

EE-2200

Winter-99

LECTURE #2

Complex Exponentials

11-Jan-99

INFORMATION

- **MATLAB: M-T-W-Th 6pm VL-456**
- **LABS start this week**
 - ┆ Attend correct section (in CoC-309)
 - ┆ Get your computer acct ASAP
 - ┆ Verification must be signed during Lab
- **RECITATIONS**
 - ┆ Attend your assigned time

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Introduction to Discrete Systems

Autumn 1998

Lecture Time: M & F 11:05-11:55

Room: W200 Yan Leer (Auditorium)

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Office: E475-C Yan Leer, or 363 GCATT **Phone:** (404) 894-8325

Office Hours: Tu-Th 12:00-2:00p; F 12:00-1:00p, or by appointment

For Recitation instructors and TAs, please refer to the [Course Information and Help](#) page below.



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REMINDERS

- **Web-CT Password:**
 - ┆ SSN(4:8), 4th thru 8th digits of SSN
- **Hard copy of Instructor Verification Sheet**
 - ┆ Get PDF file of Lab#1 from WebCT
 - ┆ Lab #1 is different from the book
- **HW #1 is due Friday (in Lecture)**

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READING ASSIGNMENTS

- This Lecture:
 - Chapter 2, pp. 17–32
- Appendix A: Complex Numbers
- Appendix B: MATLAB
- Next Lecture: finish Chap. 2, pp. 31–43

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LECTURE OBJECTIVES

- Define Sinusoid from a plot
- Relate TIME-SHIFT to PHASE
- Introduce an ABSTRACTION:
 - Complex Numbers **represent** Sinusoids
 - Complex Exponential Signal

$$z(t) = Ze^{j\omega t}$$

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SINUSOIDAL SIGNAL

$$A \cos(\omega t + \varphi)$$

- FREQUENCY ω
 - Radians/sec
 - Hertz (cycles/sec)
 - $\omega = (2\pi)f$
- AMPLITUDE A
 - Magnitude
- PERIOD (in sec) $T = \frac{1}{f} = \frac{2\pi}{\omega}$
- PHASE φ

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PLOT a COSINE SIGNAL

- Given the Formula $5 \cos(0.3\pi t + 1.2\pi)$
- Make a plot
- Formula defines A , ω , and ϕ

$$\begin{aligned} A &= 5 \\ \omega &= 0.3\pi \\ \varphi &= 1.2\pi \end{aligned}$$

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PLOTTING COSINE SIGNAL from the FORMULA

$$5\cos(0.3\pi t + 1.2\pi)$$

- Determine **period**:

$$T = 2\pi / \omega = 2\pi / 0.3\pi = 20/3$$

- Determine a **peak** location by solving

$$(\omega t + \phi) = 0$$

- Peak at $t = -4$**
- Zero crossing is $T/4$ before or after**

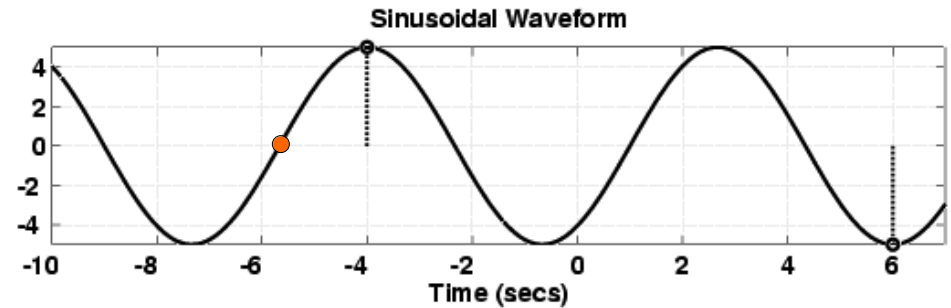
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ANSWER for the PLOT

$$5\cos(0.3\pi t + 1.2\pi)$$



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TIME-SHIFT

- In a mathematical formula replace t with $t - t_1$
- For example, $x(t - t_1) = \cos(\omega(t - t_1))$
- Then the $t=0$ point moves to $t=t_1$

$$x(t - t_1) = A \cos(\omega(t - t_1))$$

- Peak value of $\cos(\omega(t - t_1))$ is at $t=t_1$

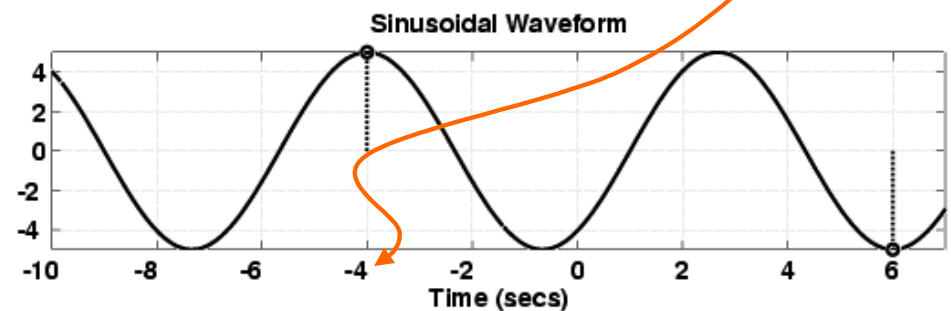
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TIME-SHIFTED SINUSOID

$$x(t) = 5\cos(0.3\pi(t+4)) = 5\cos(0.3\pi(t - (-4)))$$



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PHASE <--> TIME-SHIFT

- Equating the formulas:

$$A \cos(\omega(t - t_1)) = A \cos(\omega t + \phi)$$

- and we obtain: $-\omega t_1 = \phi$

- or, $t_1 = \frac{-\phi}{\omega}$

EXAMPLE: Phase from Time-Shift

- Frequency: $\omega = 30\pi$
- Phase: $\phi = -0.2\pi$
- What is the time shift?
 - Also called the "time delay"
 - $t_1 = -\phi/\omega = -(-0.2\pi)/30\pi$
 - $t_1 = 1/150$ sec.
 - Note: $T = 1/15$ sec. (period)

PHASE is AMBIGUOUS

- The cosine signal is periodic

- Period is 2π

$$A \cos(\omega t + \phi + 2\pi) = A \cos(\omega t + \phi)$$

- Thus adding any multiple of 2π leaves $x(t)$ unchanged

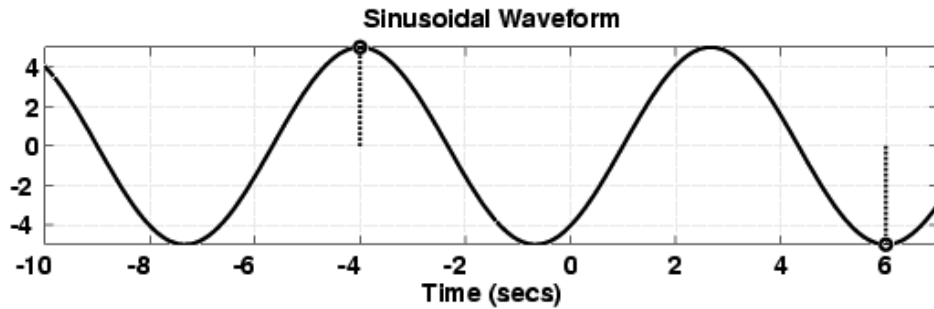
- How much does t_1 change, when phase changes by 2π ?

$$t_1 = \frac{-\phi}{\omega} = \left(\frac{-\phi}{2\pi}\right)T$$

SINUSOID from a PLOT

- Measure the period, T
 - Between peaks or zero crossings
 - Compute frequency: $\omega = 2\pi/T$
- Measure time of peak: t_1
 - Compute phase: $\phi = -\omega t_1$
- Measure height of positive peak: A

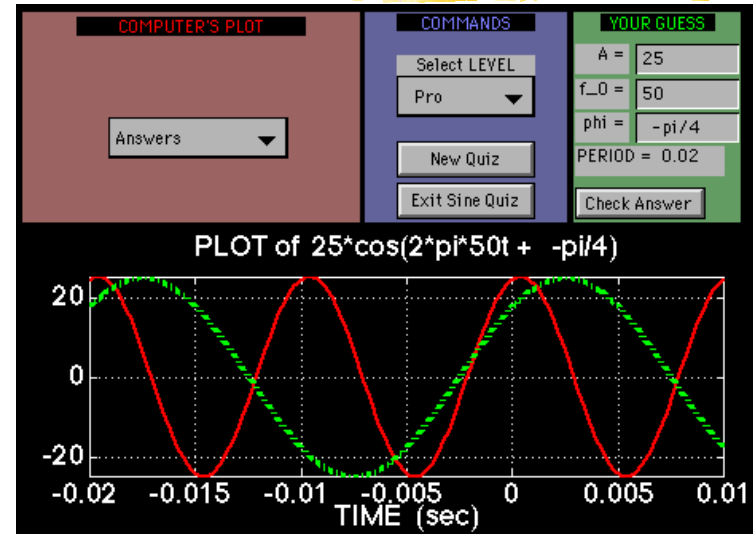
(A, ω , ϕ) from a PLOT



$$T = 10 / (1.5) = 20/3 \quad \longrightarrow \quad \omega = 2\pi / T = 0.3\pi$$

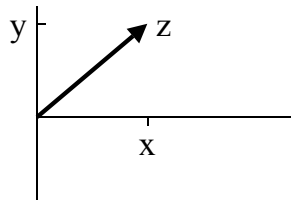
$$t_1 = -4 \quad \longrightarrow \quad \phi = -(-4)(0.3\pi) = 1.2\pi$$

SINE DRILL (MATLAB GUI)



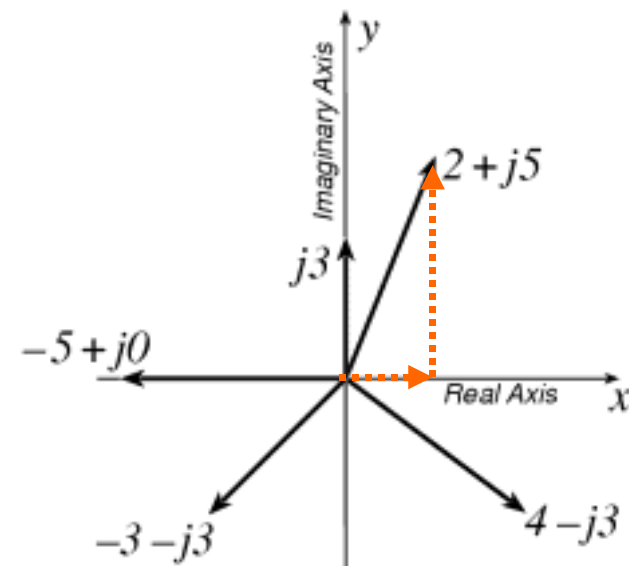
COMPLEX NUMBERS

- To solve: $z^2 = -1$
 - $z = j$
 - Math and Physics use $z = i$
- Complex number: $z = x + jy$



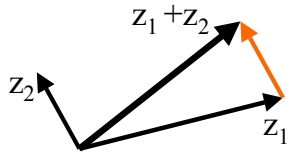
Cartesian coordinate system

EX: COMPLEX NUMBERS



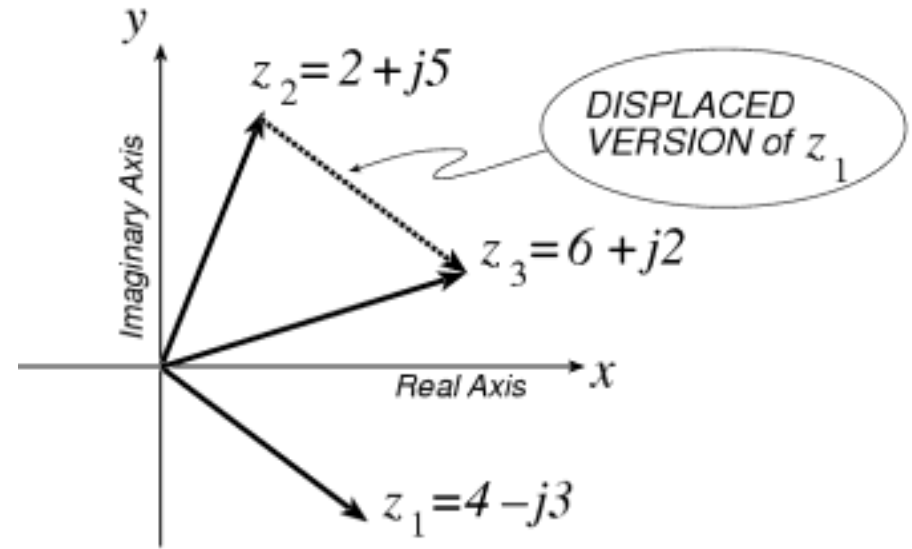
ADD COMPLEX NUMBERS

- VECTOR Addition is necessary



- Example: $z = 4 - j3$, $w = 2 + j5$
 - $z + w = (4 + 2) + j(-3 + 5) = 6 + j2$
- Add sinusoids = add complex nums

EX: COMPLEX ADDITION



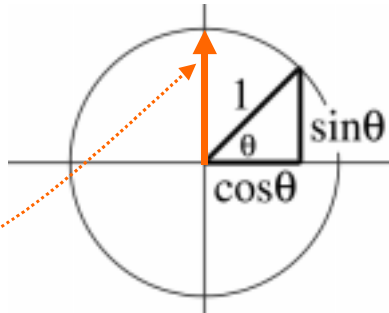
*** POLAR FORM ***

- Vector Form

- Length = 1
- Angle = θ

- Common Values

- j has angle of 0.5π
- -1 has angle of π
- $-j$ has angle of 1.5π
- or, its angle is $-0.5\pi = 1.5\pi - 2\pi$



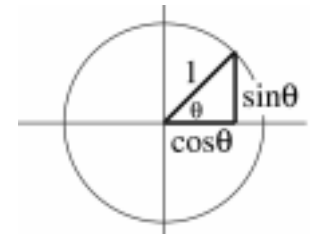
Euler's FORMULA

- Complex Exponential

- Real part is cosine
- Imaginary part is sine
- Magnitude is one

$$e^{j\theta} = \cos(\theta) + j \sin(\theta)$$

$$re^{j\theta} = r \cos(\theta) + jr \sin(\theta)$$

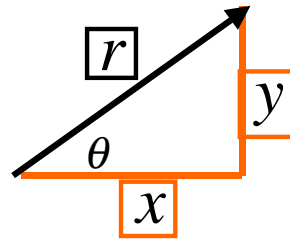


POLAR <--> RECTANGULAR

- Relate (x,y) to (r,θ)

$$z = x + jy = re^{j\theta}$$

$$r^2 = x^2 + y^2$$
$$\theta = \text{Tan}^{-1}\left(\frac{y}{x}\right)$$



COMPLEX EXPONENTIAL

$$e^{j\omega t} = \cos(\omega t) + j \sin(\omega t)$$

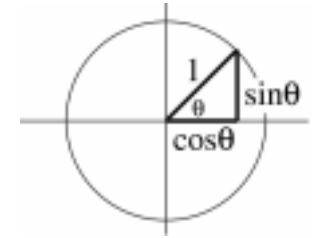
- Rotating Vector

- Angle changes vs. time

- $\theta = \omega t$

- ex: $\omega = 10\pi$

- Rotates 0.1π in **0.01** secs



$$e^{j\theta} = \cos(\theta) + j \sin(\theta)$$

Cos = REAL PART

- Real Part of Euler's:

$$\cos(\omega t) = \Re\{e^{j\omega t}\}$$

- General Sinusoid

$$x(t) = A \cos(\omega t + \varphi)$$

- So,

$$A \cos(\omega t + \varphi) = \Re\{Ae^{j(\omega t + \varphi)}\}$$
$$= \Re\{Ae^{j\varphi} e^{j\omega t}\}$$

COMPLEX AMPLITUDE

- General Sinusoid

$$x(t) = A \cos(\omega t + \varphi) = \Re\{Ae^{j\varphi} e^{j\omega t}\}$$

- Complex Exponential

$$z(t) = Ze^{j\omega t} \quad Z = Ae^{j\varphi}$$

- Sinusoid is REAL PART of $e^{j\omega t}$

$$x(t) = \Re\{z(t)\} = \Re\{Ze^{j\omega t}\}$$