EE-2200 Winter-99

Lecture 8

D-to-A Conversion

8-Feb-99

Information

- Prob Set #4 due THIS FRIDAY
  - In Lecture, before 2–PM
- Lab #5: AM and FM signals
  - Also Sampling and Reconstruction
- Lab QUIZ on 16 & 18 Feb

READING ASSIGNMENTS

- This Lecture:
  - Chapter 4, pp. 100–111
- Other Reading:
  - Recitation: Chapter 4, pp. 90–100
    - Strobe Demo
  - Next Lecture: Chapter 5 (beginning)

LECTURE OBJECTIVES

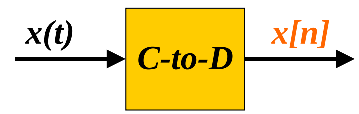
- DIGITAL–to–ANALOG CONVERSION is
  - Reconstruction from samples
    - SAMPLING THEOREM applies
  - Smooth Interpolation
- Mathematical Model of D–to–A
  - SUM of SHIFTED PULSES
    - Linear Interpolation example

SIGNAL TYPES

- A–to–D
  - Convert x(t) to numbers stored in memory
- D–to–A
  - Convert y[n] back to a "continuous–time" signal, x(t)
  - y[n] is called a "discrete–time" signal

SAMPLING x(t)

- UNIFORM SAMPLING at $t = nT_s$
  - IDEAL: $x[n] = x(nT_s)$



Shannon Sampling Theorem

A continuous-time signal $x(t)$ with frequencies no higher than f_{max} can be reconstructed exactly from its samples $x[n] = x(nT_s)$, if the samples are taken at a rate $f_s = 1/T_s$ that is greater than $2f_{max}$.

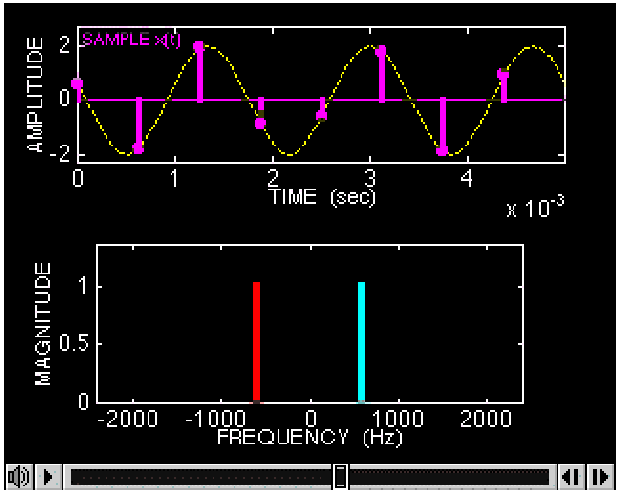
NYQUIST RATE

- "Nyquist Rate" Sampling
  - f_s = TWICE THE HIGHEST FREQUENCY in x(t)
  - "Sampling above the Nyquist rate"
- BANDLIMITED SIGNALS
  - x(t) has a HIGHEST FREQUENCY COMPONENT in its SPECTRUM
  - COUNTER–EXAMPLE:
    - TRIANGLE WAVE is NOT BANDLIMITED

DEMOS from CHAPTER 4

- SAMPLING DEMO
  - Different Sampling Rates
    - Aliasing of a Sinusoid
- STROBE DEMO
  - Synthetic vs. Real
  - Movie Camera SAMPLING at 15 fps
- Sampling & Reconstruction

SAMPLING DEMO (Ch. 4)



ALIASING & FOLDING

- $x(t)$ = SINUSOID @ f_o
- SAMPLED SIGNAL: $x[n] = x(n/f_s)$
- ALIASING:
  - x[n] COULD HAVE COME FROM
  - $(f_o + f_s)$
  - or $(f_o - f_s)$
  - or $(f_o + 2f_s)$
  - or $(f_o - 2f_s)$, etc.
- FOLDING:
  - A type of ALIASING
  - x[n] COULD BE:
  - $(-f_o + f_s)$
  - or $(-f_o - f_s)$
  - or $(-f_o + 2f_s)$
  - or $(-f_o - 2f_s)$, etc.

D-to-A MIGHT FAIL !

- ALIASING
  - INFINITE NUMBER of x(t)
  - WHICH ONE DO WE PICK?
  - D–to–A RECONSTRUCTION MUST CHOOSE
- RECONSTRUCT THE SMOOTHEST ONE
  - THE LOWEST FREQ, if x(t) = sinusoid

FOUR FREQUENCY AXES

- ANALOG FREQUENCY: f, ω
- DIGITAL FREQUENCY

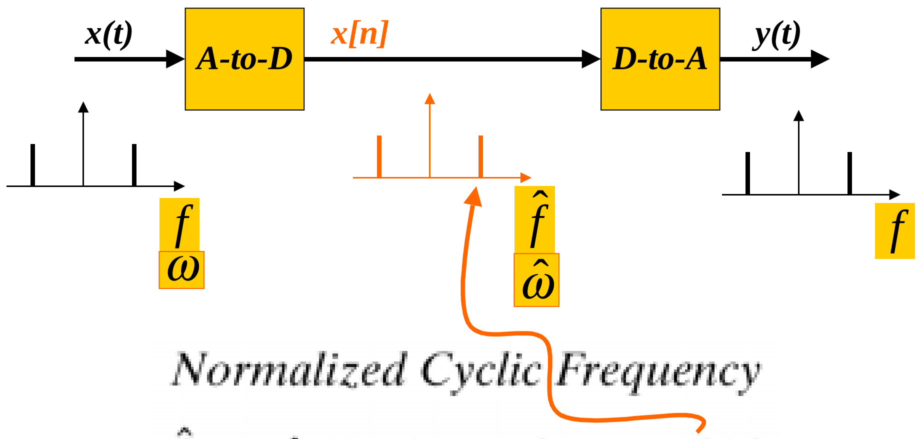
Normalized Radian Frequency

$$\hat{\omega} = \omega T_s$$

Normalized Cyclic Frequency

$$\hat{f} = \hat{\omega}/(2\pi) = f T_s = f/f_s$$

FREQUENCY DOMAINS



$$\hat{f} = \hat{\omega}/(2\pi) = f T_s = f/f_s$$

SPECTRUM for x[n]

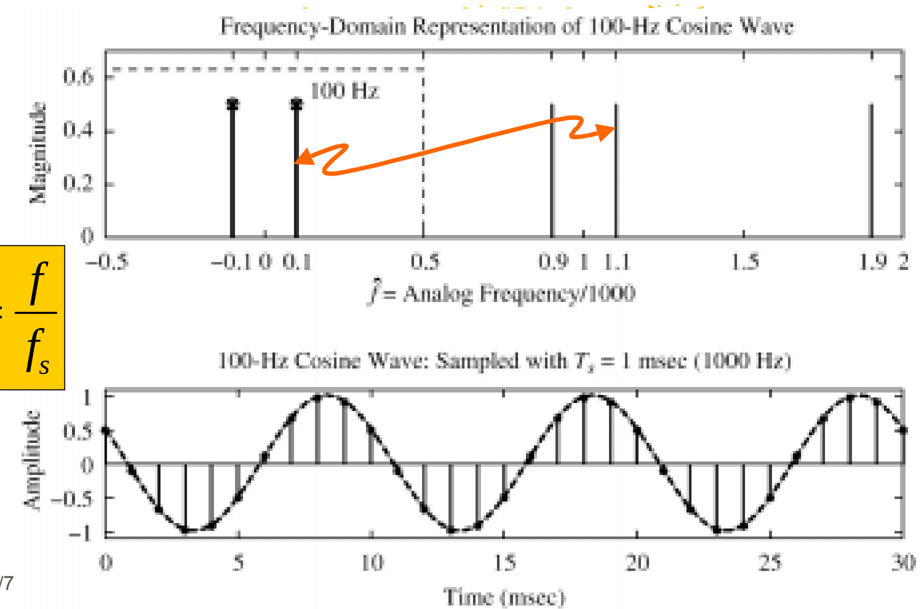
- INCLUDE ALL SPECTRUM LINES
  - ALIASES
    - ADD INTEGER MULTIPLES of f_s and $-f_s$
  - FOLDED ALIASES
    - ALIASES of NEGATIVE FREQS
- PLOT versus NORMALIZED FREQUENCY
  - CONVERT f_o and $-f_o$

EXAMPLE: SPECTRUM

- $x[n] = A\cos(0.2\pi n+\phi)$
- FREQS @ 0.2π and -0.2π
- CONVERT to NORMALIZED CYCLIC FREQ
  - 0.2π --> 0.1 and -0.2π --> -0.1
- ALIASES (and FOLDING):
  - {1.1, 2.1, 3.1, ...} & {−0.9,−1.9,−2.9,...}
  - {0.9,1.9,2.9,...} & {−1.1, −2.1, −3.1, ...}

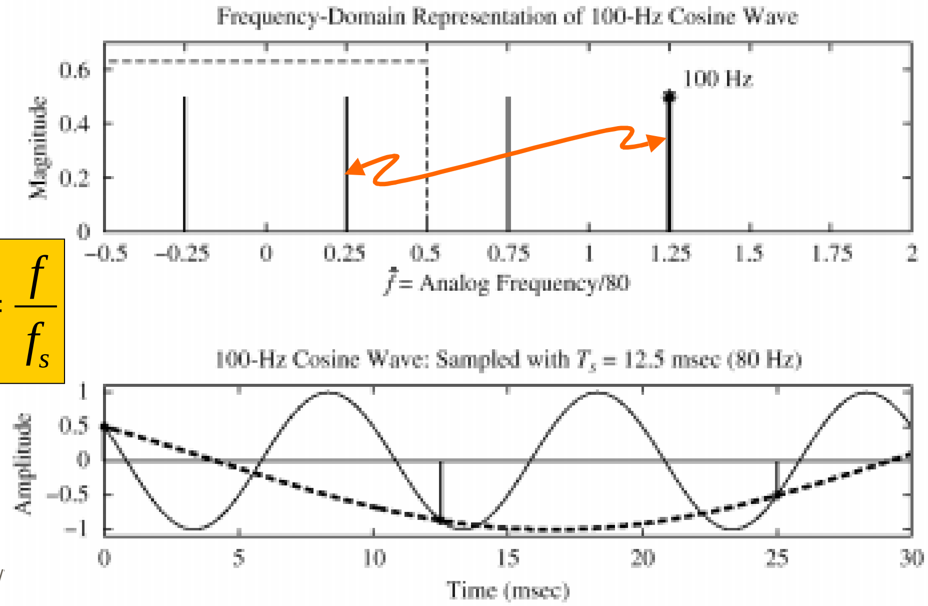
SPECTRUM (DIGITAL)

$$\hat{f} = \frac{f}{f_s}$$

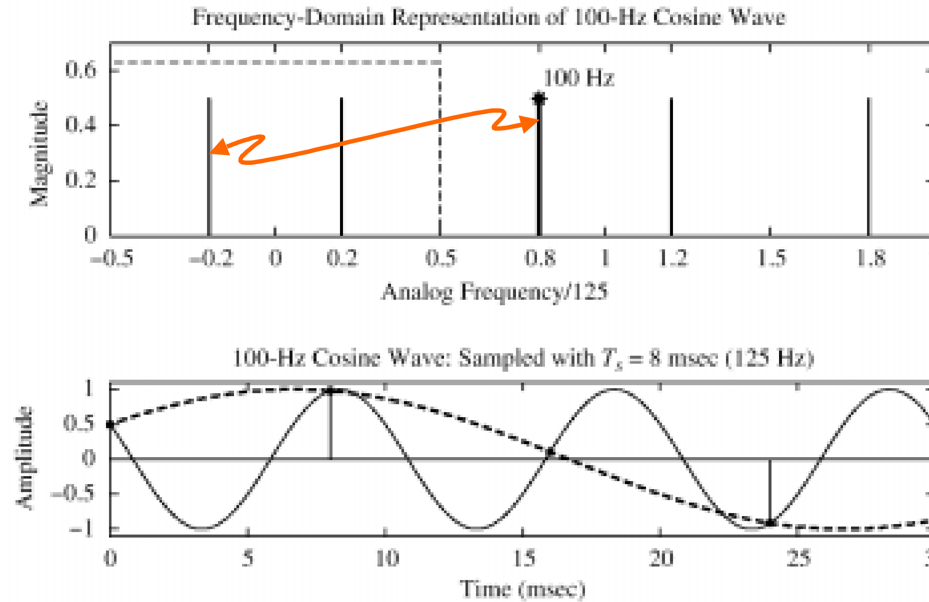


SPECTRUM of x[n]
ALIASING CASE

$$\hat{f} = \frac{f}{f_s}$$

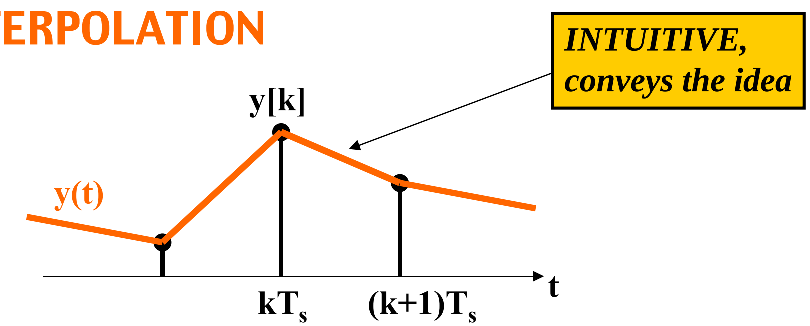


SPECTRUM of x[n]
FOLDING CASE



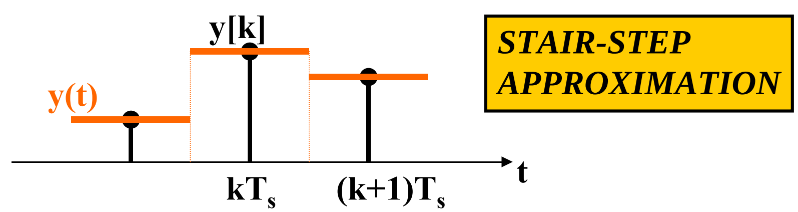
Reconstruction (D-to-A)

- CONVERT STREAM of NUMBERS to x(t)
- "CONNECT THE DOTS"
- INTERPOLATION



SAMPLE & HOLD DEVICE

- CONVERT y[n] to y(t)
  - y[k] should be the value of y(t) at $t = kT_s$
  - Make y(t) equal to y[k] for
    - $kT_s - 0.5T_s < t < kT_s + 0.5T_s$



MATH MODEL for D-to-A

$$y(t) = \sum_{n=-\infty}^{\infty} y[n]p(t - nT_s)$$

SQUARE PULSE:

$$p(t) = \begin{cases} 1 & -\frac{1}{2}T_s < t \leq \frac{1}{2}T_s \\ 0 & \text{otherwise} \end{cases}$$

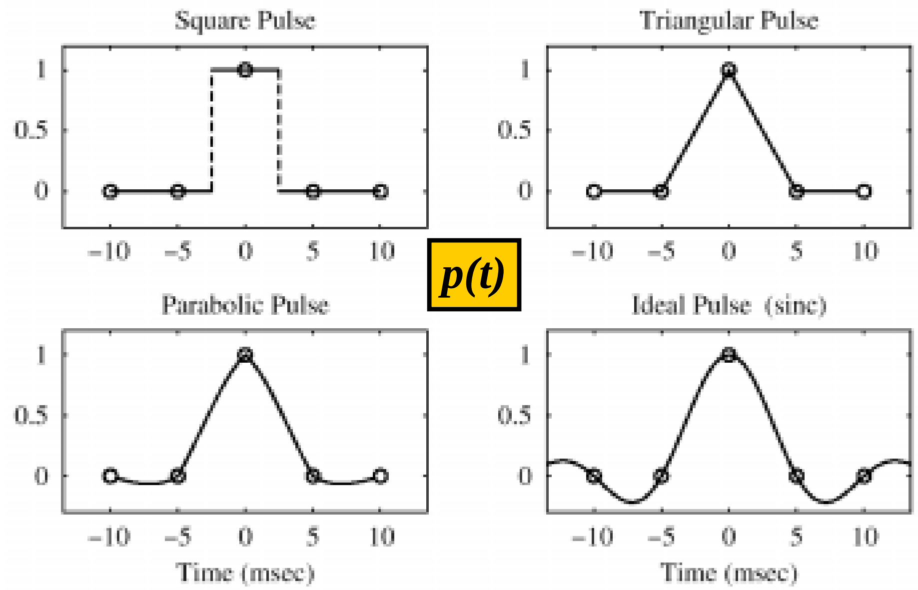


Figure 4.17 Four different pulses for D-to-C conversion. The sampling period is $T_s = 0.005$, i.e., $f_s = 200$ Hz. Note that the duration of each pulse is approximately one or two times T_s.

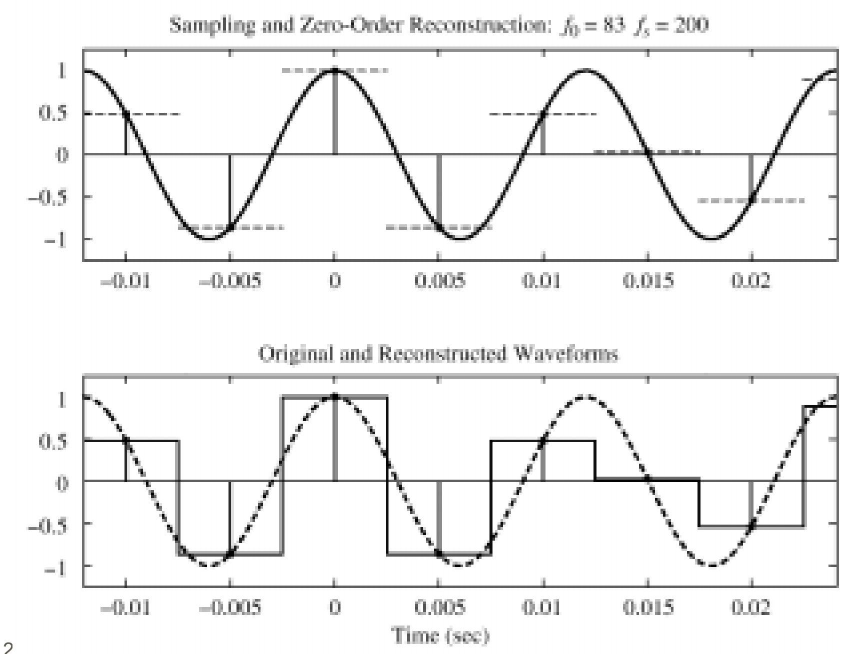
EXPAND the SUMMATION

$$\sum_{n=-\infty}^{\infty} y[n]p(t - nT_s) =$$

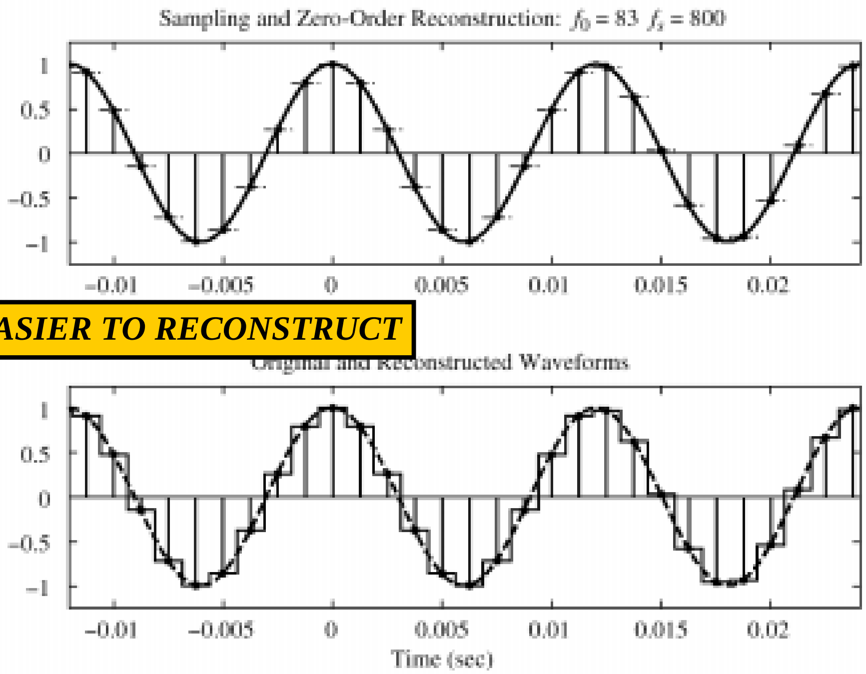
$$\ldots + y[0]p(t) + y[1]p(t - T_s) + y[2]p(t - 2T_s) + \ldots$$

- SUM of SHIFTED PULSES $p(t-nT_s)$
  - "WEIGHTED" by y[n]
  - CENTERED at $t=nT_s$
  - SPACED by T_s
    - RESTORES "REAL TIME"

SQUARE PULSE CASE

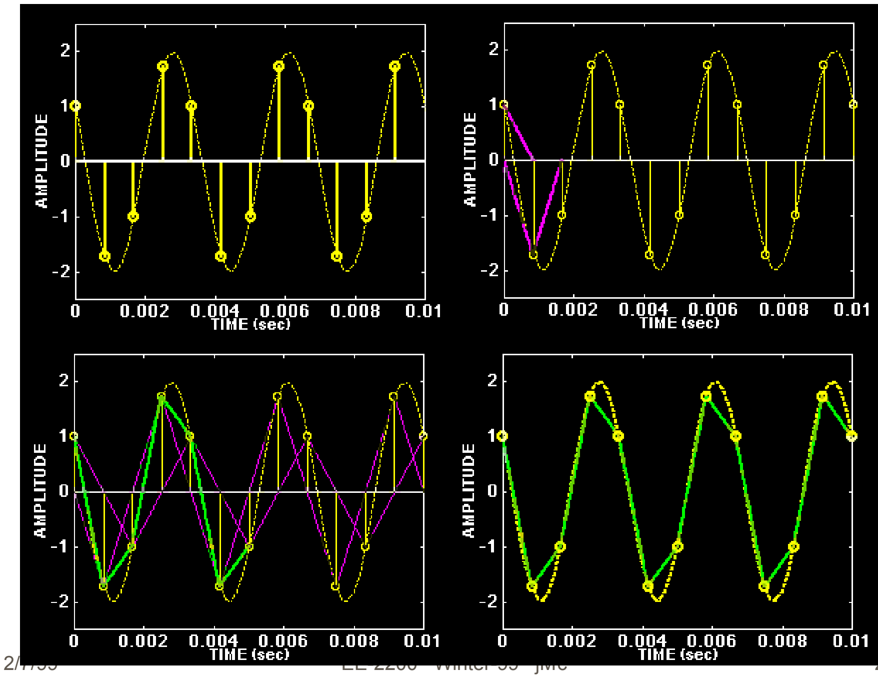


OVER-SAMPLING CASE



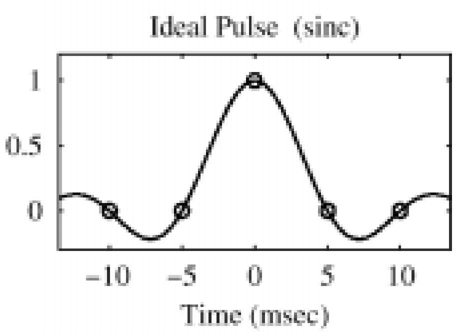
EASIER TO RECONSTRUCT

TRIANGULAR PULSE (2X)



OPTIMAL PULSE ?

CALLED "BANDLIMITED INTERPOLATION"



$$p(t) = \frac{\sin \frac{\pi}{T_s} t}{\frac{\pi}{T_s} t} \qquad \text{for } -\infty < t < \infty$$

$$p(t) = 0 \qquad \text{for } t = 0, \pm T_s, \pm 2T_s$$