EE-2200 Winter-99

Lecture 10

Linearity & Time-Invariance

15-Feb-99

Info: Web-CT, Lab, HW

- Lab Quiz this week
- Quiz #2 on 1–March (Monday)
- Prob Set #5 due FRIDAY
- Lab #6 on Filtering
  - Echoes & Deconvolution

READING ASSIGNMENTS

- This Lecture:
  - Chapter 5, pp. 133–152
- Other Reading:
  - Recitation: Ch. 5, pp. 127–133, 142–146
    - CONVOLUTION
  - Next Lecture: Chapter 6, start

LECTURE OBJECTIVES

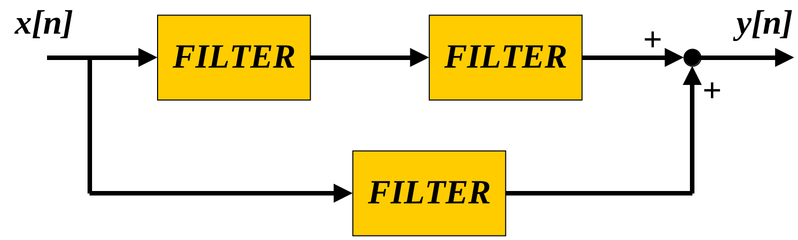
- BLOCK DIAGRAM REPRESENTATION
  - Components for Hardware
  - Connect Simple Filters Together to Build More Complicated Systems
- GENERAL PROPERTIES of FILTERS
  - LINEARITY
  - TIME–INVARIANCE
  - ==> CONVOLUTION

LTI SYSTEMS

DIGITAL FILTERING

- CONCENTRATE on the FILTER (DSP)
- DISCRETE–TIME SIGNALS
  - FUNCTIONS of n, the "time index"
  - INPUT x[n]
  - OUTPUT y[n]

BUILDING BLOCKS



- BUILD UP COMPLICATED FILTERS
  - FROM SIMPLE MODULES
  - Ex: FILTER MODULE MIGHT BE 3–pt FIR

GENERAL FIR FILTER

- FILTER COEFFICIENTS $\{b_k\}$
  - DEFINE THE FILTER

$$y[n] = \sum_{k=0}^{M} b_k\, x[n-k]$$

  - For example, $\{b_k\} = \{3, -1, 2, 1\}$

$$y[n] = \sum_{k=0}^{3} b_k x[n-k]$$

$$= 3x[n] - x[n-1] + 2x[n-2] + x[n-3]$$

MATLAB for FIR FILTER

- `yy = filter(bb,1,xx)`
  - VECTOR bb contains Filter Coefficients
  - DSP–First: `yy = firfilt(bb,xx)`
- FILTER COEFFICIENTS $\{b_k\}$

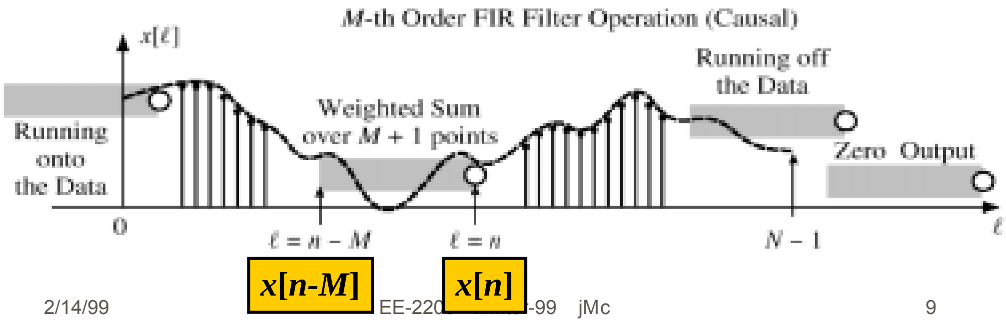
Conv2( ) *for images*

$$y[n] = \sum_{k=0}^{M} b_k\, x[n-k]$$

GENERAL FIR FILTER

- SLIDE a Length-L WINDOW over x[n]

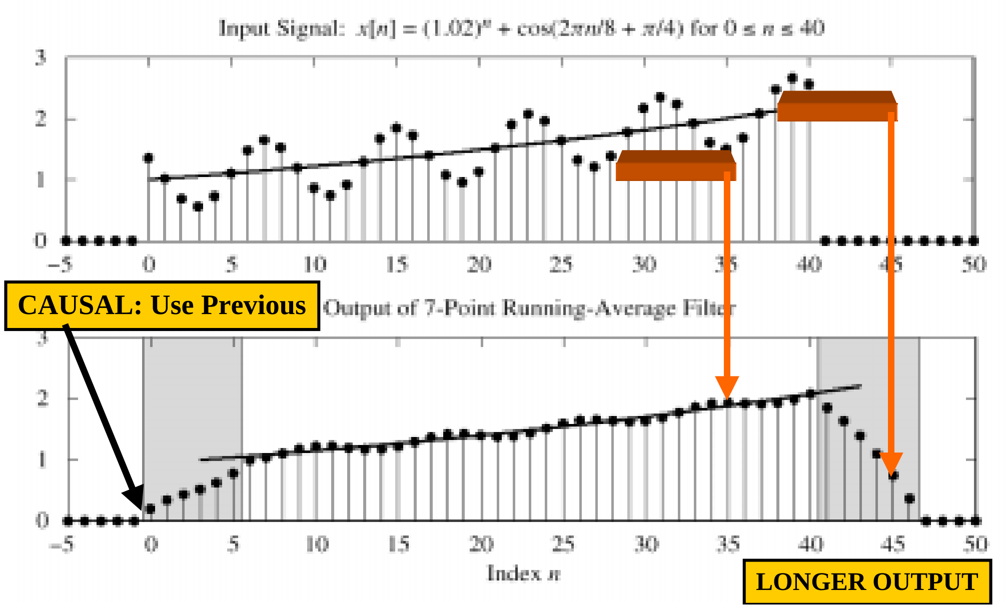
$$y[n] = \sum_{k=0}^{M} b_k\, x[n-k]$$



FILTERING EXAMPLE

- 7-point AVERAGER $\quad y_7[n] = \frac{1}{7}\left(\sum_{k=0}^{6} x[n-k]\right)$
  - Removes cosine
    - By making its amplitdue (A) smaller
- 3-point AVERAGER $\quad y_3[n] = \frac{1}{3}\left(\sum_{k=0}^{2} x[n-k]\right)$
  - Changes A slightly

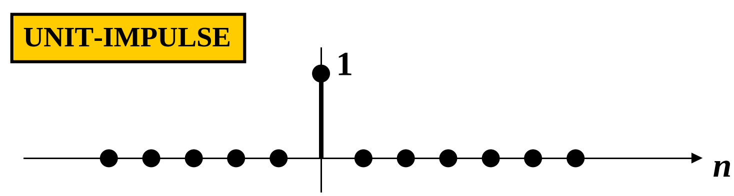
7-pt FIR EXAMPLE (AVG)



SPECIAL INPUT SIGNALS

- x[n] = SINUSOID **FREQUENCY RESPONSE**
- x[n] has only one NON-ZERO VALUE

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$



UNIT IMPULSE SIGNAL δ[n]

n	...	−2	−1	0	1	2	3	4	5	6	...
$\delta[n]$	0	0	0	1	0	0	0	0	0	0	0
$\delta[n-3]$	0	0	0	0	0	0	1	0	0	0	0

n=3

δ[n] is NON-ZERO When its argument is equal to ZERO

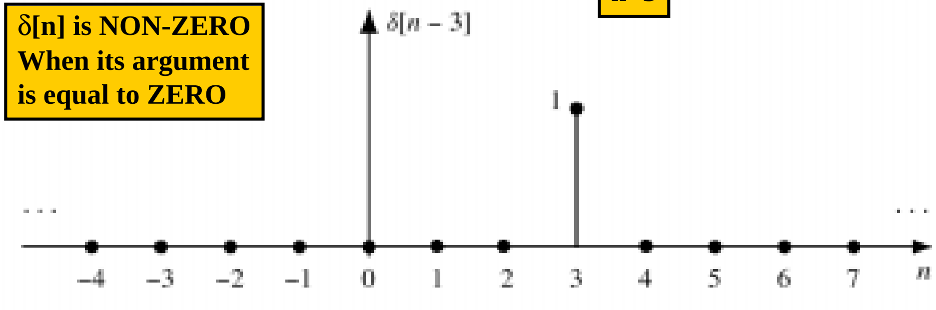
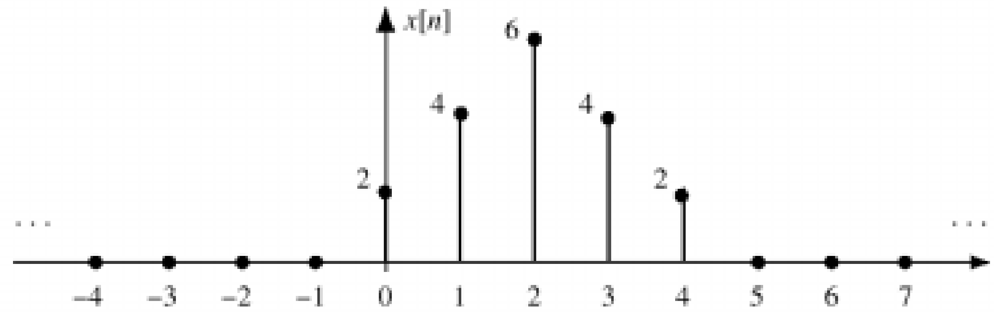


Figure 5.7 Shifted impulse sequence, $\delta[n-3]$.

MATH FORMULA for x[n]

- Use SHIFTED IMPULSES to write x[n]

$$x[n] = 2\delta[n] + 4\delta[n-1] + 6\delta[n-2] + 4\delta[n-3] + 2\delta[n-4]$$



SUM of SHIFTED IMPULSES

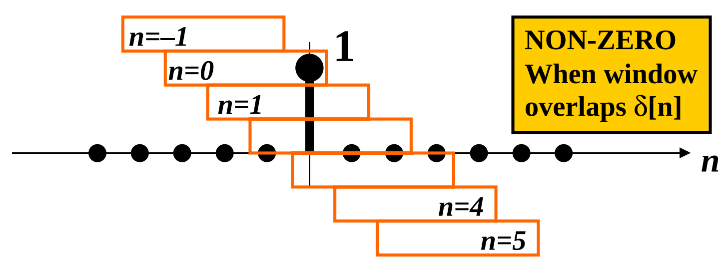
n	...	−2	−1	0	1	2	3	4	5	6	...
$2\delta[n]$	0	0	0	2	0	0	0	0	0	0	0
$4\delta[n-1]$	0	0	0	0	4	0	0	0	0	0	0
$6\delta[n-2]$	0	0	0	0	0	6	0	0	0	0	0
$4\delta[n-3]$	0	0	0	0	0	0	4	0	0	0	0
$2\delta[n-4]$	0	0	0	0	0	0	0	2	0	0	0
$x[n]$	0	0	0	2	4	6	4	2	0	0	0

$$x[n] = \sum_k x[k]\delta[n-k]$$ ← This formula ALWAYS works

$$= \ldots + x[-1]\delta[n+1] + x[0]\delta[n] + x[1]\delta[n-1] + \ldots \quad (5.3.6)$$

4-pt AVERAGER

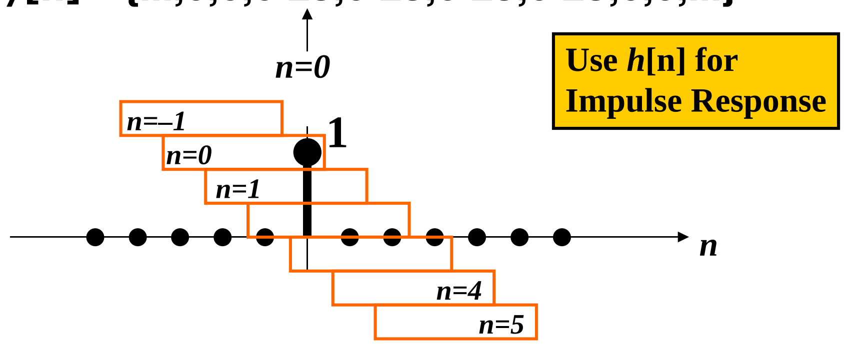
- CAUSAL SYSTEM: USE PAST VALUES
- INPUT = UNIT IMPULSE SIGNAL = δ[n]
- OUTPUT is called "IMPULSE RESPONSE"



NON-ZERO When window overlaps δ[n]

4-pt Avg Impulse Response

- $y[n] = 0.25(x[n]+x[n-1]+x[n-2]+x[n-3])$
- "READS OUT" the FILTER COEFFICIENTS
- $y[n] = \{\ldots,0,0,0.25,0.25,0.25,0.25,0,0,\ldots\}$



FIR IMPULSE RESPONSE

- Convolution = Filter Definition
  - Filter Coeffs = Impulse Response

n	$n<0$	0	1	2	3	...	M	$M+1$	$n>M+1$
$x[n]=\delta[n]$	0	1	0	0	0	0	0	0	0
$y[n]=h[n]$	0	b_0	b_1	b_2	b_3	...	b_M	0	0

$$y[n] = \sum_{k=0}^{M} h[k]\, x[n-k]$$

HARDWARE STRUCTURES

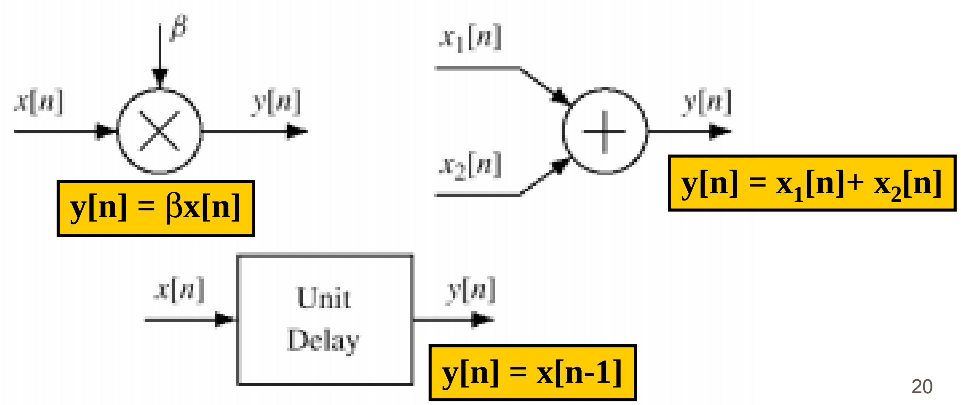
$$y[n] = \sum_{k=0}^{M} b_k\, x[n-k]$$

- INTERNAL STRUCTURE of "FILTER"
  - WHAT COMPONENTS ARE NEEDED?
  - HOW DO WE "HOOK" THEM TOGETHER?
- SIGNAL FLOW GRAPH NOTATION

HARDWARE ATOMS

- Add, Multiply & Store

$$y[n] = \sum_{k=0}^{M} b_k\, x[n-k]$$



FIR STRUCTURE

- Direct Form

$$y[n] = \sum_{k=0}^{M} b_k\, x[n-k]$$

SIGNAL FLOW GRAPH

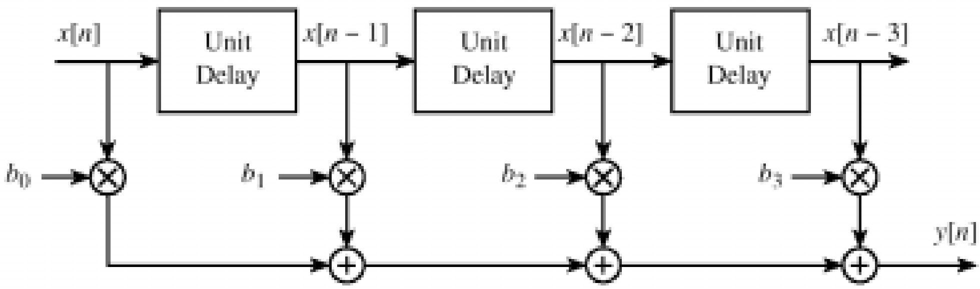


Figure 5.13 Block-diagram structure for the Mth order FIR filter.

ALTERNATE STRUCTURE

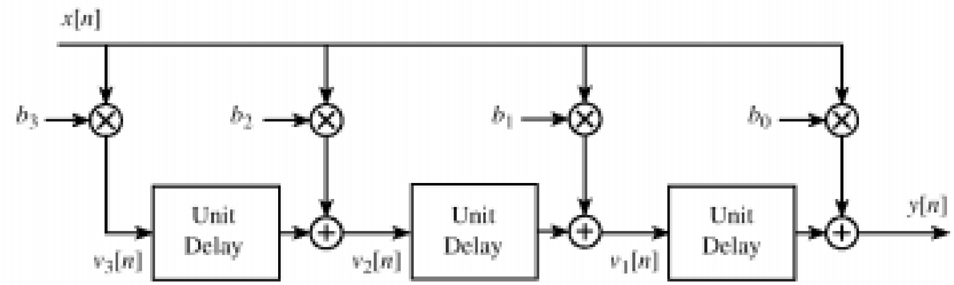


Figure 5.14 Transposed form block diagram structure for the Mth order FIR filter.

$$y[n] = b_0 x[n] + v_1[n-1]$$
$$v_1[n] = b_1 x[n] + v_2[n-1]$$
$$v_2[n] = b_2 x[n] + v_3[n-1]$$
$$v_3[n] = b_3 x[n]$$

DIFFERENT COMPUTATION

SYSTEM PROPERTIES

- TIME–INVARIANCE
- LINEARITY
- CAUSALITY
  - “No output prior to input”

TIME-INVARIANCE

- IDEA:
  - “Time–Shifting the input will cause the same time–shift in the output”
- EQUIVALENTLY,
  - We can prove that
    - The time origin (n=0) is arbitrary

TESTING Time-Invariance

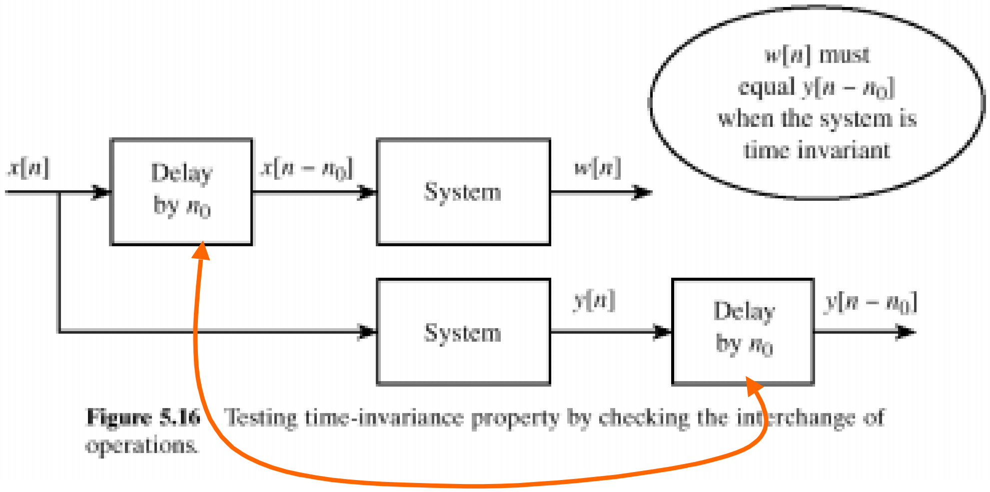


Figure 5.16 Testing time-invariance property by checking the interchange of operations.

LINEAR SYSTEM

- SCALING
  - "Doubling x[n] will double y[n]"
- SUPERPOSITION:
  - "Adding two inputs gives an output that is the sum of the individual outputs"

TESTING LINEARITY

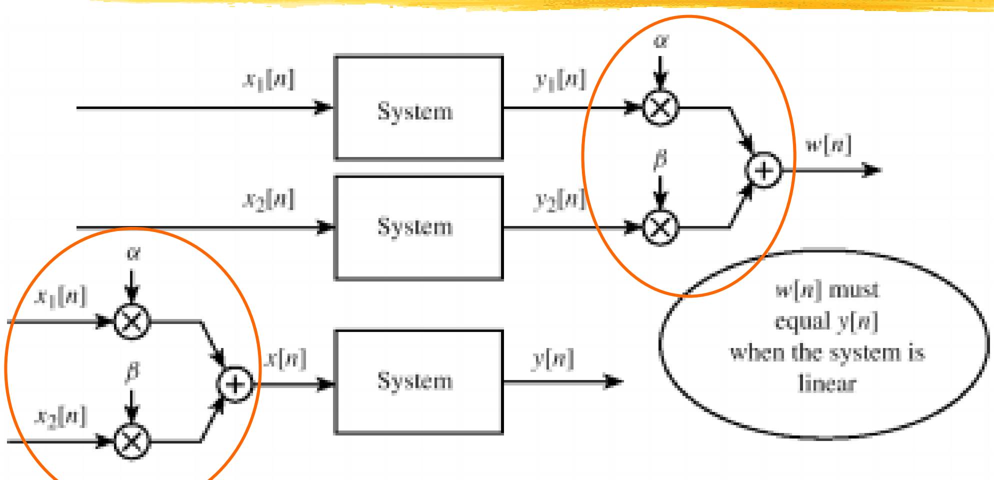


Figure 5.17 Testing linearity by checking the interchange of operations.

LTI SYSTEMS

- LTI: **L**inear & **T**ime–**I**nvariant
- COMPLETELY CHARACTERIZED by:
  - CONVOLUTION: y[n] = x[n]*h[n]
  - IMPULSE RESPONSE h[n]
    - The "rule" can be re-written as convolution
- FIR Example: h[n] is same as b_k

LTI: Convolution

- Output = Convolution of x[n] & h[n]
  - NOTATION: y[n] = x[n]*h[n]
  - Here is the FIR case:

$$y[n] = \sum_{k=0}^{M} h[k]\, x[n-k]$$

FINITE LIMITS

FINITE LIMITS

Same as b_k

CONVOLUTION Example

n	0	1	2	3	4	5	6	7	8
x[n]	2	4	6	4	2				
h[n]	3	-1	2	1					
h[0]x[n-0]	6	12	18	12	6				
h[1]x[n-1]		-2	-4	-6	-4	-2			
h[2]x[n-2]			4	8	12	8	4		
h[3]x[n-3]				2	4	6	4	2	
y[n]	6	10	18	16	18	12	8	2	

CASCADE SYSTEMS

- Does the order of S_1 & S_2 matter?
  - NO, LTI SYSTEMS can be rearranged !!!
  - WHAT ARE THE FILTER COEFFS? $\{b_k\}$

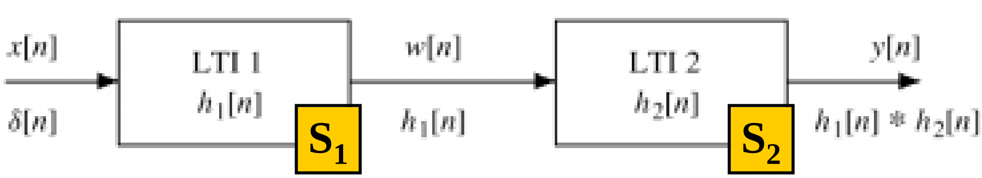


Figure 5.19 A Cascade of Two LTI Systems.

CASCADE EQUIVALENT

- Find "overall" h[n] for a cascade ?

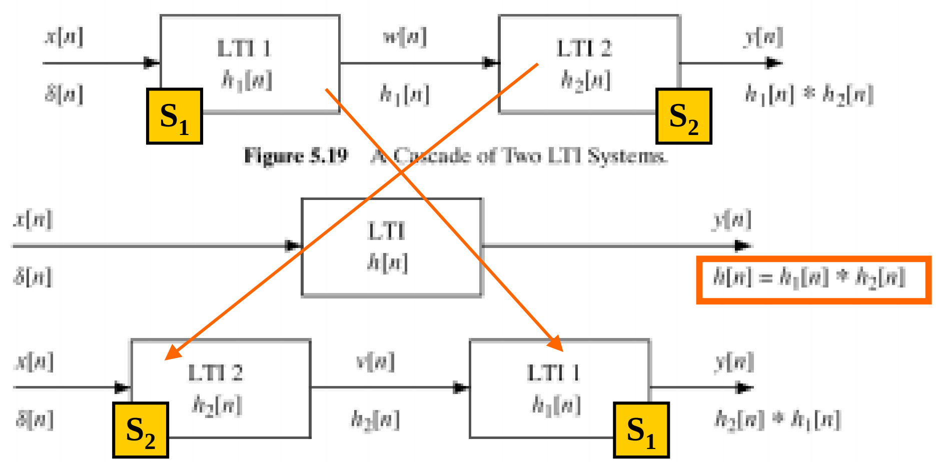


Figure 5.19 A Cascade of Two LTI Systems.

Figure 5.20 Switching the order of cascaded LTI systems.