EE-2200 Winter-99

LECTURE #3

Phasor Addition Theorem

15-Jan-99

Web-CT Info

- Check the Bulletin Board for msgs
  - MAKE YOUR OWN POSTINGS
- Lectures are being posted
  - PDF format (4 per page)
- Quiz Dates:
  - Quiz #1 on 1–Feb (Monday)
  - Quiz #2 on 1–March (Monday)

Homework Info

- Prob Set #1 due TODAY in CLASS
- HW will be posted on Friday/Saturday
  - Due the following Friday
  - HW #2 is on the web site
- Solutions will be posted on Weekend
  - HW #1 should be there today

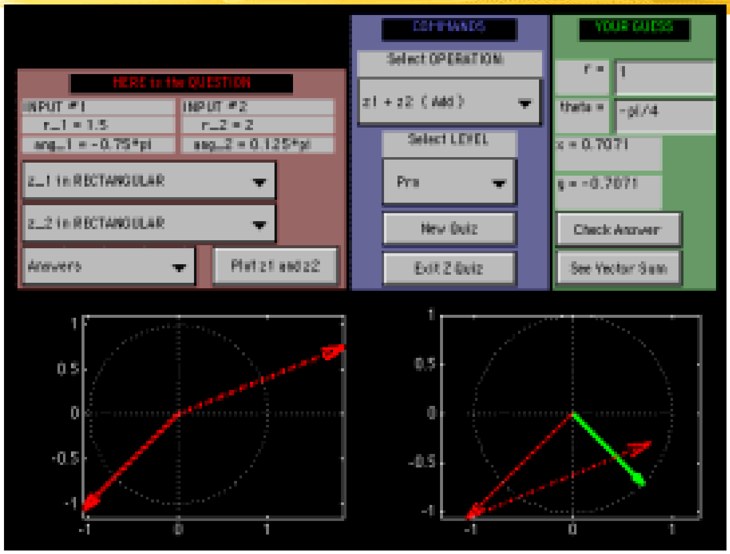
Lab Info

- LAB QUIZ next week (short)
- MATLAB Help: T–W–Th 6PM VL–456
- Lab #1 Report
  - Turn in during your lab time
  - Write–up sections 2 and 3
  - Include INSTRUCTOR VERIFICATION
- Lab #2 will be posted later today

READING ASSIGNMENTS

- This Lecture:
  - Chapter 2, pp. 31–43
- Other Reading:
  - Appendix A: Complex Numbers
  - Appendix B: MATLAB
  - Next Lecture: start Chapter 3

Z DRILL (Complex Arith)



LECTURE OBJECTIVES

- Phasors = Complex Amplitude
  - Add Sinusoids = Complex Addition
  - PHASOR ADDITION THEOREM

$$z(t) = Ze^{j\omega t} = (Ae^{j\varphi})e^{j\omega t}$$

- Develop the ABSTRACTION:
  - Complex Numbers represent Sinusoids

AVOID Trigonometry

- Algebra, even complex, is EASIER !!!
- Can you recall $\cos(\theta_1+\theta_2)$?
- Use the real part of $e^{j\theta_1} e^{j\theta_2}$

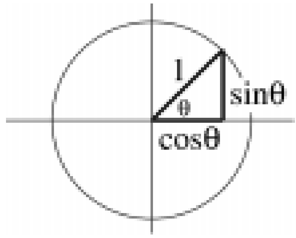
$$e^{j(\theta_1+\theta_2)} = e^{j\theta_1} e^{j\theta_2}$$

$$= (\cos\theta_1 + j\sin\theta_1)(\cos\theta_2 + j\sin\theta_2)$$

$$= \boxed{\cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2} + j(\ldots)$$

Euler's FORMULA

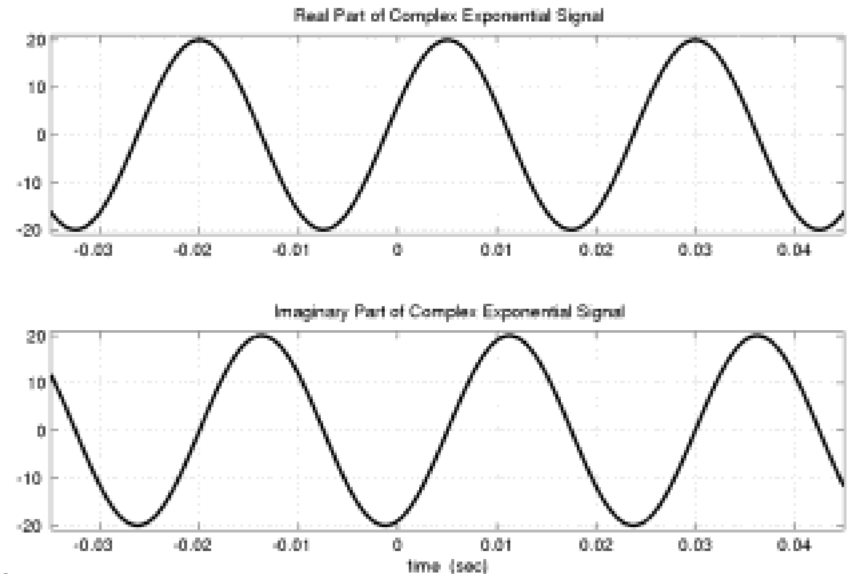
- **Complex Exponential**
  - Real part is cosine
  - Imaginary part is sine
  - Magnitude is one



$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

$$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$$

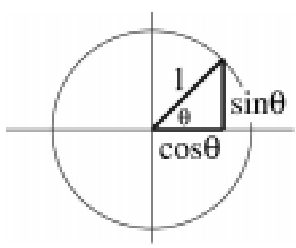
Real & Imaginary Part Plots



COMPLEX EXPONENTIAL

$$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$$

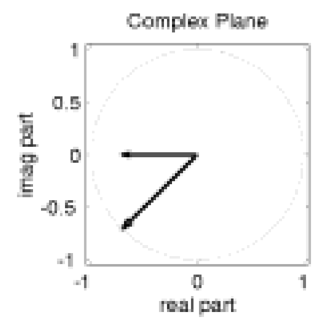
- **Rotating Vector**
  - Angle changes vs. time
  - $\theta = \omega t$
  - ex: $\omega = 10\pi$
  - Rotates 0.1π in 0.01 secs

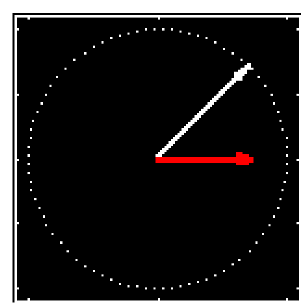


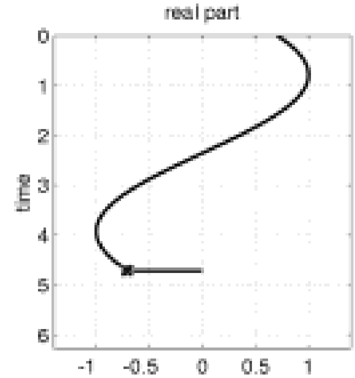
$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

Rotating Phasor

See Demo on CD-ROM Chapter 2







Cos = REAL PART

- Real Part of Euler's:

$$\cos(\omega t) = \Re e\{e^{j\omega t}\}$$

- General Sinusoid

$$x(t) = A\cos(\omega t + \varphi)$$

- So,

$$A\cos(\omega t + \varphi) = \Re e\{Ae^{j(\omega t+\varphi)}\}$$

$$= \Re e\{Ae^{j\varphi}e^{j\omega t}\}$$

COMPLEX AMPLITUDE

- General Sinusoid

$$x(t) = A\cos(\omega t + \varphi) = \Re e\{Ae^{j\varphi}e^{j\omega t}\}$$

- Complex Exponential

$$z(t) = Ze^{j\omega t} \qquad Z = Ae^{j\varphi}$$

- Sinusoid is REAL PART of $e^{j\omega t}$

$$x(t) = \Re e\{z(t)\} = \Re e\{Ze^{j\omega t}\}$$

WANT to ADD SINUSOIDS

- ALL SINUSOIDS HAVE SAME FREQUENCY
- HOW to GET {Amp,Phase} of RESULT ?

$$x_1(t) = 1.7\cos(2\pi(10)t + 70\pi/180)$$

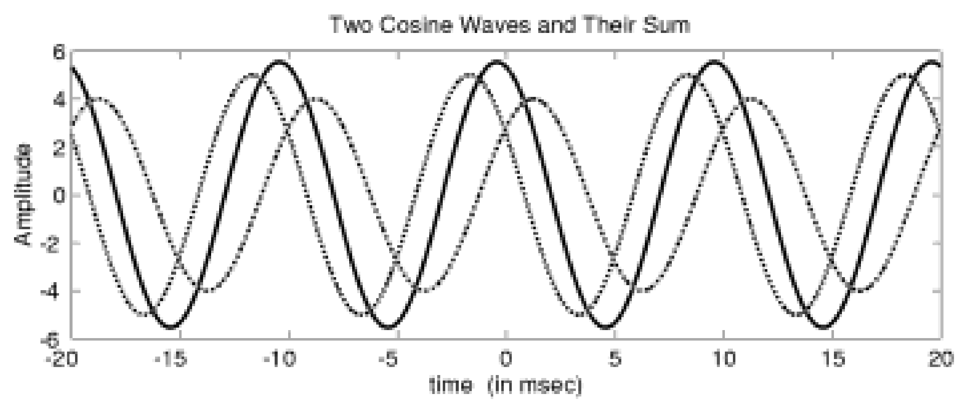
$$x_2(t) = 1.9\cos(2\pi(10)t + 200\pi/180)$$

$$x_3(t) = x_1(t) + x_2(t)$$

$$= 1.532\cos(2\pi(10)t + 141.79\pi/180)$$

ADD SINUSOIDS

- Sum Sinusoid has same Frequency



PHASOR ADDITION RULE

$$x(t) = \sum_{k=1}^{N} A_k \cos(\omega_0 t + \phi_k)$$

$$= A\cos(\omega_0 t + \phi)$$

Get the new complex amplitude by addition

$$\sum_{k=1}^{N} A_k e^{j\phi_k} = Ae^{j\phi}$$

Phasor Addition Proof

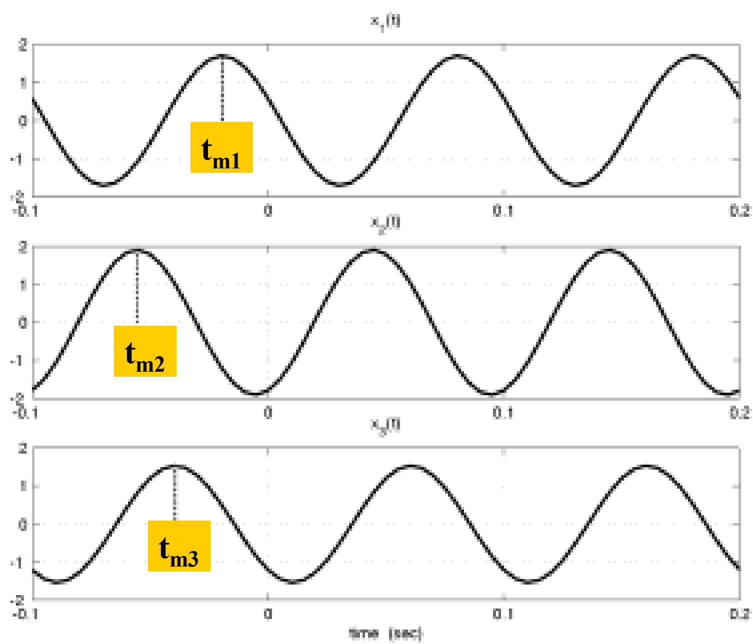
$$\sum_{k=1}^{N} A_k \cos(\omega_0 t + \phi_k) = \sum_{k=1}^{N} \Re e\left\{A_k e^{j(\omega_0 t + \phi_k)}\right\}$$

$$= \Re e\left\{\sum_{k=1}^{N} A_k e^{j\phi_k} e^{j\omega_0 t}\right\}$$

$$= \Re e\left\{\left(\sum_{k=1}^{N} A_k e^{j\phi_k}\right) e^{j\omega_0 t}\right\}$$

$$= \Re e\left\{\left(Ae^{j\phi}\right) e^{j\omega_0 t}\right\} = A\cos(\omega_0 t + \phi)$$

ADD SINUSOIDS EXAMPLE

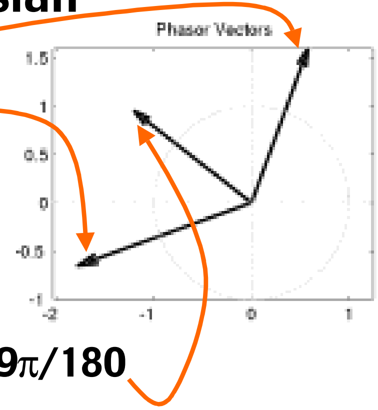


Convert Time-Shift to Phase

- Measure **peak times**:
  - $t_{m1}=-0.0194$, $t_{m2}=-0.0556$, $t_{m3}=-0.0394$
- Convert to **phase** (T=0.1)
  - $\phi_1=-2\pi(t_{m1}/T)=70\pi/180$,
  - $\phi_2= 200\pi/180$
- Amplitudes
  - $A_1=1.7$, $A_2=1.9$, $A_3=1.532$

Phasor Add: Numerical

- Convert Polar to Cartesian
  - $X_1 = 0.5814 + j1.597$
  - $X_2 = -1.785 - j0.6498$
  - $X_3 = -1.204 + j0.9476$



- Convert back to Polar
  - $X_3 = 1.532$ at angle $141.79\pi/180$

ADD SINUSOIDS

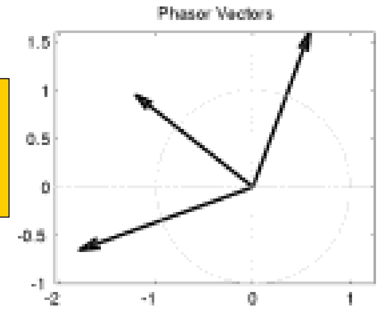
$$x_1(t) = 1.7\cos(2\pi(10)t + 70\pi/180)$$

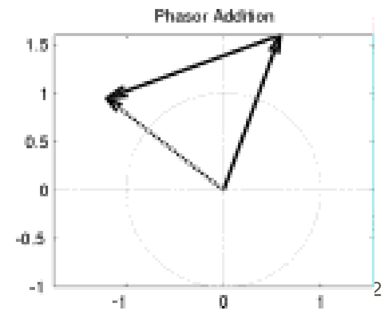
$$x_2(t) = 1.9\cos(2\pi(10)t + 200\pi/180)$$

$$x_3(t) = x_1(t) + x_2(t)$$

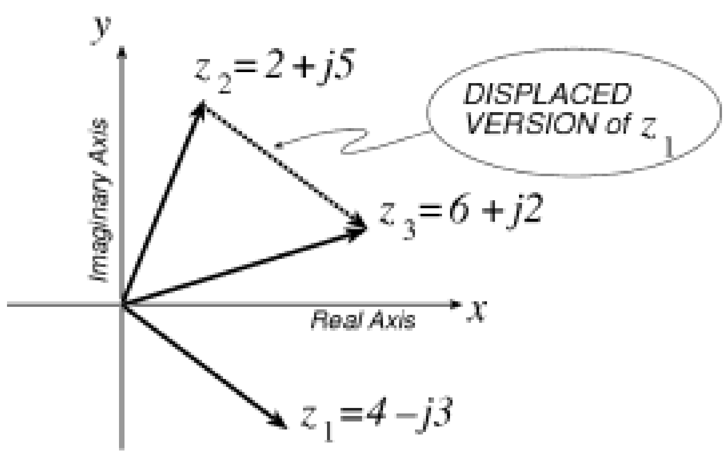
$$= 1.532\cos(2\pi(10)t + 141.79\pi/180)$$

VECTOR (PHASOR) ADD





COMPLEX ADDITION



POP QUIZ

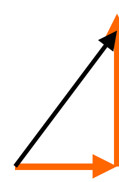
- ADD THESE 2 SINUSOIDS:

$$x_1(t) = \cos(77\pi t)$$

$$x_2(t) = \sqrt{3}\cos(77\pi t + 0.5\pi)$$

- COMPLEX ADDITION:

$$1 + j\sqrt{3} = 2e^{j\pi/3}$$



$$j\sqrt{3} = \sqrt{3}e^{j0.5\pi}$$