EE-2200 Winter-99

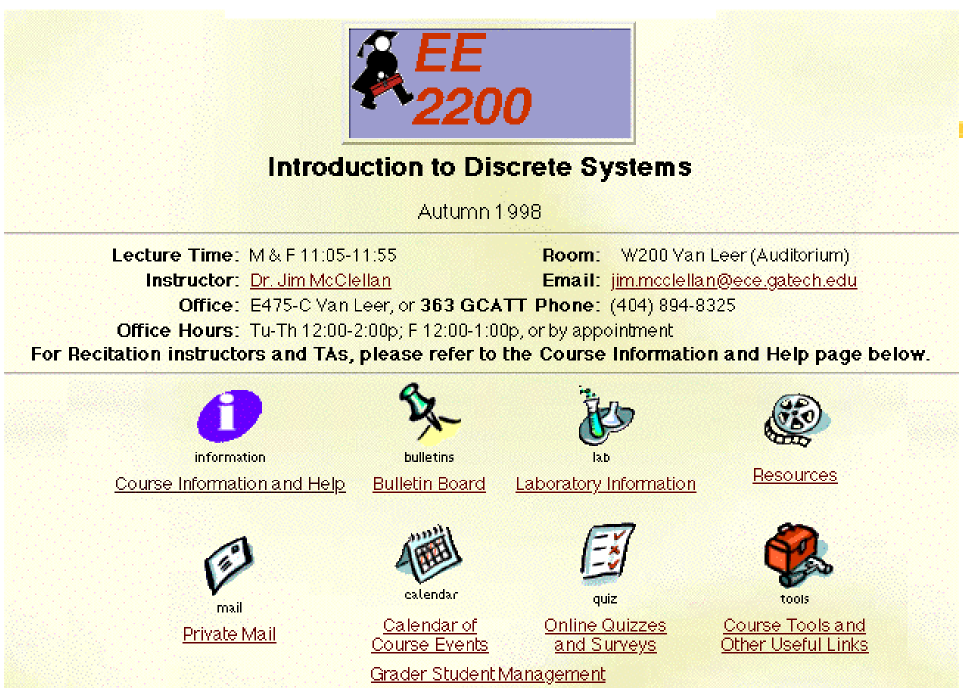
LECTURE #2

Complex Exponentials

11-Jan-99

INFORMATION

- MATLAB: M–T–W–Th 6pm VL–456
- LABS start this week
  - Attend correct section (in CoC–309)
  - Get your computer acct ASAP
  - Verification must be signed during Lab
- RECITATIONS
  - Attend your assigned time



REMINDERS

- Web–CT Password:
  - SSN(4:8), 4th thru 8th digits of SSN
- Hard copy of Instructor Verification Sheet
  - Get PDF file of Lab#1 from WebCT
  - Lab #1 is different from the book
- HW #1 is due Friday (in Lecture)

READING ASSIGNMENTS

- This Lecture:
  - Chapter 2, pp. 17–32
- Appendix A: Complex Numbers
- Appendix B: MATLAB
- Next Lecture: finish Chap. 2, pp. 31–43

LECTURE OBJECTIVES

- Define Sinusoid from a plot
- Relate TIME-SHIFT to PHASE
- Introduce an ABSTRACTION:
  - Complex Numbers represent Sinusoids
  - Complex Exponential Signal

$$z(t) = Ze^{j\omega t}$$

SINUSOIDAL SIGNAL

$$A\cos(\omega t + \varphi)$$

- FREQUENCY ω
  - Radians/sec
  - Hertz (cycles/sec)

$$\omega = (2\pi)f$$

- PERIOD (in sec)

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

- AMPLITUDE A
  - Magnitude
- PHASE φ

PLOT a COSINE SIGNAL

- Given the Formula

$$5\cos(0.3\pi t + 1.2\pi)$$

- Make a plot
- Formula defines A, ω, and ϕ

$$A = 5$$
$$\omega = 0.3\pi$$
$$\varphi = 1.2\pi$$

PLOTTING COSINE SIGNAL from the FORMULA

$$5\cos(0.3\pi t + 1.2\pi)$$

- Determine **period**:

$$T = 2\pi/\omega = 2\pi/0.3\pi = 20/3$$

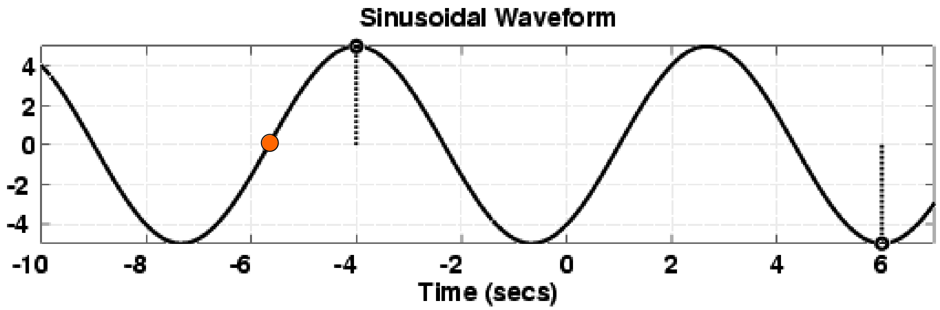
- Determine a **peak** location by solving

$$(\omega t + \varphi) = 0$$

- **Peak at t = –4**
- **Zero** crossing is T/4 before or after

ANSWER for the PLOT

$$5\cos(0.3\pi t + 1.2\pi)$$



TIME-SHIFT

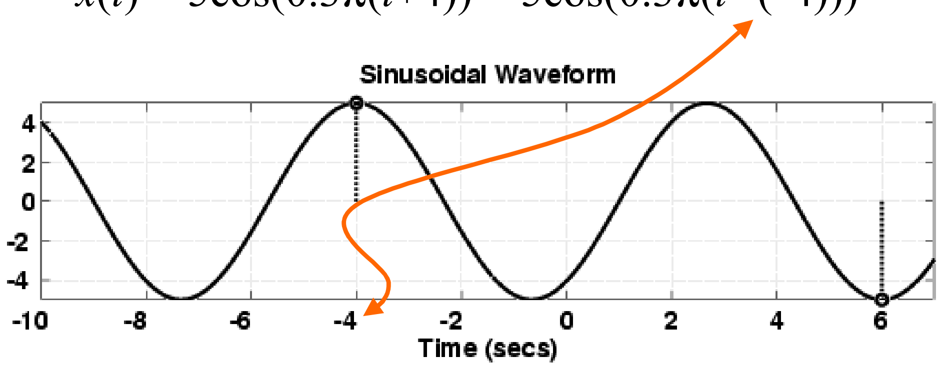
- In a mathematical formula replace t with $t - t_1$
- For example, $x(t-t_1) = \cos(\omega(t-t_1))$
- **Then the t=0 point moves to $t=t_1$**

$$x(t - t_1) = A\cos(\omega(t - t_1))$$

- Peak value of $\cos(\omega(t-t_1))$ is at $t=t_1$

TIME-SHIFTED SINUSOID

$$x(t) = 5\cos(0.3\pi(t+4)) = 5\cos(0.3\pi(t-(-4)))$$



PHASE <--> TIME-SHIFT

- Equate the formulas:

$$A\cos(\omega(t-t_1)) = A\cos(\omega t+\varphi)$$

- and we obtain: $-\omega t_1 = \varphi$
- or, $t_1 = \frac{-\varphi}{\omega}$

EXAMPLE: Phase from Time-Shift

- Frequency : $\omega = 30\pi$
- Phase: $\phi = -0.2\pi$
- What is the time shift?
  - Also called the "time delay"
  - $t_1 = -\phi/\omega = -(-0.2\pi)/30\pi$
  - $t_1 = 1/150$ sec.
  - Note: T = 1/15 sec. (period)

PHASE is AMBIGUOUS

- The cosine signal is periodic
  - Period is 2π

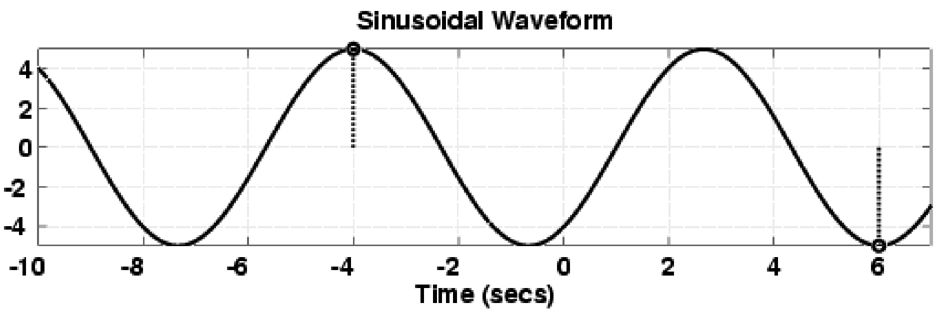
$$A\cos(\omega t+\varphi+2\pi) = A\cos(\omega t+\varphi)$$

  - Thus adding any multiple of 2π leaves x(t) unchanged
  - How much does t_1 change, when phase changes by 2π ? $t_1 = \frac{-\varphi}{\omega} = (\frac{-\varphi}{2\pi})T$

SINUSOID from a PLOT

- Measure the period, T
  - Between peaks or zero crossings
  - Compute frequency: $\omega = 2\pi/T$
- Measure time of peak: t_1
  - Compute phase: $\phi = -\omega t_1$
- Measure height of positive peak: A

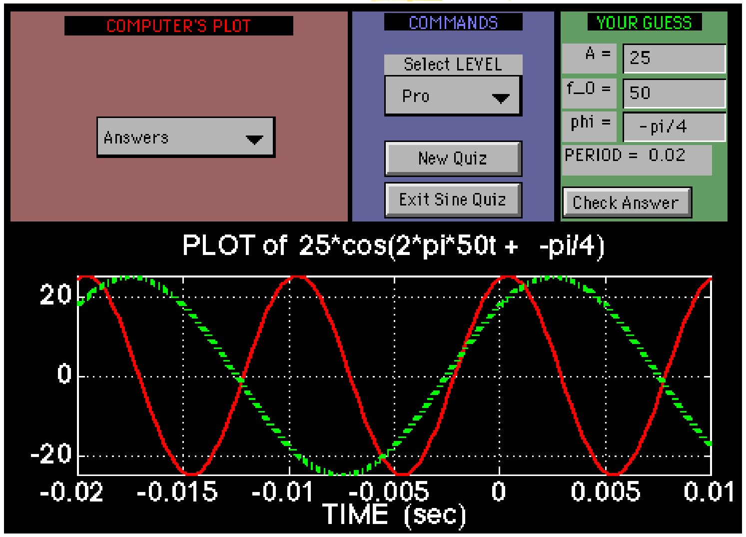
(A, ω, φ) from a PLOT



$T = 10/(1.5) = 20/3$ → $\omega = 2\pi/T = 0.3\pi$

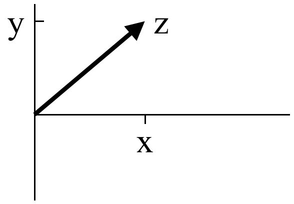
$t_1 = -4$ → $\phi = -(-4)(0.3\pi) = 1.2\pi$

SINE DRILL (MATLAB GUI)



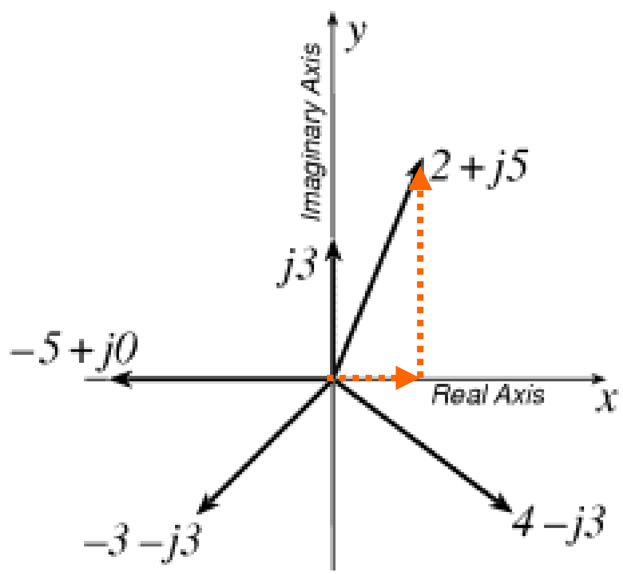
COMPLEX NUMBERS

- To solve: $z^2 = -1$
  - $z = j$
  - Math and Physics use $z = i$
- Complex number: $z = x + jy$



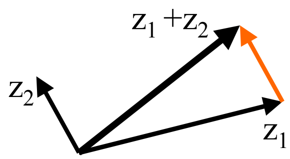
Cartesian coordinate system

EX: COMPLEX NUMBERS



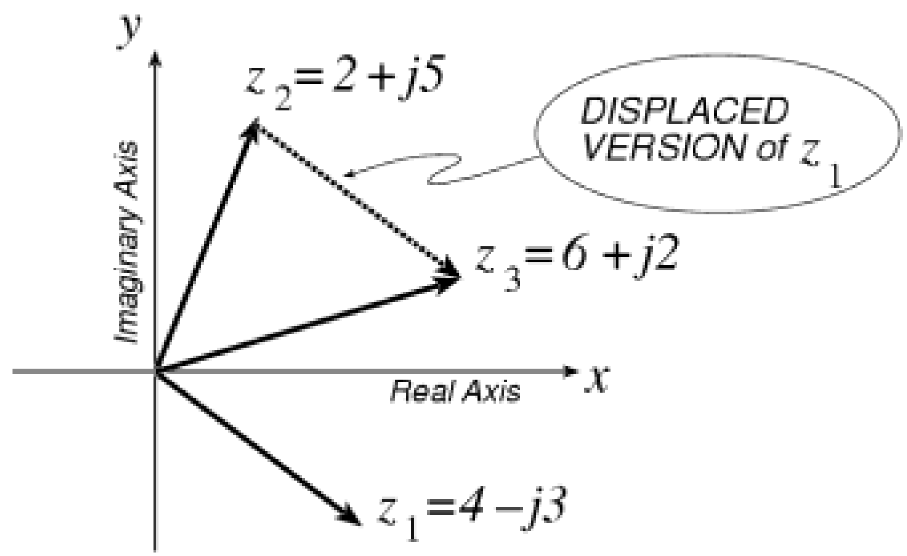
ADD COMPLEX NUMBERS

- **VECTOR Addition is necessary**



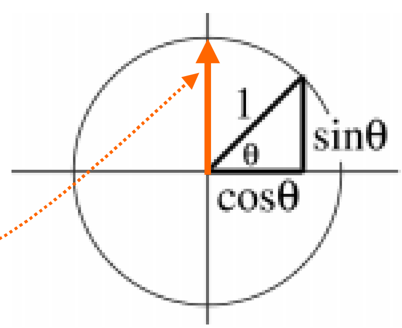
- **Example: z = 4−*j*3, w = 2+*j* 5**
  - **z+w = (4+2) + *j*(−3+5) = 6 +*j*2**
- **Add sinusoids = add complex nums**

EX: COMPLEX ADDITION



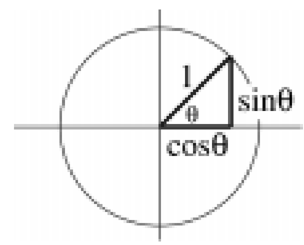
*** POLAR FORM ***

- **Vector Form**
  - **Length =1**
  - **Angle =** θ
- **Common Values**
  - **j has angle of** 0.5π
  - -1 **has angle of** π
  - $-$j **has angle of** 1.5π
  - **or, its angle is** $-0.5\pi = 1.5\pi - 2\pi$



Euler's FORMULA

- **Complex Exponential**
  - **Real part is cosine**
  - **Imaginary part is sine**
  - **Magnitude is one**



$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

$$re^{j\theta} = r\cos(\theta) + jr\sin(\theta)$$

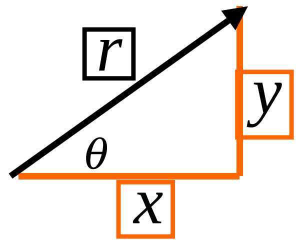
POLAR <--> RECTANGULAR

- Relate (x,y) to (r,θ)

$$z = x + jy = re^{j\theta}$$

$$r^2 = x^2 + y^2$$

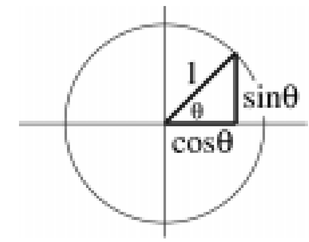
$$\theta = \mathrm{Tan}^{-1}\left(\frac{y}{x}\right)$$



COMPLEX EXPONENTIAL

$$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$$

- Rotating Vector
  - Angle changes vs. time
  - $\theta = \omega t$
  - ex: $\omega = 10\pi$
  - Rotates 0.1π in 0.01 secs



$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

Cos = REAL PART

- Real Part of Euler's:

$$\cos(\omega t) = \Re e\{e^{j\omega t}\}$$

- General Sinusoid

$$x(t) = A\cos(\omega t + \varphi)$$

- So,

$$A\cos(\omega t + \varphi) = \Re e\{Ae^{j(\omega t + \varphi)}\}$$
$$= \Re e\{Ae^{j\varphi}e^{j\omega t}\}$$

COMPLEX AMPLITUDE

- General Sinusoid

$$x(t) = A\cos(\omega t + \varphi) = \Re e\{Ae^{j\varphi}e^{j\omega t}\}$$

- Complex Exponential

$$z(t) = Ze^{j\omega t} \qquad Z = Ae^{j\varphi}$$

- Sinusoid is REAL PART of $e^{j\omega t}$

$$x(t) = \Re e\{z(t)\} = \Re e\{Ze^{j\omega t}\}$$