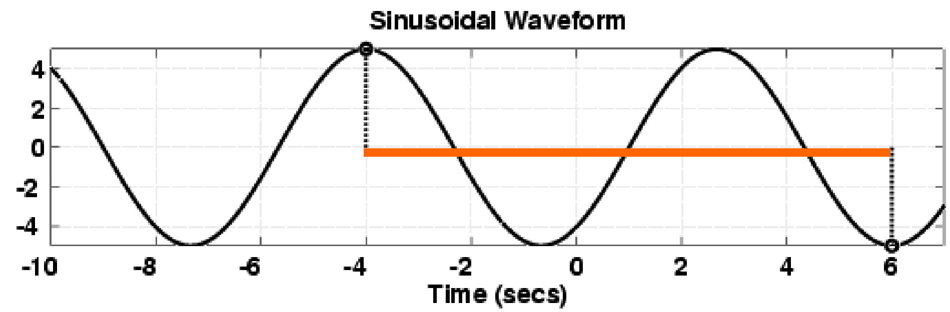
$$t_2 = \frac{-(\varphi+2\pi)}{\omega} = \frac{-\varphi}{\omega} - \frac{2\pi}{\omega} = t_1 - T$$

# SINUSOID from a PLOT

- **Measure the period, T**
  - **Between peaks or zero crossings**
  - **Compute frequency: $\omega = 2\pi/T$**
- **Measure time of peak: $t_1$**
  - **Compute phase: $\phi = -\omega t_1$**
- **Measure height of positive peak: A**



# (A, ω, φ) from a PLOT

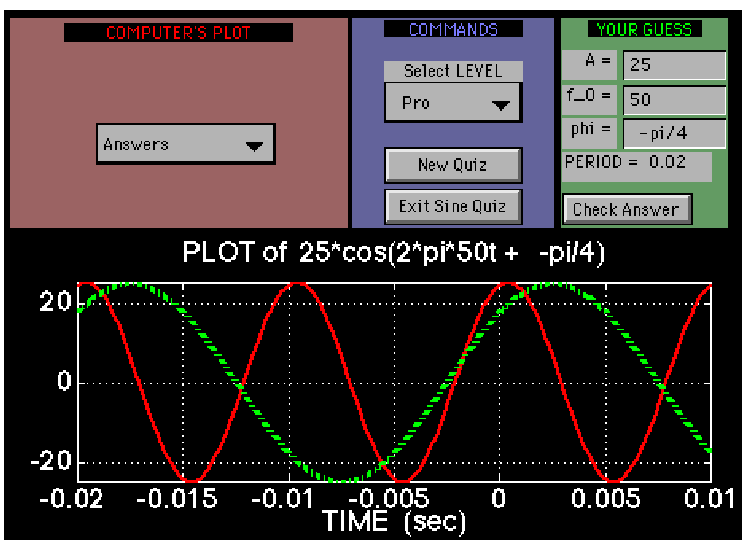



$T = 10/(1.5) = 20/3$ → $\omega = 2\pi/T = 0.3\pi$

$t_1 = -4$ → $\phi = -(-4)(0.3\pi) = 1.2\pi$



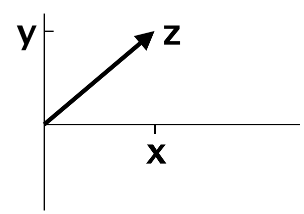
# SINE DRILL (MATLAB GUI)





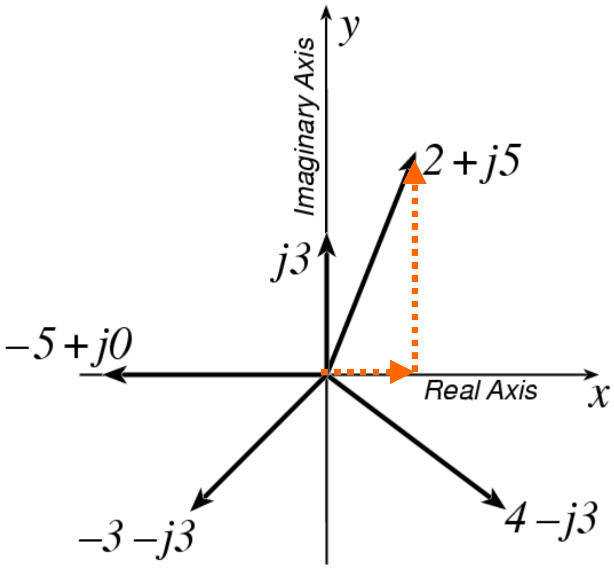

# COMPLEX NUMBERS

- **To solve: $z^2 = -1$**
  - **$z = j$**
  - **Math and Physics use $z = i$**
- **Complex number: $z = x + j\,y$**




Cartesian coordinate system

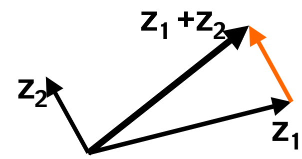
# EX: COMPLEX NUMBERS






# ADD COMPLEX NUMBERS

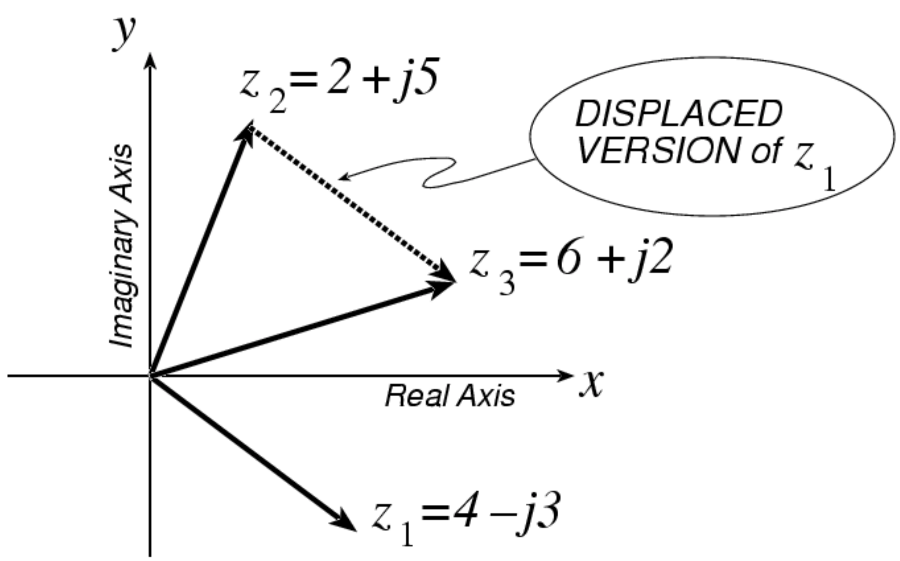
- **VECTOR Addition is necessary**




- **Example: z = 4−*j*3, w = 2+*j*5**
  - **z+w = (4+2) + *j*(−3+5) = 6 +*j*2**
- **Add sinusoids = add complex nums**

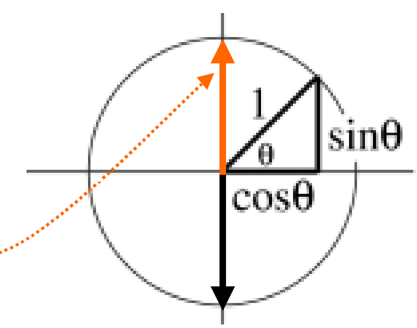


# EX: COMPLEX ADDITION



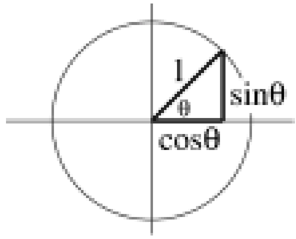

# *** POLAR FORM ***

- **Vector Form**
  - **Length =1**
  - **Angle = θ**
- **Common Values**
  - **j has angle of 0.5π**
  - **−1 has angle of π**
  - **−j has angle of 1.5π**
  - **or, its angle is −0.5π = 1.5π − 2π**



# Euler's FORMULA

- **Complex Exponential**
  - **Real part is cosine**
  - **Imaginary part is sine**
  - **Magnitude is one**




$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

$$re^{j\theta} = r\cos(\theta) + jr\sin(\theta)$$



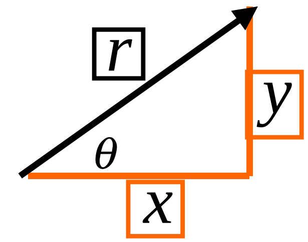
# POLAR <--> RECTANGULAR

- **Relate (x,y) to (r,θ)**

$$z = x + jy = re^{j\theta}$$

$$r^2 = x^2 + y^2$$

$$\theta = \mathrm{Tan}^{-1}\left(\tfrac{y}{x}\right)$$

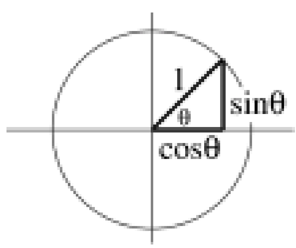





# COMPLEX EXPONENTIAL

$$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$$

- **Rotating Vector**
  - **Angle changes vs. time**
  - **θ=ω*t***
  - **ex:** $\omega = 10\pi$
  - **Rotates** $0.1\pi$ **in 0.01 secs**




$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$



# Cos = REAL PART

- **Real Part of Euler's:**

$$\cos(\omega t) = \Re e\left\{e^{j\omega t}\right\}$$

- **General Sinusoid**

$$x(t) = A\cos(\omega t + \varphi)$$

- **So,**

$$A\cos(\omega t + \varphi) = \Re e\left\{Ae^{j(\omega t + \varphi)}\right\}$$

$$= \Re e\left\{Ae^{j\varphi}e^{j\omega t}\right\}$$

# COMPLEX AMPLITUDE

- General Sinusoid

$$x(t) = A\cos(\omega t + \varphi) = \Re e\left\{Ae^{j\varphi}e^{j\omega t}\right\}$$

- Complex Exponential

$$z(t) = Ze^{j\omega t} \qquad Z = Ae^{j\varphi}$$

- Sinusoid is REAL PART of $e^{j\omega t}$

$$x(t) = \Re e\{z(t)\} = \Re e\left\{Ze^{j\omega t}\right\}$$