EE-2025 Fall-99

Lecture 10

Frequency Response of FIR

1-Oct-99

Info: Web-CT, Lab, HW

- PDF PRINTING PROBLEMS ??????????
- Calendar:
  - Quiz #2 on 25-Oct (Monday)
- Prob Set #5 due TODAY
  - On-Line Problems due Saturday
- Lab #6: Frequency Response
  - Lab Quiz during Lab #6

READING ASSIGNMENTS

- This Lecture:
  - Chapter 6, pp. 157–165, 169–176
- Other Reading:
  - Recitation: Ch. 6, pp. 176–188
    - FREQUENCY RESPONSE EXAMPLES
  - Next Lecture: Chapter 6, pp. 188–194

LAB IMAGES (TRUE)

- Getting TRUE SIZE comparisons is hard: use an image display program

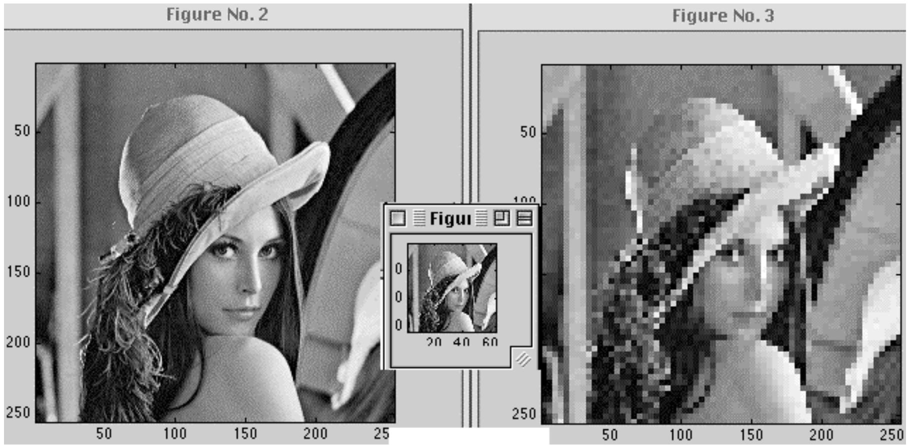


Image Display Procedure

- Use 256 by 256 Lenna (512 is OK)
- Make MATLAB Figures in separate windows
- ALT–PRINT–SCREEN captures the active window (Win–95)
- Paste into “Paint” program
  - Under Win–95 Accessories
- Print after arranging images

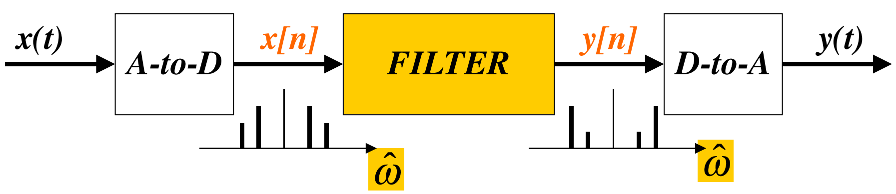
LECTURE OBJECTIVES

- SINUSOIDAL INPUT SIGNAL
  - DETERMINE the FIR FILTER OUTPUT
- FREQUENCY RESPONSE of FIR
  - PLOTTING vs. Frequency
  - MAGNITUDE vs. Freq
  - PHASE vs. Freq

MAG — PHASE

$$H(\hat{\omega}) = \left|H(\hat{\omega})\right| e^{j\varphi(\hat{\omega})}$$

DIGITAL “FILTERING”



- CONCENTRATE on the SPECTRUM
- SINUSOIDAL INPUT
  - INPUT x[n] = SUM of SINUSOIDS
  - Then, OUTPUT y[n] = SUM of SINUSOIDS

GENERAL FIR FILTER

- FILTER COEFFICIENTS $\{b_k\}$
  - DEFINE THE FILTER

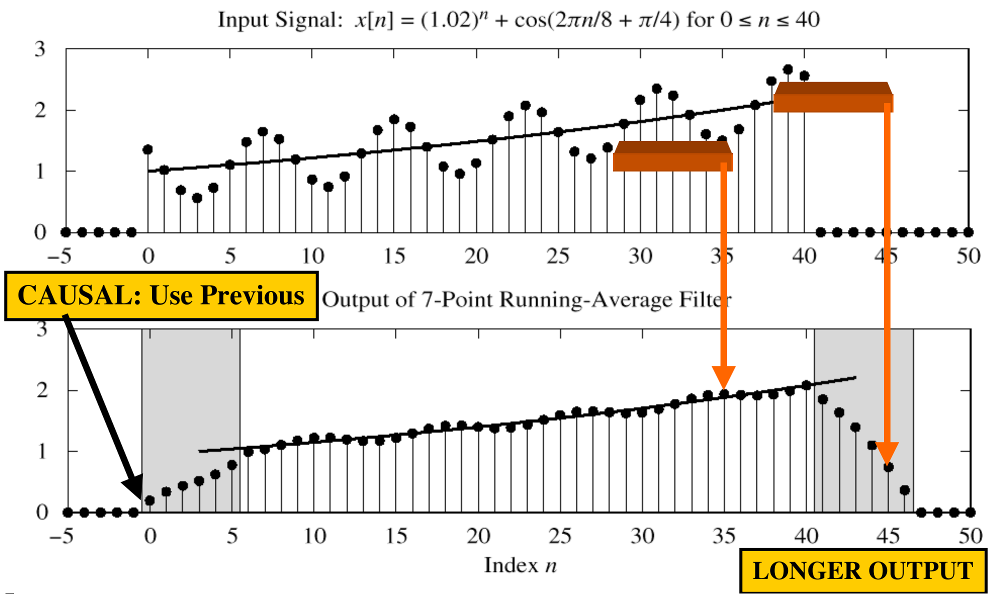
$$y[n] = \sum_{k=0}^{M} b_k\, x[n-k]$$

  - For example, $\{b_k\} = \{3, -1, 2, 1\}$

$$y[n] = \sum_{k=0}^{3} b_k x[n-k]$$

$$= 3x[n] - x[n-1] + 2x[n-2] + x[n-3]$$

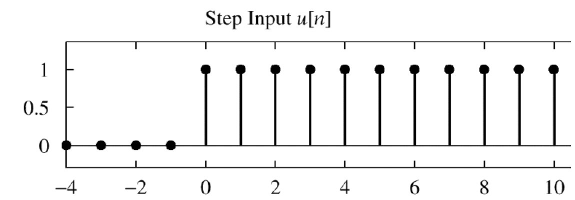
ANIMATION of FIR FILTER



POP QUIZ

- FIR Filter is "FIRST DIFFERENCE"
  - y[n] = x[n] − x[n−1]
- INPUT is "UNIT STEP"

$$u[n] = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$



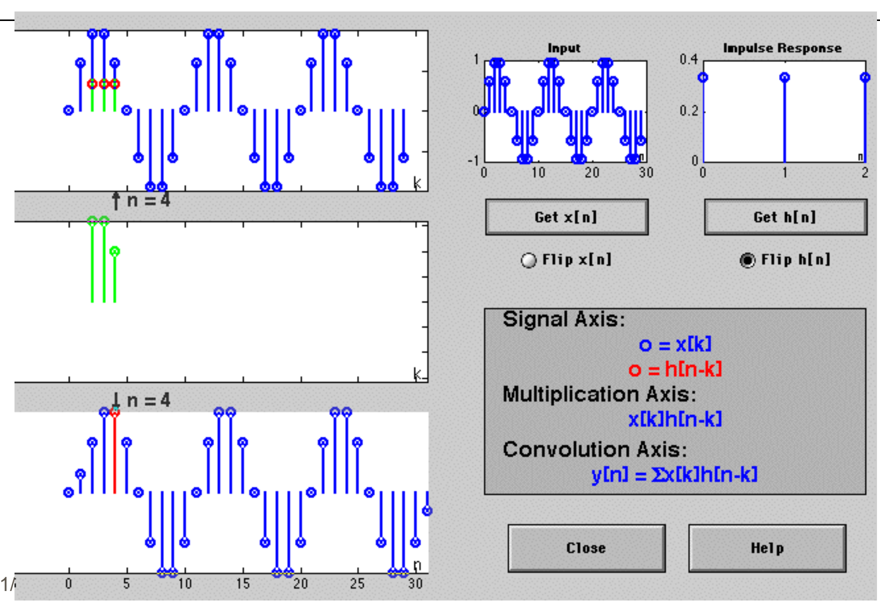
- Find y[n]

$$y[n] = u[n] - u[n-1] = \delta[n]$$

INPUT = SINUSOID

- INPUT: x[n] = SINUSOID
- OUTPUT: y[n] will also be a SINUSOID
  - Different Amplitude and Phase
  - SAME Frequency
- AMPLITUDE & PHASE CHANGE
  - Called the FREQUENCY RESPONSE

CONVDEMO: MATLAB GUI



COMPLEX EXPONENTIAL

$$x[n] = Ae^{j\phi}e^{j\hat{\omega}n} \qquad -\infty < n < \infty$$

x[n] is the input signal—a complex exponential

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] = \sum_{k=0}^{M} h[k]x[n-k]$$

FIR DIFFERENCE EQUATION

COMPLEX EXP OUTPUT

- Use the FIR "Difference Equation"

$$y[n] = \sum_{k=0}^{M} b_k Ae^{j\phi}e^{j\hat{\omega}(n-k)}$$

x[n-k]

$$= \left(\sum_{k=0}^{M} b_k e^{-j\hat{\omega}k}\right) Ae^{j\phi}e^{j\hat{\omega}n}$$

$$= \mathcal{H}(\hat{\omega}) Ae^{j\phi}e^{j\hat{\omega}n} \qquad -\infty < n < \infty$$

FREQUENCY REPONSE

- At each frequency, we can DEFINE

$$\mathcal{H}(\hat{\omega}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k}$$

FREQUENCY RESPONSE

- Complex-valued formula
  - Has MAGNITUDE vs. frequency
  - And PHASE vs. frequency

EXAMPLE 6.1

Example 6.1

$$\{b_k\} = \{1,\ 2,\ 1\}$$

$$\mathcal{H}(\hat{\omega}) = 1 + 2e^{-j\hat{\omega}} + e^{-j\hat{\omega}2}$$

To obtain formulas for the magnitude and phase of the frequency response

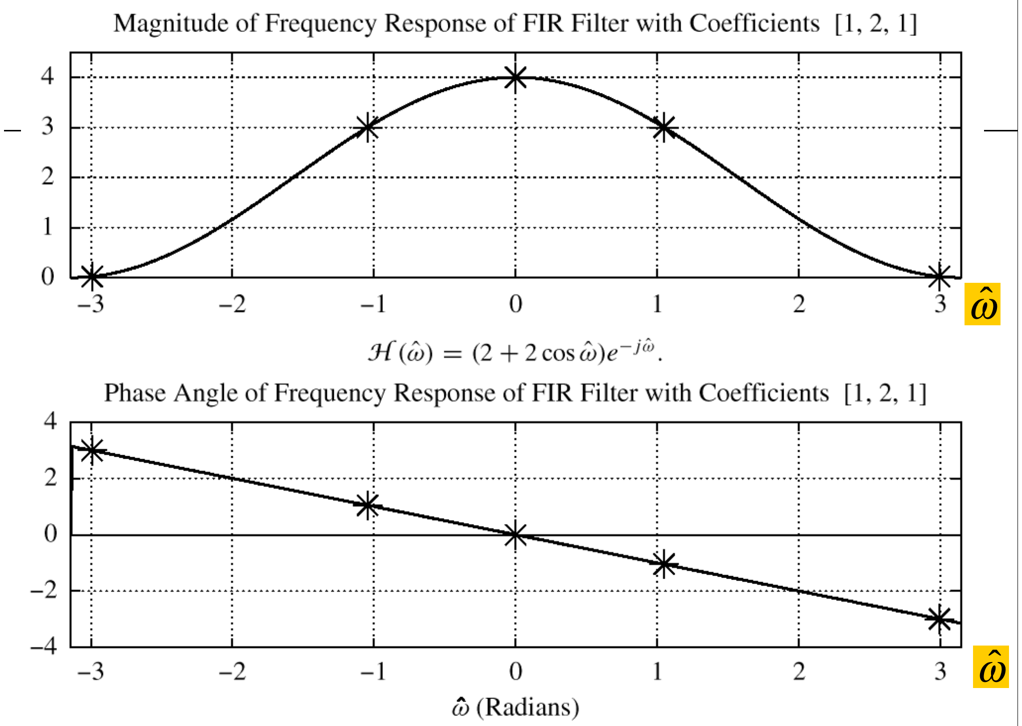
$$\mathcal{H}(\hat{\omega}) = 1 + 2e^{-j\hat{\omega}} + e^{-j\hat{\omega}2}$$

$$= e^{-j\hat{\omega}}\left(e^{j\hat{\omega}} + 2 + e^{-j\hat{\omega}}\right)$$

$$= e^{-j\hat{\omega}}\left(2 + 2\cos\hat{\omega}\right)$$

EXPLOIT SYMMETRY

Since $(2 + 2\cos\hat{\omega}) \geq 0$ for frequencies $-\pi < \hat{\omega} \leq \pi$,
the magnitude is $|\mathcal{H}(\hat{\omega})| = (2 + 2\cos\hat{\omega})$
and the phase is $\angle\mathcal{H}(\hat{\omega}) = -\hat{\omega}$.



MATLAB: FREQUENCY RESPONSE

- `HH = freqz(bb,1,ww)`
  - VECTOR **bb** contains Filter Coefficients
  - DSP-First: `HH = freekz(bb,1,ww)`
- FILTER COEFFICIENTS $\{b_k\}$

$$\mathcal{H}(\hat{\omega}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k}$$

LTI SYSTEMS

- LTI: Linear & Time-Invariant
- COMPLETELY CHARACTERIZED by:
  - FREQUENCY RESPONSE, or
  - IMPULSE RESPONSE h[n]
- Sinusoid IN -----> Sinusoid OUT
  - At the SAME Frequency

TWO DOMAINS

- TIME DOMAIN: Difference Equation

$$y[n] = x[n] - x[n-1]$$

  - Impulse Response

$$h[n] = \delta[n] - \delta[n-1]$$

- FREQUENCY DOMAIN: $H(\hat{\omega})$

Time & Frequency Relation

- Get Frequency Response from h[n]
  - Here is the FIR case:

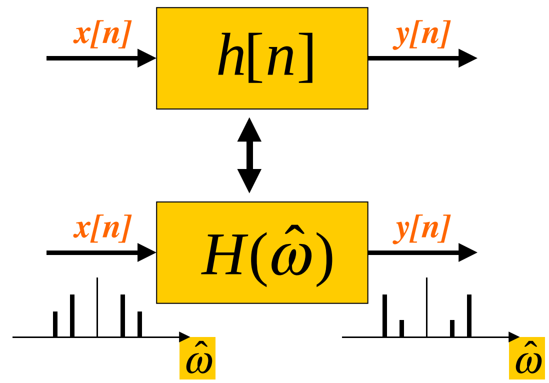
The frequency response of an LTI system

$$\mathcal{H}(\hat{\omega}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k} = \sum_{k=0}^{M} h[k] e^{-j\hat{\omega}k} \qquad (6.1.4)$$

IMPULSE RESPONSE

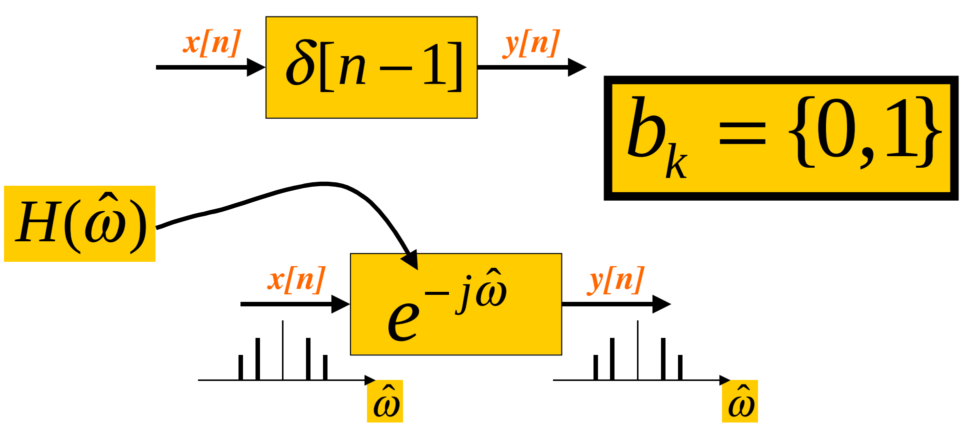
BLOCK DIAGRAMS

- Equivalent Representations



UNIT-DELAY SYSTEM

Find $h[n]$ and $H(\hat{\omega})$ for $y[n] = x[n-1]$



FIRST DIFFERENCE SYSTEM

Find $h[n]$ and $H(\hat{\omega})$ for the Diff. Eqn:

$$y[n] = x[n] - x[n-1]$$