

EE-2200

Winter-99

Lecture 4

Spectrum Representation

22-Jan-99

Web-CT Info

- Check the Bulletin Board for msgs
- Lectures are being posted
 - HTML format
 - PDF format (4 per page)
- Calendar has entries:
 - Quiz #1 on 1-Feb (Monday)
 - Quiz #2 on 1-March (Monday)

Homework Info

- Prob Set #2 due TODAY
 - In Lecture, before 2 PM
- HW #3 will be posted Today
 - Due Next Friday, 29-Jan
- Solutions this weekend
 - HW #1 Solutions were posted last Friday.

Lab Info

- Lab #2 Report
 - Turn in during your lab time
 - Include INSTRUCTOR VERIFICATION
- Lab #3 will be posted Saturday
 - Lab #4 will be Music Synthesis

READING ASSIGNMENTS

- This Lecture:
 - ▮ Chapter 3, pp. 48–61
- Other Reading:
 - ▮ Appendix A: Complex Numbers
 - ▮ Appendix B: MATLAB
 - ▮ Next Lecture: Chapter 3

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HISTORY

- What company's first successful product was a sine-wave generator?
 - ▮ Variable frequency
 - ▮ Lab Instrument

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LECTURE OBJECTIVES

- Sinusoids with **DIFFERENT** Frequencies
 - ▮ Add Sinusoids

$$x(t) = \sum_{k=1}^N A_k \cos(2\pi f_k t + \varphi_k)$$

- **SPECTRUM** Representation
 - ▮ **Graphical** Form shows Different Freqs

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FREQUENCY DIAGRAM

- Plot Complex Amplitude vs. Freq

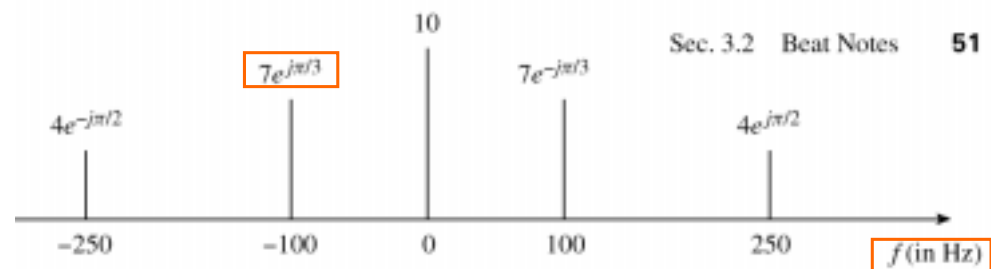


Figure 3.1 Spectrum of the signal $x(t) = 10 + 14 \cos(200\pi t - \pi/3) + 8 \cos(500\pi t + \pi/2)$. Positive and negative frequency components must be included even though the negative-frequency ones are the conjugate of the positive-frequency components.

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Another FREQ. Diagram



Figure 3.18 Sheet-music notation is a time–frequency diagram.

Time is the horizontal axis

MOTIVATION

■ Synthesize **Complicated** Signals

■ Musical Notes

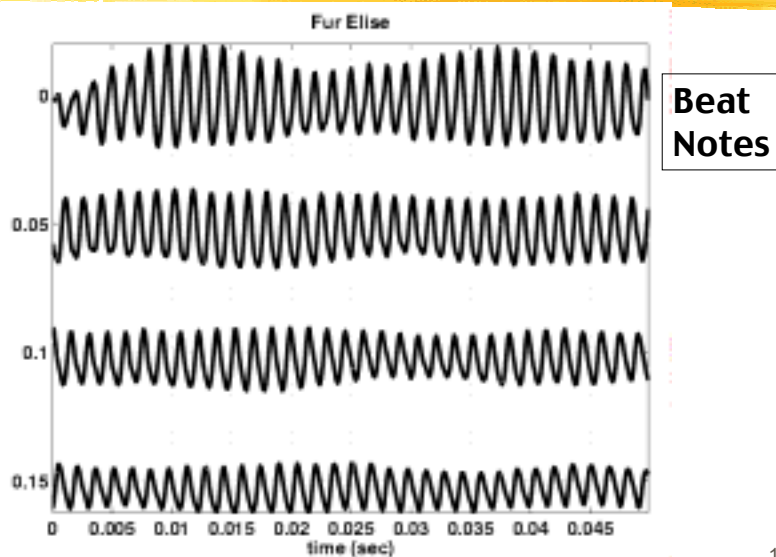
- Piano uses 3 strings for many notes
- Chords: play several notes simultaneously

■ Human Speech

- Vowels have dominant frequencies 
- Application: computer generated speech

■ Can **all** signals be generated this way?

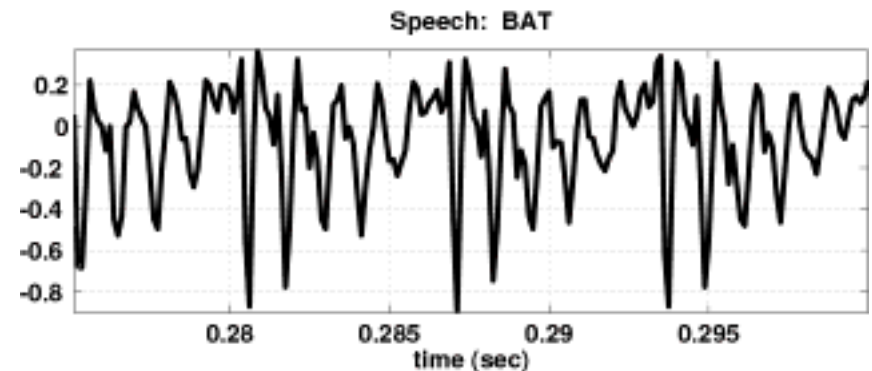
Für Elise WAVEFORM



Speech Signal: BAT

■ Nearly Periodic in Vowel Region

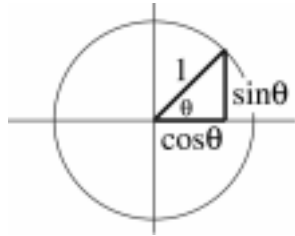
- Period is (Approximately) $T = 0.0065$ sec



Euler's FORMULA

Complex Exponential

- Real part is cosine
- Imaginary part is sine
- Magnitude is one



$$e^{j\theta} = \cos(\theta) + j \sin(\theta)$$

$$e^{j\omega t} = \cos(\omega t) + j \sin(\omega t)$$

INVERSE Euler's Formula

Solve for cosine (or sine)

$$e^{j\omega t} = \cos(\omega t) + j \sin(\omega t)$$

$$e^{-j\omega t} = \cos(-\omega t) + j \sin(-\omega t)$$

$$e^{-j\omega t} = \cos(\omega t) - j \sin(\omega t)$$

$$e^{j\omega t} + e^{-j\omega t} = 2 \cos(\omega t)$$

$$\cos(\omega t) = \frac{1}{2} (e^{j\omega t} + e^{-j\omega t})$$

INVERSE Euler's Formula

Solve for cosine (or sine)

$$\cos(\omega t) = \frac{1}{2} (e^{j\omega t} + e^{-j\omega t})$$

$$\sin(\omega t) = \frac{1}{2j} (e^{j\omega t} - e^{-j\omega t})$$

SPECTRUM Interpretation

Cosine = sum of 2 complex exponentials:

$$A \cos(7t) = \frac{A}{2} e^{j7t} + \frac{A}{2} e^{-j7t}$$

- One has a positive frequency
- The other has **negative** freq.
- Amplitude of each is half as big

SPECTRUM of SINE

- Sine = sum of 2 complex exponentials:

$$A \sin(7t) = \frac{A}{2j} e^{j7t} - \frac{A}{2j} e^{-j7t}$$

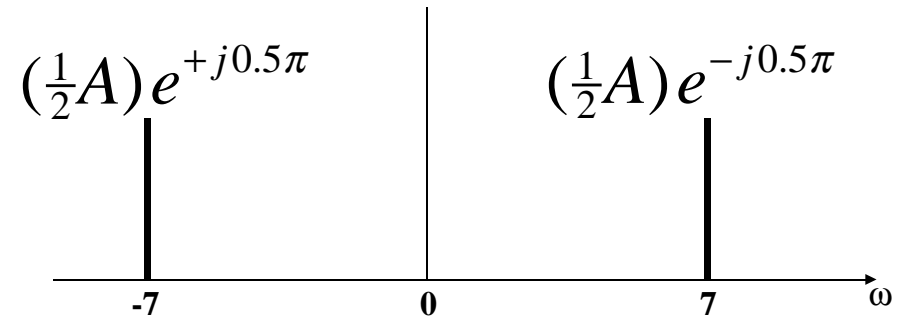
$$= \frac{1}{2} A e^{-j0.5\pi} e^{j7t} + \frac{1}{2} A e^{j0.5\pi} e^{-j7t}$$

$$\frac{-1}{j} = j = e^{j0.5\pi}$$

- Positive freq has phase = -0.5π
- Negative freq. has phase = $+0.5\pi$

GRAPHICAL SPECTRUM

EXAMPLE of SINE



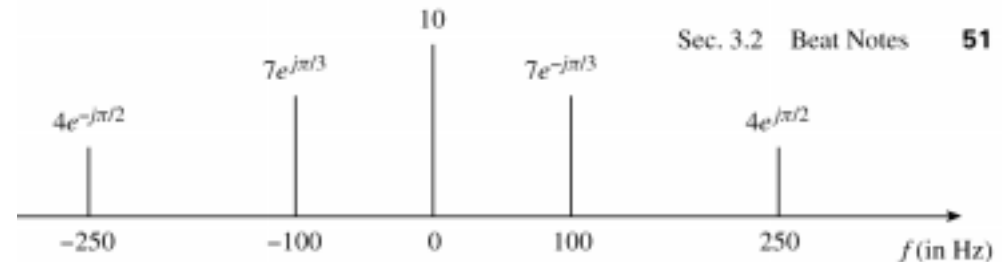
AMPLITUDE, PHASE & FREQUENCY are shown

NEGATIVE FREQUENCY

- Is negative frequency real?
- Doppler Radar provides an example
 - Police radar measures speed by using the Doppler shift principle
 - Let's assume 400Hz <----> 60 mph
 - +400Hz** means towards the radar
 - 400Hz** means away (opposite **direction**)
 - Think of a train whistle

SPECTRUM ---> SINUSOID

- Add the spectrum components:



What is the formula for the signal $x(t)$?

Gather (A, ω, ϕ) information

Frequencies:

- | -250 Hz
- | -100 Hz
- | 0 Hz
- | 100 Hz
- | 250 Hz

Amplitude & Phase

- | | | |
|----|----------|---|
| 4 | $-\pi/2$ | |
| 7 | $+\pi/3$ | ← |
| 10 | 0 | |
| 7 | $-\pi/3$ | ← |
| 4 | $+\pi/2$ | |

Note the conjugate phase

Zero freq always has zero phase (for real $x(t)$)

DC is another name for zero-freq component

Add All the Spectrum Components

$$\begin{aligned}
 x(t) = & 10 \\
 & + 7e^{-j\pi/3} e^{j2\pi(100)t} + 7e^{j\pi/3} e^{-j2\pi(100)t} \\
 & + 4e^{j\pi/2} e^{j2\pi(250)t} + 4e^{-j\pi/2} e^{-j2\pi(250)t}
 \end{aligned}$$

Use Euler's Formula to get **REAL** sinusoids:

$$A \cos(\omega t + \phi) = \frac{A}{2} e^{j\phi} e^{j\omega t} + \frac{A}{2} e^{-j\phi} e^{-j\omega t}$$

FINAL ANSWER

$$\begin{aligned}
 x(t) = & 10 + 14 \cos(200\pi t - \pi/3) \\
 & + 8 \cos(500\pi t + \pi/2)
 \end{aligned}$$

So, we get the general form:

$$x(t) = \sum_{k=1}^N A_k \cos(2\pi f_k t + \phi_k)$$

Summary: GENERAL FORM

$$x(t) = A_0 + \sum_{k=1}^N A_k \cos(2\pi f_k t + \phi_k)$$

$$x(t) = X_0 + \sum_{k=1}^N \Re \{ X_k e^{j2\pi f_k t} \}$$

$$X_k = A_k e^{j\phi_k}$$

frequency is f_k .

$$x(t) = X_0 + \sum_{k=1}^N \left\{ \frac{X_k}{2} e^{j2\pi f_k t} + \frac{X_k^*}{2} e^{-j2\pi f_k t} \right\}$$

Example: Synthetic Vowel

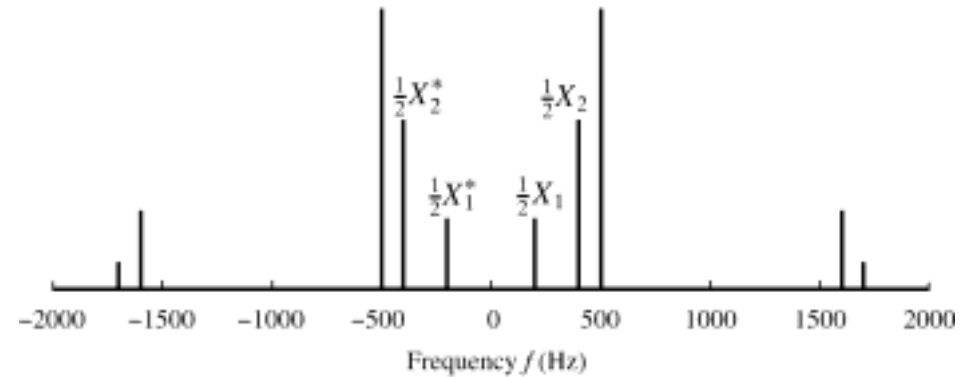
Sum of 5 Frequency Components

f_k (Hz)	X_k	Mag	Phase (rad)
200	$(771 + j12202)$	12,226	1.508
400	$(-8865 + j28048)$	29,416	1.876
500	$(48001 - j8995)$	48,836	-0.185
1600	$(1657 - j13520)$	13,621	-1.449
1700	$4723 + j0$	4723	0

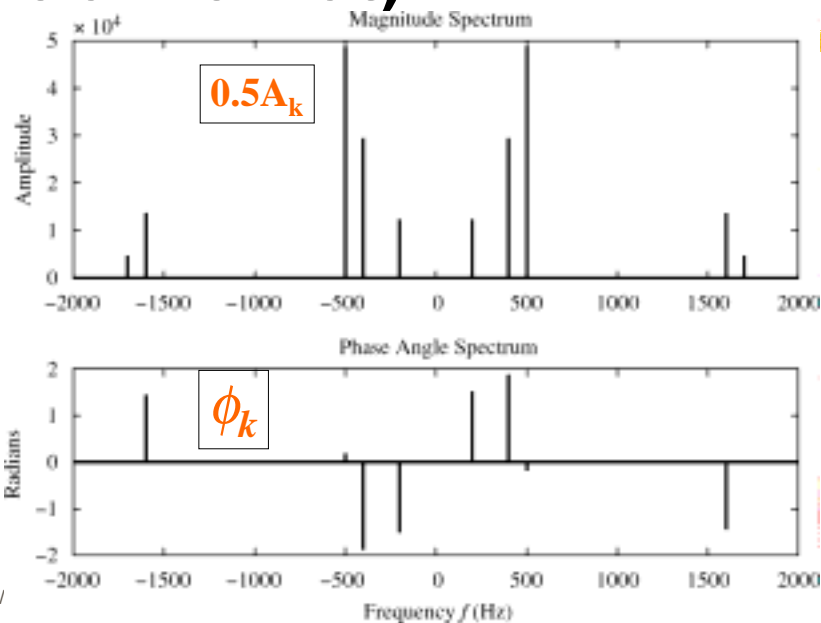
Table 3.1: Complex amplitudes for harmonic signal that approximates the vowel sound "ah".

SPECTRUM of VOWEL

- Note: Spectrum has $0.5X_k$ (except X_{DC})
- Conjugates in negative frequency



SPECTRUM of VOWEL (Polar Format)



Vowel Waveform (sum of all 5 components)

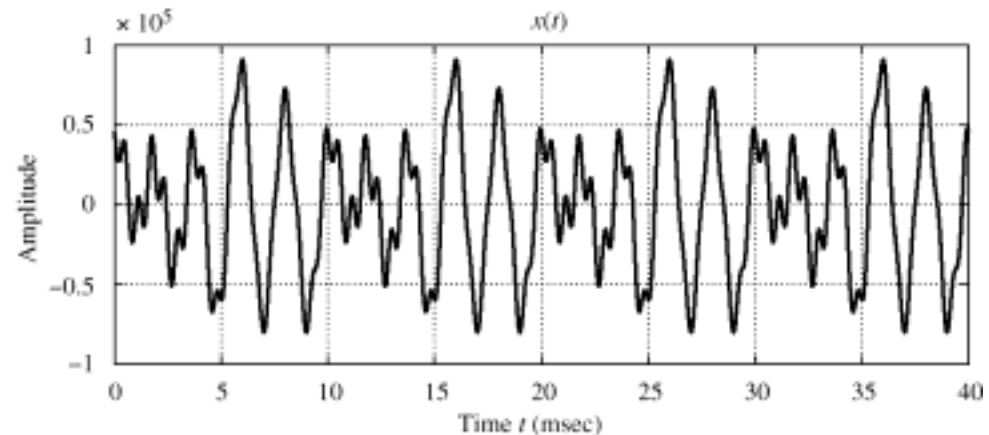


Figure 3.11 Sum of all of the terms in (3.3.4). Note that the period is 10 msec, which equals $1/f_0$.