Managing Morning Commute Congestion with a Tradable Credit Scheme Under Commuter Heterogeneity and Market Loss Aversion Behavior

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Abstract

This study investigates the impact of a tradable credit scheme (TCS) on managing morning commute congestion by considering commuters’ value-of-time and schedule delay heterogeneities, and loss aversion behavior in purchasing credits. It illustrates that total value of traded credits and credit price approach zero as commuters’ loss sensitivity increases. Further, the initial credit allocation method can impact the credit price and commuters’ departure rate. The study insights show that if commuters’ loss sensitivity is not considered, the system-optimal TCS design can lead to a less effective scheme to minimize the total system travel cost.

Keywords: Single bottleneck model; Tradable credit scheme; Heterogeneous commuters; Market loss aversion; System optimal.
1. Introduction

The severity of traffic congestion has significantly increased in metropolitan areas in recent decades. Commuters experience significant traffic delays during their morning and evening peak-period commutes. In the United States, they incurred an extra thirty eight hours of delay during peak periods annually in 2011 compared to 1982 (Schrank, Eisele, and Lomax 2012). Market-based instruments are proposed as efficient mechanisms to manage traffic demand. In the context of traffic congestion mitigation, they can be classified into price-based and quantity-based instruments.

A price-based instrument, widely known as congestion pricing, was first proposed by Pigou (1920). It has been implemented in different forms, ranging from congestion pricing on the San Francisco-Oakland Bay Bridge to Singapore’s area licensing scheme. Vickrey (1969) developed the simple deterministic bottleneck model to describe traffic congestion during the morning peak period where commuters pass a route segment with fixed capacity (bottleneck). In Vickrey’s model, commuters choose their departure times to reduce their own travel costs including schedule delay and travel time costs. Commuters cannot reduce their travel costs by unilaterally changing their departure times at user equilibrium (UE). Arnott, de Palma, and Lindsey (1990) proposed a system optimal (SO) time-varying tolling scheme where the total system travel cost, which includes the costs of schedule delay and travel time, is minimized during the morning peak period.

Several efforts have sought to improve the realism of Vickrey’s and Arnott’s models by relaxing assumptions, in terms of multi-step tolling (Laih 1994) and heterogeneous commuters in terms of schedule delay penalty, travel time penalty and desired arrival time (Newell 1987). These studies can be classified into two groups. The first group develops continuous-time analytical formulations to determine the departure rates of commuters under the UE and SO conditions. For example, Vickrey (1973) extended his earlier work to derive the equilibrium condition under road pricing where value of time and travel and schedule delay cost parameters of commuters have a fixed ratio. Daganzo (1985) investigated managing morning commute congestion with only two groups. Arnott, de Palma, and Lindsey (1994) examined the equilibrium condition and commuters’ welfare for special cases of heterogeneity in which commuters have different
absolute values of time but same ratio of late arrival to early arrival penalties and same preferred arrival times. Lindsey (2004) demonstrated the existence of equilibrium solution with heterogeneity of commuters without providing an algorithm to obtain the equilibrium solution. Xiao, Qian, and Zhang (2011) investigated the effect of single-step tolling on morning departure rates where commuters have different values of time and schedule delay penalty. However, the desired arrival times and ratio of schedule delay penalty to value of time is assumed to be constant across travelers. Van den Berg and Verhoef (2011) analyzed the efficiency of congestion pricing in managing morning commute congestion under the assumption that commuters have identical desired arrival time and continuous distributions of value of time and schedule delay penalty. While the ratio of schedule delay penalty to value of time is varied across travelers, they assumed that the ratio of late arrival penalty to early arrival penalty is constant across travelers. In another study (Van den Berg and Verhoef 2011), they investigated the effect of congestion pricing by assuming a continuous distribution of value of time across commuters. Since commuters are heterogeneous in the real world, assumptions such as constant late arrival penalty to early arrival penalty or desired arrival time across commuters, are limiting in the real world.

The second group investigates the departure rates of commuters under the UE and SO conditions using the single bottleneck model in a discrete time setting. Ramadurai et al. (2010) proposed a linear complementarity formulation for the single bottleneck model without a congestion pricing strategy. They prove the solution existence and uniqueness assuming both homogeneous and heterogeneous commuters in terms of schedule delay penalty, travel time penalty and desired arrival time. An advantage is its ability to solve for heterogeneous commuter groups without restrictive assumptions, unlike previous studies that use a continuous time setting. Along this line of research, Doan, Ukkusuri, and Han (2011) constructed a linear complementarity formulation to investigate the effect of a time-varying tolling scheme on commuter’s departure rate. They developed a linear program to obtain SO departure rates that minimize the total system travel cost including schedule delay and travel time costs.

Despite several studies on congestion pricing, it has been sparsely implemented in practice due to public resistance. For example, an official petition in 2007 collected 1.8
million signatures in the U.K. to end plans for a national charging scheme (de Palma and Lindsey 2011). Travelers tend to view toll as another de facto flat tax imposed by the central authority. Further, if no compensation is provided under congestion pricing, it benefits only those who weigh the reduced delay more than the paid toll. Thereby, a significant portion of travelers may be worse off under congestion pricing (Hau 1998). Hence, congestion pricing is perceived as an inequitable policy. These issues illustrate challenges with deploying congestion pricing schemes in practice. To overcome the aforementioned issues, some studies (Guo and Yang 2010) suggest rewarding travelers either using the revenue generated through tolls or subsidy from the central authority. However, even if the revenue of congestion pricing is redistributed, the central authority is the sole toll collector, and its claim of revenue-neutrality is difficult to verify and believe by the public (Yang and Wang 2011).

Quantity-based instruments, such as a tradable credit scheme (TCS), have recently been suggested to resolve the issues inherent to price-based instruments. They have been extensively investigated for air pollution control by the European Union in energy-intensive sectors such as oil, paper, and power generation (Böhringer, Rutherford, and Tol 2009). As a type of quantity-based instrument, Akamatsu (2007) proposes the notion of time- and link-specific tradable permits to manage traffic congestion in a road bottleneck during the pre-specified peak period. Since the permits are time-specific, this scheme is able to eliminate traffic congestion by issuing the number of permits equivalent to the bottleneck capacity. Wada and Akamatsu (2013) propose a new auction mechanism for the tradable network permit system developed by Akamatsu (2007). In this system, the central authority sells the permits in an auction market to travelers. It can be considered as a variant of congestion pricing in which link tolls are determined endogenously. Hence, it cannot resolve the issue of transferring wealth from travelers to the central authority.

Yang and Wang (2011) propose the tradable credit scheme where a central authority determines the TCS parameters including credit allocation and charging schemes. The credit allocation scheme is characterized by the total credit endowment and the method of credit allocation. The credit charging scheme, i.e., link tolls to charge travelers, is determined by the central authority. Thereby, travelers need to pay credits for traveling
in the network. Travelers can also trade credits among themselves in the market. It is demonstrated that the traffic and market equilibrium conditions depend on the total number of allocated credits, and are hence independent of the credit allocation scheme. While any congestion pricing has a TCS mirror that is equally effective in managing congestion (Nie and Yin 2013), there are potentially fewer social objections to TCS due to two reasons. First, transfer of wealth does not exist between travelers and central authority. Second, travelers with less value of time are directly compensated for higher experienced travel time by their ability to sell credits to those with higher value of time. Hence, tradable credit scheme is a promising equitable alternative to congestion pricing to reduce public resistance. The comprehensive review of the literature on TCS application, to manage parking, mobility and bottleneck congestion, is provided by Grant-Muller and Xu (2014). They show that TCS can perform better than congestion pricing especially in terms of cost and benefit distribution. Similarly, Dogterom, Ettema, and Dijst (2016) show that the further car usage reduction under TCS can be achieved compared to congestion pricing.

Numerous efforts have sought to investigate the effects of TCS on managing morning commute congestion. Tian, Yang, and Huang (2013) investigate the modal split of morning commuters between auto and transit under an optimal TCS implementation. Nie and Yin (2013) propose a credit allocation scheme for homogeneous commuters, in which the horizon of interest is divided into off-peak and peak periods. Commuters are rewarded for traveling during the off-peak period, and are charged credits by the central authority for traveling during the morning peak period. Xiao, Qian, and Zhang (2013) examine the effects of the initial credit allocation scheme on equity and welfare aspects of TCS, in the context of morning commute congestion. While various studies investigate TCSs for managing bottleneck congestion, they address only the homogeneous case or are limited to special heterogeneous cases (such as heterogeneity in travel time penalty). For example, while Nie and Yin (2013) and Tian, Yang, and Huang (2013) focus on the SO design of TCS, the former assumes commuter homogeneity and the latter considers heterogeneity only in terms of travel time penalty with a continuous cumulative distribution function. Similar to Tian, Yang, and Huang (2013), Xiao, Qian, and Zhang (2013) assume commuter heterogeneity only in terms of travel time penalty, with a
continuous cumulative distribution function to investigate the UE condition under a given TCS, and the SO design of TCS. However, for practical realism, it is essential to factor commuter heterogeneity in terms of schedule delay penalty, travel time penalty and desired arrival time to investigate TCSs for managing bottleneck congestion. It increases the ability of the central authority to exert greater control in managing morning commute congestion by accounting for the behavioral characteristics of commuters (such as value of time) because commuters are more likely to comply with the TCS in this case. Our study results also illustrate that considering the heterogeneity of commuters increases the efficiency of the TCS.

This study focuses on managing morning commute congestion during the peak period using a TCS. It integrates concepts across the domains of TCS and managing morning commute congestion. In contrast to previous studies on managing morning commute congestion using TCS, this integration allows for the determination of UE and SO conditions by considering commuter heterogeneity. These conditions are formulated in a discrete time setting using complementarity constraints. This approach has two advantages: (1) it offers a framework to study the existence and uniqueness of the UE solution under TCS, and (2) the equilibria can be computed efficiently using Lemke’s algorithm (Lemke 1965). As empirical studies suggest that commuters have different sensitivities to travel and schedule delay costs based on socioeconomic characteristics (Small 1982), we consider commuter heterogeneity in terms of travel time penalty, schedule delay penalty and desired arrival time to understand the effects of TCS implementation in a realistic setting. The previous studies using single bottleneck model in discrete time setting (Ramadurai et al. 2010; Doan, Ukkusuri, and Han 2011) investigated the effect of commuters’ heterogeneity on the UE and SO conditions, but did not consider TCS implementation. While Ramadurai et al. (2010) do not propose a congestion pricing strategy, Doan, Ukkusuri, and Han (2011) use a congestion pricing strategy to address morning commute congestion. However, their strategy can lead to higher generalized travel costs for commuters compared to a no-pricing strategy. By contrast, we show here that a pareto-improving solution can be achieved through the TCS design which makes every commuter better off. Consequently, public acceptance of TCS as a congestion mitigation strategy can be higher compared to congestion pricing.
In the proposed TCS, commuters are grouped based on their value of time, schedule delay penalty and desired arrival time. The central authority implements a group-specific credit allocation scheme where a predetermined number of credits are allocated free of cost to each commuter of a group, based on a predetermined credit allocation method by group. Then, the central authority determines a time-varying group-specific credit charging scheme in which commuters of a group use a certain number of credits based on their departure times and group index. It is assumed that the total number of credits are sufficient to address the commuters’ credit needs. Given the credit allocation and charging schemes, commuters can trade credits in the market with negligible transaction fees. They either sell credits to gain monetary benefit or purchase credits in the market by incurring monetary loss, based on their credit endowment and credits required to fulfill their travel needs.

A commuter’s departure time choice for the morning peak period depends on the travel disutility, which in this study includes travel time cost, schedule delay cost and credit consumption disutility. Commuters are assumed to have full information on the time-varying group-specific credit charging scheme for the morning peak period; hence, their departure time choice is assumed to be riskless. However, departure time choice involves commuter behavior towards monetary loss and gain associated with trading credits in the market. Kahneman and Tversky (1979) propose that people perceive the payoffs as gain or loss by benchmarking to some reference points. They demonstrate the effect of loss aversion behavior in the decision-making process when people treat losses and gains asymmetrically (Tversky and Kahneman 1991). Traveler loss-aversion behavior has been studied using cumulative prospect theory (Tversky and Kahneman 1992) to model route choices of travelers under travel time uncertainty (Gao, Frejinger, and Ben-Akiva 2010). In the context of managing traffic congestion with TCS, Bao et al. (2014) formulate a reference-dependent UE model using the value function (Tversky and Kahneman 1991) to study the impact of travelers’ loss aversion behavior in route choice in the static context. Travelers choose to trade credits in the market by paying transaction fees. They compare the charged credits with a reference point, which is assumed to be their credit endowments. So, they perceive purchasing credits as loss and selling credits as gain. In this riskless decision-making process of trading credits, the market loss
aversion behavior of travelers is considered which implies that loss looms larger than gain for them. However, they neglect the impact of the credit allocation method under TCS, and do not analytically illustrate the relationship between credit price and market loss aversion behavior of commuters. Our study follows a similar approach to model the gain and loss perception of commuters in trading credits in the market where it is assumed that the commuters perceive travel time and scheduled delay costs as a pure loss. In other words, commuters consider the absolute value of time and scheduled delay costs in the departure time choice.

The contributions of this study are fivefold. We address many aspects beyond those of papers that address single bottleneck models and others that focus on TCS, by integrating across these domains. First, this is the first study to consider the heterogeneity of commuters, in terms of value of time, schedule delay penalty and desired arrival time, in deriving the equilibrium departure rates and credit price of commuters under a TCS. This increases the TCS practicality and enables the central authority to understand the market and commuter behaviors in practice. Second, this study shows that the uniqueness of equilibrium credit price depends on the uniqueness of departure rates. The equilibrium credit price uniqueness indicates a healthy market. Without the price uniqueness, credits have to be purchased at uncertain prices. As credit price increases, commuters’ travel choices reduce (Miralinaghi and Peeta 2016; Miralinaghi and Peeta 2018). Third, this study analytically demonstrates the impacts of commuters’ loss aversion on the equilibrium credit price. The TCS may become inactive if the market loss aversion behavior of commuters is significant. This would reduce the ability of the central authority to manage morning commute congestion. Fourth, this study investigates the effect of credit allocation method on the equilibrium credit price when loss aversion behavior is considered. The numerical results demonstrate the effect of credit allocation method on the equilibrium credit price and departure rates. Finally, the SO TCS design is developed as a benchmark for planners. The numerical results demonstrate that the central authority can achieve Pareto-improving SO TCS design, which makes everybody better off through the appropriate design of group-specific credit allocation schemes.

This study first formulates the reference-dependent UE condition of commuters during the morning peak period as a linear complementarity problem (LCP) in a discrete
time setting, where the travel disutility of commuters includes the schedule delay, travel time and credit consumption disutilities. Second, this study investigates the solution existence and uniqueness of the departure rates and credit price. Third, we investigate the effect of market loss aversion behavior of commuters on the equilibrium credit price. Fourth, we demonstrate the effect of credit allocation method on equilibrium credit price and departure rates. Finally, the SO TCS design is developed to minimize the total travel cost, which includes the costs of schedule delay and travel time, while incorporating the heterogeneity of commuters. The proposed TCS design determines the SO group-specific credit allocation and time-varying group-specific credit charging schemes by applying linear programming duality.

The remainder of the paper is organized as follows. Section 2 includes preliminaries for the single bottleneck model with heterogeneous commuters in a discrete time setting. Section 3 formulates the reference-dependent heterogeneous single bottleneck model in discrete time with a TCS. Also, it illustrates the proofs for solution existence and uniqueness. Section 4 develops a primal-dual method to determine the SO TCS design. Section 5 provides the results of computational experiments. Section 6 discusses some concluding comments.

2. Preliminaries

As preliminaries, we provide a brief description of the analysis of the morning commute congestion in a discrete time setting, proposed in Ramadurai et al. (2010) and later extended in Doan, Ukkusuri, and Han (2011). Consider a highway with a bottleneck connecting the residential area and workplace, where commuters travel from home to workplace during the morning peak period. Each commuter is assumed to travel in his/her own vehicle to his/her workplace. A certain number of commuters pass the segment of highway with limited deterministic capacity, \(s\), in a discrete peak period with \(T\) time intervals indexed by \(t = 0, ..., T - 1\). The set of time intervals is denoted by \(\Gamma\). Let \(|G|\) denote the cardinality of commuter groups \(g \in G\), where commuters are classified based on their value of time, schedule delay penalty and desired arrival time. A commuter of group \(g\) incurs \(\alpha_g\) unit cost of travel time (expressed in $/(time interval)/(vehicle))

Though travel time includes free flow travel time and queuing delay, the former is
assumed to be zero. Thereby, in this study, the terms “travel time” and “queuing delay” are used interchangeably.

Commuters of group $g$ have a desired arrival time interval of $t_g^*$ to their workplace, where they incur a “schedule delay cost” for arriving earlier or later. The travel demand of group $g$ during the morning peak period is denoted by $N_g$. It is assumed that $N_g, t_g^*$ and $s$ are strictly greater than zero. If commuters arrive earlier, they incur $\beta_g$ unit cost for early arrival. If they arrive later, they incur $\gamma_g$ unit cost for late arrival (expressed in $/(time interval)/(vehicle)$). Based on empirical studies (Small 1982), it is assumed that $\alpha_g > \beta_g$. Commuters choose their departure times, $t$, based on the total travel cost, which includes two components: (1) the travel time ($TT_t$), and (2) the schedule delay. Travel time and schedule delay are expressed in units of time intervals, as the average value of time and schedule delay, respectively, over all commuters departing in time interval $t$. The departure rate of commuters of group $g$ in time interval $t$ is represented by $r_{t,g}$ (expressed in (vehicles)/(time interval)). Upon arriving at the queue, vehicles are served in a first-in-first-out order. The travel time of commuters is given by (Ramadurai et al. 2010):

$$TT_0 = \max \left( 0, \frac{\sum g r_{0,g} - s}{s} \right)$$

$$TT_t = \max \left( 0, TT_{t-1} + \frac{\sum g r_{t,g} - s}{s} \right) \quad \forall t \in \Gamma$$

Equation (1) denotes the travel time of commuters departing in time interval 0. Equation (2) is a recursive function implying that if departure rates are greater than bottleneck capacity, queue builds up over time and accumulates until the bottleneck capacity clears the queue. The early arrival delay, $e_{t,g}$, of commuters of group $g$ departing in $t$ is determined as follows (Ramadurai et al. 2010):

$$e_{t,g} = \max(0, t_g^* - t - TT_t) \quad \forall t \in \Gamma, \forall g \in G$$

Equation (3) denotes that early arrival delay of commuters of group $g$ departing in $t$ is zero if they arrive late to the workplace. The travel cost, $\sigma_{t,g}$, of commuters (expressed in $/(vehicle)$) of group $g$ departing in $t$ is as follows (Ramadurai et al. 2010):
At equilibrium, no commuter can reduce his/her total travel cost by unilaterally changing departure time.

3. Reference-dependent user equilibrium under TCS

The proposed TCS is characterized by its initial credit allocation and charging schemes. The credit allocation and charging schemes are predetermined and credits are allocated to the commuters initially. Let \( \mathbf{n} = \{n_g, g \in G\} \) denote the credit allocation scheme where \( n_g \) is the number of allocated credits to each commuter of group \( g \) per day. These credits are consumed during morning commute trips and commuters are assumed to travel once from home to workplace in morning. Let \( \mathbf{k} = \{k_{t,g}, t \in \Gamma, g \in G\} \) denote the credit charging scheme, where \( k_{t,g} \) is the number of credits charged for group \( g \) commuters departing in \( t \). In this context, \( (\mathbf{n}, \mathbf{k}) \) represents the TCS with initial group-specific allocation of credits \( \mathbf{n} \) and time-varying group-specific credit charging scheme \( \mathbf{k} \). Let \( K \) denote the total number of credits allocated to commuters, which is given by \( \sum_{t \in \Gamma} \sum_{g \in G} r_{t,g} n_g \). Commuters trade credits in the market based on their initial credit endowments \( n_g \) and travel needs for each unit of credit. In the competitive market, the credit price, \( \rho \), depends on the interaction between the credit and travel markets during the peak period. In the context of the managing morning commute congestion using TCS, the credit is a commodity generated by central authority, and its supply is predetermined prior to the peak period. The credit demand depends on the credit charging scheme and commuter travel demand. Since the credit supply and commuter travel demand are constant, credit price is assumed to be constant through the peak period (Xiao, Qian, and Zhang 2013).

In the competitive market, commuters gain benefit by selling excess credits if their initial credit endowments are greater than the number of charged credits. However, if their initial credit endowments are not sufficient to address travel needs, commuters purchase required credits from the market, which is considered as a loss. If commuters value losses more heavily than gains, they have different behaviors in purchasing or selling credits in the market. In this study, the value function proposed by Tversky and
Kahneman (1991) is applied to determine the outcome of trading credits in the market. It incorporates the following two features:

(i) The value of outcome is defined as gain or loss compared to a reference point;
(ii) The value function is linear, and steeper for losses than gains to reflect that people are more sensitive to potential losses (Thaler et al. 1997).

In this context, the credit outcome is considered as gain by commuters if they sell excess credits in the market. Otherwise, it is considered as a loss. Thereby, the reference point for a group $g$ is the initial credit endowment for a commuter of that group. Then, the corresponding credit consumption disutility, $\phi_{t,g}$, of a commuter of group $g$ departing in time interval $t$ is as follows:

$$
\phi_{t,g} = \begin{cases} 
-\rho(n_g - k_{t,g}), & k_{t,g} < n_g \\
\eta \rho(k_{t,g} - n_g), & k_{t,g} \geq n_g 
\end{cases} \quad \forall t \in \Gamma, \forall g \in G
$$

(5)

where parameter $\eta \geq 1$ denotes the “loss aversion” coefficient, indicating that commuters are more sensitive to loss than gain. Let $z_{t,g}$ denote the gain of a commuter of group $g$ departing in time interval $t$ by selling excess credits in the market when credit charge is less than initial endowment. The monetary gain of selling excess credits can be obtained as follows:

$$
z_{t,g} = \rho[n_g - k_{t,g}]_+ \quad \forall t \in \Gamma, \forall g \in G
$$

(6)

where $[n_g - k_{t,g}]_+ = (n_g - k_{t,g})$ if $(n_g - k_{t,g}) \geq 0$, and 0 otherwise. Equation (6) can be written as:

$$
z_{t,g} = \max(0, \rho(n_g - k_{t,g})) \quad \forall t \in \Gamma, \forall g \in G
$$

(7)

Using this notation, equation (5) can be rewritten as:

$$
\phi_{t,g} = -z_{t,g} + (\eta z_{t,g} - \eta \rho(n_g - k_{t,g})) \quad \forall t \in \Gamma, \forall g \in G
$$

(8)

Under the TCS, commuters choose their departure times based on the total travel disutilities, which include three components: (1) queuing delay cost, (2) schedule delay cost, and (3) credit consumption disutility. The travel disutility, $C_{t,g}$, of a commuter of group $g$ departing in time interval $t$ is as follows:
Since the credit consumption disutility can be negative, the travel disutility of a commuter of group $g$ departing in time interval $t$ can be either negative or positive. A negative travel disutility implies that the monetary gain of a commuter by selling his/her excess credits in the market is higher than the summation of the experienced queuing delay costs and schedule delay costs. The mixed-linear complementarity problem (MLCP) for the equilibrium problem with TCS $(n, k)$ is as follows:

\[ 0 \leq \alpha_{t,g} \perp \alpha_{t,g}TT_t + \beta_{t,g}e_{t,g} + \gamma_{t,g}[e_{t,g} - (t^*_g - t - TT_t)] + \phi_{t,g} - C^*_g \geq 0 \quad \forall t \in \Gamma, \forall g \in G \tag{9} \]

\[ 0 \leq r_{0,g} \perp \frac{\sum_{g \in G} r_{0,g} - s}{s} \geq 0 \tag{10} \]

\[ 0 \leq TT_t \perp TT_t - (TT_{t-1} + \frac{\sum_{g \in G} r_{t,g} - s}{s}) \geq 0 \tag{11} \]

\[ 0 \leq e_{t,g} \perp e_{t,g} - (t^*_g - t - TT_t) \geq 0 \quad \forall t \in \Gamma, \forall g \in G \tag{12} \]

\[ 0 \leq z_{t,g} \perp z_{t,g} - (\rho)(n_g - k_{t,g}) \geq 0 \quad \forall t \in \Gamma, \forall g \in G \tag{13} \]

\[ \sum_{t \in \Gamma} r_{t,g} - N_g = 0 \quad \forall g \in G \tag{14} \]

\[ 0 \leq \rho \perp \left( \sum_{t \in \Gamma} \sum_{g \in G} r_{t,g}n_g - \sum_{t \in \Gamma} \sum_{g \in G} r_{t,g}k_{t,g} \right) \geq 0 \tag{15} \]

Here, $C^*_g$ is the equilibrium travel disutility of group $g$. Mathematically, \( \perp \) means “perpendicular”, i.e., vectors $x \perp y$ if and only if $x^T y = 0$. Constraint (10) is the complementarity condition to ensure the dynamic UE, labeled as the DUE. The departure rate of commuters of group $g$ in time interval $t$ is positive, only if the sum of travel time cost, schedule delay cost and credit consumption disutility of commuters is equal to the equilibrium travel disutility of group $g$. The travel time of commuters departing in time interval $t \in \Gamma$ is computed using constraints (11) and (12). The early arrival cost of a commuter of group $g \in G$ departing in time interval $t \in \Gamma$ is determined using constraint (13). Constraint (14) computes the monetary gain of selling excess credits for a commuter of group $g \in G$ departing in time interval $t \in \Gamma$. Constraint (15) denotes the travel
demand conservation condition. Constraint (16) denotes the credit market equilibrium condition in which the equilibrium credit price is greater than zero only if commuters consume allocated credits during the peak period. In constraint (16), the total number of allocated credits, $K$, is reformulated as $\sum_t \sum_g r_{t,g} n_g$.

The MLCP (10)-(16) is reformulated as a pure LCP to leverage existing theorems in the LCP context to investigate solution properties such as solution existence in terms of equilibrium departure rates, credit price and travel disutility. It is written as the equivalent linear complementarity problem (ELCP):

\[ 0 \geq r_{t,g} \perp \alpha_g T T_t + \beta_g e_{t,g} + \gamma_g [e_{t,g} - (t_g^* - t - T T_t)] + \phi_{t,g} - (R C_g^* - \xi) \geq 0 \quad \forall t \in \Gamma, \forall g \in G \quad (17) \]

\[ 0 \geq (R C_g^*) \perp \sum_{t \in \Gamma} r_{t,g} - N_g \geq 0 \quad \forall g \in G \quad (18) \]

(11)-(14), (16)

where $\xi$ is a sufficiently large constant. Notation $R C_g^*$ denotes the revised equilibrium travel disutility of group $g$. The following theorem proves the equivalence between MLCP and ELCP.

**Theorem 1.** MLCP and ELCP are equivalent. In other words, every solution to ELCP is a solution of MLCP and every solution to MLCP is a solution to ELCP.

**Proof.** MLCP and ELCP are equivalent if every solution to the ELCP satisfies two conditions:

(i) It has a strictly positive value for $R C_g^*$.

(ii) $R C_g^*$ is equal to $\xi + C_g^*$.

Condition (i) ensures that the demand conservation constraint (15) is satisfied. Constraint (17) indicates that

\[ R C_g^* = \min_{t \in \Gamma} (\alpha_g T T_t + \beta_g e_{t,g} + \gamma_g [e_{t,g} - (t_g^* - t - T T_t)] + \phi_{t,g} + \xi) \quad \forall g \in G \quad (19) \]

which verifies that $R C_g^*$ is always greater than zero because it is the minimum value of the summation of large parameter $\xi$ and travel disutility of group $g$ departing at time $t$,
Since $RC^*_g$ is strictly positive, complementarity constraint (18) implies that the travel demand is met exactly, and hence the demand conservation constraint (15) is satisfied.

Condition (ii) indicates the relationship between the equilibrium travel disutility $C^*_g$ obtained from MLCP and the revised equilibrium travel disutility $RC^*_g$ obtained from LCP. If this condition is satisfied, the equilibrium travel disutility $C^*_g$ in constraint (10) is replaced by $(RC^*_g - \xi)$. To illustrate this relationship, the complementary constraint (10) indicates that

$$C^*_g = \min_{t \in \Gamma} (\alpha_g T T_t + \beta_g e_{t,g} + \gamma_g [e_{t,g} - (t^*_g - t - TT_t)] + \phi_{t,g})$$

∀ $g \in G$  \hspace{1cm} (20)

Because $\xi$ is a constant parameter, equation (19) is written as follows

$$RC^*_g = \min_{t \in \Gamma} (\alpha_g T T_t + \beta_g e_{t,g} + \gamma_g [e_{t,g} - (t^*_g - t - TT_t)] + \phi_{t,g}) + \xi$$

∀ $g \in G$  \hspace{1cm} (21)

Then,

$$RC^*_g = C^*_g + \xi$$

∀ $g \in G$  \hspace{1cm} (22)

This concludes the proof. ■

3.1. Solution existence

In this subsection, the solution existence for MLCP can be established by applying a fundamental existence result for linear complementarity problem to ELCP. To simplify the analysis, ELCP is normalized by multiplying the right-hand side of the complementarity constraints (11) and (12) by $s$. Also, equation (8) is reformulated to include it in constraint (10) as follows:

$$\phi_{t,g} = (\eta - 1) z_{t,g} + \eta \rho (k_{t,g} - n_g)$$

∀ $t \in \Gamma$, ∀ $g \in G$  \hspace{1cm} (23)

ELCP is equivalent to the following form of $LCP(q, M)$:

$$0 \preceq x \perp Mx + q \succeq 0$$

∀ $t \in \Gamma$, ∀ $g \in G$  \hspace{1cm} (24)

In complementarity constraint (24), $M$ is a $n \times n$ real matrix, $q$ is a vector in $\mathbb{R}^n$ and superscript $T$ denotes the transpose of matrix. In the context of ELCP, the vector of variables $x$ in constraint (24) is as follows:
\[ x^T = (r^T \ TT^T \ e^T \ z^T \ RC^T \ \rho) \]  

where \( r = (r_{t,g})_{(t,g)\in T \times G} \), \( TT \equiv (TT_t)_{t\in T} \), \( e \equiv e_{t,g \in T \times G} \), \( z \equiv z_{t,g \in T \times G} \) and \( RC^* \equiv (RC^*_g)_{g \in G} \). In constraint (24), \( q \) is the constant vector with the following form:

\[ q = \left( \frac{1}{\alpha_1 + \gamma_1}(-\gamma_1(t^*_1 - t)_{t \in T} + \xi), \ldots, \frac{1}{\alpha_g + \gamma_g}(-\gamma_g(t^*_g - t)_{t \in T} + \xi), s1, -(t^*_1 - t)_{t \in T}, \ldots, -(t^*_g - t)_{t \in T}, 0, \ldots, 0, \ldots, \right)^T \]

where \( 1 \) is a vector of ones. Matrix \( M \), partitioned in accordance with the vectors \( x \) and \( q \) is given by

\[
M = \begin{bmatrix}
0 & M_1 & M_2 & (\eta - 1)M_5 & -M_3^T & -\eta M_4 \\
-M_1^T & S & 0 & 0 & 0 & 0 \\
0 & M_1 & I & 0 & 0 & 0 \\
0 & 0 & 0 & M_5 & 0 & -M_4 \\
M_3 & 0 & 0 & 0 & 0 & 0 \\
M_4 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

where \( I \in \mathbb{R}^{(|T| \times |G|)} \times \mathbb{R}^{(|T| \times |G|)} \) is the identity matrix and

\[
M_1 = \begin{bmatrix}
I \\
I
\end{bmatrix} \in \mathbb{R}^{(|T| \times |G|)} \times \mathbb{R}^{(|T| \times |G|)},
\]

\[
M_2 = \begin{bmatrix}
(\bar{\beta}_1 + \gamma_1)I & \ldots & 0 \\
\vdots & \ddots & \vdots \\
0 & \ldots & (\bar{\beta}_g + \gamma_g)I
\end{bmatrix} \in \mathbb{R}^{(|T| \times |G|)} \times \mathbb{R}^{(|T| \times |G|)},
\]

\[
M_3 = \begin{bmatrix}
1^T & \ldots & 0 \\
\alpha_1 + \gamma_1 & \vdots & \vdots \\
0 & \ldots & 1^T
\end{bmatrix} \in \mathbb{R}^{(|G|)} \times \mathbb{R}^{(|T| \times |G|)},
\]

\[
M_4 = \begin{bmatrix}
1^T & \ldots & 0 \\
\alpha_1 + \gamma_1 & \vdots & \vdots \\
0 & \ldots & 1^T
\end{bmatrix} \in \mathbb{R}^{(|G|)} \times \mathbb{R}^{(|T| \times |G|)}.
\]
The solution existence of $LCP(q, M)$ has been investigated in different studies (Cottle and Dantzig 1968; Lemke 1965; Cottle, Pang, and Stone 1992). The solution set of $LCP(q, M)$ is denoted by $SOL(q, M)$. Cottle et al. (1992) proved that $SOL(q, M) \neq \emptyset$ if $M$ belongs to both of the following matrix classes:

(i) Let $M \in \mathbb{R}^{n \times n}$. Then, $M$ is called an $R_0$-matrix if $SOL(\mathbf{0}, M) = \{0\}$. The class of such matrices is denoted by $R_0$.

(ii) $M$ is copositive if $x^T M x \geq 0$ for every nonnegative vector $x \geq 0$.

**Theorem 2.** Let $\varphi$ denote the feasible set of departure rates defined by $\varphi = \{ r \mid \sum_t r_{t,g} = N_g, \forall t \in \Gamma, \forall g \in G \}$. $MLCP(q, M)$ has a solution if a TCS $(n, k) \in \Psi = \{(n, k) \mid \exists r \in \varphi \text{ such that } \sum_{t \in \Gamma} \sum_g r_{t,g} n_g \geq \sum_{t \in \Gamma} \sum_g r_{t,g} k_{t,g} \}$. The proof of this theorem is given in Appendix A. Theorem 2 ensures that if the central authority implements the TCS to mitigate morning commute congestion, travelers are able to use the credits to address their travel needs as there exists at least a feasible
departure rate. After demonstrating the solution existence to \textit{MLCP}, it is necessary to illustrate the solution uniqueness condition in the next section.

\textbf{3.2. Uniqueness of equilibrium solution}

In this section, we investigate the uniqueness of equilibrium credit price and travel disutility. Further, the effect of market loss aversion behavior of commuters on the equilibrium credit price is examined. The uniqueness of the equilibrium credit price is important to investigate as it is a sign of a healthy market. The uniqueness is described in the following theorem:

\textbf{Theorem 3.} Let $TT_t^*$, $e_{t,g}^*$ and $z_{t,g}^*$ denote the equilibrium travel time of commuters, early arrival delay of commuters, and monetary gain of selling excess credits by group $g$ departing in time interval $t$, respectively. Given a TCS $(n, k)$, the equilibrium credit price $\rho^*$ is unique if the following two conditions are satisfied:

(i) Equilibrium departure rates are unique.

(ii) There exists at least one group of commuters whose equilibrium departure rates are positive in at least two different time intervals with different credit charges during the peak period.

\textbf{Proof.} To prove the uniqueness of the equilibrium credit price, consider the case where commuters consume all allocated credits during peak period. Otherwise, constraint (16) becomes inactive and the equilibrium credit price is zero and unique. If there exist two time intervals $t$ and $t'$ in which commuters of group $g$ depart, then:

\begin{align*}
\alpha_g TT_t^* + \beta_g e_{t,g}^* + \gamma_g \left[ e_{t,g}^* - (t_g^* - t - TT_t^*) \right] + (\eta - 1)z_{t,g}^* + \eta \rho^* (k_{t,g} - n_g) &= C_g^* \\
\alpha_g TT_{t'}^* + \beta_g e_{t',g}^* + \gamma_g \left[ e_{t',g}^* - (t_g^* - t' - TT_{t'}^*) \right] + (\eta - 1)z_{t',g}^* + \eta \rho^* (k_{t',g} - n_g) &= C_g^* 
\end{align*}

Hence, the equilibrium credit price can be written as:

\begin{equation}
\rho^* =
\end{equation}

\begin{align*}
&\frac{\alpha_g (TT_{t'}^* - TT_t^*) + \beta_g \left( e_{t',g}^* - e_{t,g}^* \right) + \gamma_g \left( e_{t',g}^* - (t_g^* - t' - TT_{t'}^*) \right) - \left( e_{t,g}^* - (t_g^* - t - TT_t^*) \right) - (\eta - 1)(z_{t',g}^* - z_{t,g}^*)}{\eta (k_{t,g} - k_{t',g})}
\end{align*}
Combining equations (6) and (30) yields:

$$\rho^* = \frac{\alpha_g(TT^*_t - TT^*_i) + \beta_g(e^*_{t,g} - e^*_{i,g}) + \gamma_g(e^*_{t,g} - (t^*_g - t^* - TT^*_t)) - \left(e^*_{i,g} - (t^*_g - t - TT^*_i)\right)}{\eta(k_{t,g} - k_{t',g}) + (\eta - 1) \left[ n_g - k_{i,g} \right]_+ - \left[ n_g - k_{i',g} \right]_+}$$

There are four possible cases related to $k_{t,g}, k_{t',g}$ and $n_g$ where $k_{t,g} \neq k_{t',g}$. In the first case, $k_{t,g}, k_{t',g} \geq n_g$. Then, equation (31) can be rewritten as follows:

$$\rho^* = \frac{\alpha_g(TT^*_t - TT^*_i) + \beta_g(e^*_{t,g} - e^*_{i,g}) + \gamma_g(e^*_{t,g} - (t^*_g - t^* - TT^*_t)) - \left(e^*_{i,g} - (t^*_g - t - TT^*_i)\right)}{\eta(k_{t,g} - k_{t',g})}$$

In the second case, $k_{t,g} > n_g > k_{t',g}$. Then, equation (31) can be rewritten as follows:

$$\rho^* = \frac{\alpha_g(TT^*_t - TT^*_i) + \beta_g(e^*_{t,g} - e^*_{i,g}) + \gamma_g(e^*_{t,g} - (t^*_g - t^* - TT^*_t)) - \left(e^*_{i,g} - (t^*_g - t - TT^*_i)\right)}{\eta(k_{t,g} - n_g) + (n_g - k_{i,g})}$$

In the third case, $k_{t',g} > n_g > k_{t,g}$. Then, equation (31) can be expressed as:

$$\rho^* = \frac{\alpha_g(TT^*_t - TT^*_i) + \beta_g(e^*_{t,g} - e^*_{i,g}) + \gamma_g(e^*_{t,g} - (t^*_g - t^* - TT^*_t)) - \left(e^*_{i,g} - (t^*_g - t - TT^*_i)\right)}{\eta(n_g - k_{t',g}) - (n_g - k_{t,g})}$$

In the fourth case, $n_g \geq k_{t,g}, k_{t',g}$. Then, equation (31) can be expressed as:

$$\rho^* = \frac{\alpha_g(TT^*_t - TT^*_i) + \beta_g(e^*_{t,g} - e^*_{i,g}) + \gamma_g(e^*_{t,g} - (t^*_g - t^* - TT^*_t)) - \left(e^*_{i,g} - (t^*_g - t - TT^*_i)\right)}{\eta(k_{t,g} - k_{t',g})}$$

Since (i) implies unique departure rates, it can be concluded that the equilibrium credit price is unique in any of these four cases. Therefore, the equilibrium credit price is
unique if departure rates of commuters are unique and strictly positive in at least two
different time intervals during the peak period. ■

To provide an alternative expression of the equilibrium credit price, we sum
constraint (10) over \( g \in G \) and \( t \in \Gamma \). By combining this summation with equation (6)
and constraint (15), it follows:

\[
\sum_{t \in \Gamma} \sum_{g \in G} r_{t,g} \left( a_g T^*_t + \beta_g e^*_t + \gamma_g \left[ e^*_{t,g} - (t^*_g - t - TT_t) \right] \right) + \sum_{t \in \Gamma} \sum_{g \in G} r_{t,g} \rho^* \left( (\eta - 1)[n_{g} - k_{t,g}]_+ - \eta(n_{g} - k_{t,g}) \right) = \sum_{g \in G} N_g C^*_g \tag{36}
\]

If \( \eta = 1 \), the equilibrium credit price cannot be derived using equation (36) because:

\[
\sum_{t \in \Gamma} \sum_{g \in G} r_{t,g} \rho^* (n_{g} - k_{t,g}) = 0 \tag{37}
\]

Hence, the equilibrium credit price \( \rho^* \) is removed from equation (36). If \( \eta > 1 \), then
equilibrium credit price is given by

\[
\rho^* = \frac{\sum_g (N_g C^*_g) - \lambda^*}{\sum_{t \in \Gamma} \sum_{g \in G} r_{t,g} \left( (\eta - 1)[n_{g} - k_{t,g}]_+ - \eta(n_{g} - k_{t,g}) \right)} \tag{38}
\]

where \( \lambda^* \) denotes the summation of the equilibrium schedule and queuing delay costs of
commuters. Equation (38) is valid for both the cases in terms of whether commuters
consume or do not consume all issued credits during the peak period. If they do not
consume all issued credits, then the TCS does not impact the equilibrium cost of
commuters, and \( \sum_g (N_g C^*_g) = \lambda^* \). Therefore, the equilibrium credit price is equal to zero
if commuters do not consume all credits during the peak period.

The first condition of theorem 3 requires the uniqueness of equilibrium departure
rates. However, Doan, Ukkusuri, and Han (2011) showed that departure rates of travelers
may not be unique under congestion pricing. Because each TCS has a congestion pricing
mirror (Nie and Yin 2013), departure rates of commuters may not be unique under TCS.
For example, non-uniqueness can be illustrated if we reformulate constraint (10) as
follows:
Equation (39) shows that departure rates depend on ratios of $\frac{\beta_g}{a_g}$, $\frac{\gamma_g}{a_g}$ and $\frac{\phi_{t,g}}{a_g}$. Hence, if two groups have identical ratios $\frac{\beta_g}{a_g}$, $\frac{\gamma_g}{a_g}$ and $\frac{\phi_{t,g}}{a_g}$ during the morning peak period, any portion of one group of travelers can be replaced by the other group to generate the departure rates that solve equation (39).

While the equilibrium credit price in equations (32), (33) and (34) depends on the market loss sensitivity of commuters, the equilibrium credit price under the fourth case in the proof of theorem 3 (equation (35)) is independent of commuters’ loss aversion coefficient $\eta$. This is because the loss aversion coefficient $\eta$ does not impact the travel disutility of commuters of group $g$ in time interval $t$ if the initial credit endowment of group $g$ is higher than or equal to the charged credits in that period. Consequently, if the equilibrium credit price is solely determined by the departure rates of commuters whose initial credit endowments are higher than credit charges while commuters of other groups depart in only one time interval, then the loss aversion of commuters does not impact the equilibrium credit price. However, this special case would rarely occur in practice because commuters of different groups are likely to depart in more than one time interval. This special case is ignored in this study to analyze the effect of loss aversion behavior of commuters on equilibrium credit price and total value of traded credits in the market.

In Proposition 1, the effect of market loss sensitivity of commuters is investigated on equilibrium credit price in the market.

**Proposition 1.** If the loss aversion coefficient increases, then the equilibrium credit price approaches zero.

**Proof.** If commuters’ sensitivity to loss increases, it follows that

$$\lim_{\eta \to \infty} \rho^* =$$

$$\frac{\alpha_g \left( TT_{t'} - TT_t \right) + \beta_g \left( e_{t',g} - e_{t,g} \right) + \gamma_g \left( \left( e_{t',g} - (t_g^* - t - TT_{t'}) \right) - \left( e_{t,g} - (t_g^* - t - TT_t) \right) \right)}{\eta \left( k_{t,g} - k_{t',g} \right) + (\eta - 1) \left( \left[ n_g - k_{t,g} \right]_+ - \left[ n_g - k_{t',g} \right]_+ \right)}$$

(40)
Equations (32), (33) and (34) can be substituted into equation (40). As can be seen in these equations, as the loss sensitivity of commuters increases, the equilibrium credit price approaches zero if $TT^*_t$ and $e^*_{t,g}$ are bounded. Using equation (2), it follows:

$$TT_t \leq TT_{t-1} + \frac{\sum g r_{t,g}}{s}$$

\forall t \in \Gamma \tag{41}$$

Inequality (41) can be simplified as follows:

$$TT_t \leq \frac{\sum_t \sum g r_{t,g} - (t + 1)s}{s}$$

\forall t \in \Gamma \tag{42}$$

It follows that $TT_t$ is bounded and less than or equal to $\frac{\sum g N_g (t+1)s}{s}$. Hence, $\lim_{\eta \to \infty} TT_t$ is less than or equal to $\frac{\sum g N_g (t+1)s}{s}$. Further, it follows from equation (3) that $e^*_{t,g} \leq t^*_g$ and hence, $\lim_{\eta \to \infty} e^*_{t,g}$ is less than or equal to $t^*_g$. Since $TT^*_t$ and $e^*_{t,g}$ are bounded, the equilibrium credit price approaches zero as the loss sensitivity of commuters increases. This completes the proof. ■

Proposition 1 implies that if commuters’ sensitivity to monetary loss of purchasing credits in the market increases, then commuters change departure times such that they purchase less credits in the market. Therefore, credit demand in the market reduces while credit supply remains constant. Consequently, equilibrium credit price decreases as commuters’ loss sensitivity increases. This trend continues until equilibrium credit price approaches zero. If the equilibrium credit price becomes zero, then the TCS becomes inactive and the central authority fails to manage the morning commute congestion using the TCS. Since credit price approaches zero, the total value of traded credits in the market approaches zero. Therefore, we have the following proposition.

**Proposition 2.** Total value of traded credits in the market approaches zero as the loss aversion coefficient increases.

**Proof.** Since the number of traded credits in the market is finite and the equilibrium credit price approaches zero as commuters’ loss aversion increases, it follows

$$\lim_{\eta \to \infty} \sum_{t \in \Gamma} \sum_{g \in G} \rho^* r^*_{t,g} \left[ n^*_g - k^*_{t,g} \right] = 0$$

(43)$$

This concludes the proof. ■
The next proposition investigates the effect of the credit allocation scheme on credit price when commuters are equally sensitive to the monetary loss and gain associated with trading credits in the market. Previous studies on managing traffic congestion using TCS ignore the effects of loss aversion behavior on the travel equilibrium conditions. Due to this assumption, they conclude that the equilibrium condition is independent of the initial credit allocation scheme. This is illustrated in Proposition 3, where if commuters are equally sensitive to loss and gain, the departure rates and credit price are independent of the initial credit allocation method. However, as illustrated in our numerical results, this assumption may not be realistic as the loss aversion behavior impacts the departure rates and credit prices under different credit allocation methods.

**Proposition 3.** If commuters are sensitive to loss and gain equally, the total allocated credits are the only factor in the credit allocation scheme to determine departure rates and credit price. In other words, the credit allocation method does not impact the equilibrium departure rates and credit price.

**Proof.** If commuters are equally sensitive to loss and gain, then \( \eta \) is equal to 1. Credit consumption disutility in equation (8) can be simplified as follows:

\[
\phi_{t,g} = \rho (k_{t,g} - n_g) \quad \forall t \in \Gamma, \forall g \in G
\]

As the number of allocated credits to commuters of each group is constant, it follows from equation (31) that the equilibrium credit price is independent of the credit allocation scheme when \( \eta \) is equal to 1. Therefore, the credit consumption disutility can be further simplified as:

\[
\phi_{t,g} = \rho (k_{t,g}) \quad \forall t \in \Gamma, \forall g \in G
\]

However, the total number of allocated credits in constraint (16) affects the departure rates and equilibrium credit price. This concludes the proof.

The next theorem investigates the uniqueness of equilibrium travel disutility.

**Theorem 4.** Equilibrium travel disutility is unique if equilibrium credit price is unique.

**Proof.** To prove the uniqueness of equilibrium travel disutility, assume that equilibrium credit price is greater than zero. If it is zero, then TCS becomes inactive and so, model (10)-(16) reduces to UE formulation of no-toll morning commute congestion of...
Ramadurai et al. (2010). They also prove that equilibrium travel disutility is unique under no-toll morning commute congestion.

Given the equilibrium credit price \( \rho^* \), the equilibrium credit consumption disutility \( \phi_{t,g}^* \) in equation (8) can be rewritten as

\[
\phi_{t,g}^* = \rho^* \left( (\eta - 1)[n_g - k_{t,g}]_+ - \eta(n_g - k_{t,g}) \right) \quad \forall t \in \Gamma, \forall g \in G
\]  

(46)

Hence, the equilibrium credit consumption disutility is unique if the equilibrium credit price is unique. Then, MLCP can be rewritten as follows

\[
0 \leq r_{t,g} \perp \alpha_g T T_t + \beta_g e_{t,g} + \gamma_g [e_{t,g} - (t_g^* - t - T T_t)] - (C_g^* - \phi_{t,g}^*) \geq 0 \quad \forall t \in \Gamma, \forall g \in G
\]  

(47)

(11)-(13), (15)

Following the approach of Ramadurai et al. (2010) to prove the uniqueness of equilibrium travel cost in no-toll morning commute congestion, it can be proved that \( (C_g^* - \phi_{t,g}^*) \) is unique. Because \( \phi_{t,g}^* \) is unique, the equilibrium travel disutility \( C_g^* \) is unique if the equilibrium credit price is unique. This concludes the proof. ■

4. System optimal design of TCS

This section proposes a primal-dual method to obtain the SO TCS design in terms of the initial group-specific credit allocation scheme and time-varying group-specific credit charging scheme. Doan et al. (2011) prove that, the queuing delay of commuters is equal to zero under SO condition, i.e. \( T T_t = 0 \). Using this property, we seek to determine the SO credit consumption disutility to achieve pre-specified SO departure rates (equation (50)) under the SO TCS. Let us define \( u_{t,g} = \begin{cases} \beta_g (t_g^* - t) & \text{if } t \leq t_g^* \\ \gamma_g (t - t_g^*) & \text{if } t > t_g^* \end{cases} \). If the set of equations (48)-(54) holds, then \( \rho, \phi_{t,g} \) and \( C_g \) are the optimal solution of model (10)-(16) based on the SO departure rates \( r_{t,g}^{SO} \) where the queuing delay is equal to zero.

\[
r_{t,g}(u_{t,g} + \phi_{t,g} - C_g) = 0 \quad \forall t \in \Gamma, \forall g \in G
\]  

(48)

\[
u_{t,g} + \phi_{t,g} - C_g \geq 0 \quad \forall t \in \Gamma, \forall g \in G
\]  

(49)

\[
r_{t,g} = r_{t,g}^{SO} \quad \forall t \in \Gamma, \forall g \in G
\]  

(50)
\[
\sum_{t \in T} r_{t,g} = N_g \quad \forall g \in G
\]  
(51)

\[
r_{t,g} \geq 0 \quad \forall t \in \Gamma, \forall g \in G
\]  
(52)

\[
0 \leq z_{t,g} \perp z_{t,g} - ((\rho)(n_g - k_{t,g})) \geq 0 
\]  
(53)

\[
0 \leq \rho \perp \left( \sum_{t \in T} \sum_{g \in G} r_{t,g}n_g - \sum_{t \in T} \sum_{g \in G} r_{t,g}k_{t,g} \right) \geq 0
\]  
(54)

Constraints (48), (49) and (52) satisfy constraints (10) and (13) where queuing delay is equal to zero. Constraints (11) and (12) are also satisfied since queuing delay is zero. Constraint (50) ensures that the equilibrium departure rates are identical to the SO departure rates. Constraints (51) and (53) and (54) are identical to constraints (15), (14) and (16), respectively. To regulate the SO TCS parameters, it is sufficient to first determine the SO credit consumption disutility using equations (48)-(52). Then, \( \phi_{t,g}^{SO} \) is used to determine the SO TCS parameters so that constraints (8), (53) and (54) are satisfied. As the set of equations (48)-(52) is a linear complementarity problem and difficult to solve, we develop a primal-dual program to solve it. Using equations (48)-(52), the SO model can be formulated as the following linear program:

\[
\min \sum_{g \in G} \sum_{t \in T} r_{t,g}u_{t,g} 
\]  
(55)

\[
r_{t,g} = r_{t,g}^{SO} \quad \forall t \in \Gamma, \forall g \in G
\]  
(56)

\[
\sum_{t \in T} r_{t,g} = N_g \quad \forall g \in G
\]  
(57)

\[
r_{t,g} \geq 0 \quad \forall t \in \Gamma, \forall g \in G
\]  
(58)

The objective function (55) is to minimize commuters’ total schedule delay cost. Constraint (56) restricts the departure rate of commuters of group \( g \) departing in \( t \) to be equal to the SO departure rates. Because of constraint (56), the feasible solution space contains only the desired SO departure rates. Because the feasible solution space only contains the desired SO solution, it has a unique solution in terms of departure rates. If \( \phi_{t,g} \) and \( C_g \) are Lagrangian multipliers for constraints (56)-(57), the dual problem of primal problem (55)-(58) is as follows:
\[
\max - \sum_{g \in G} \sum_{t \in T} r_{t,g} \phi_{t,g} + \sum_{g \in G} N_g C_g \tag{59}
\]

\[
C_g - \phi_{t,g} \leq u_{t,g} \quad \forall t \in \Gamma, \forall g \in G \tag{60}
\]

\[
C_g, \phi_{t,g} \text{ Unrestricted} \quad \forall t \in \Gamma, \forall g \in G \tag{61}
\]

The next theorem provides a system of equations to determine SO credit price \( \rho^{SO} \), group-specific credit allocation scheme \( n_{g}^{SO} \), and time-varying group-specific credit charging scheme \( k_{t,g}^{SO} \) using the SO credit consumption disutility of the dual problem (59)-(61).

**Theorem 5.** Let \( \phi_{t,g}^{SO} \) be an optimal solution to the dual problem (59)-(61). Given the loss aversion coefficient \( \eta \) of commuters, if \( (\rho^{SO}, n_{g}^{SO}, k_{t,g}^{SO}) \) satisfies the following set of equations:

\[
\rho^{SO} \left( (\eta - 1)[n_{g}^{SO} - k_{t,g}^{SO}]_{+} - \eta(n_{g}^{SO} - k_{t,g}^{SO}) \right) = \phi_{t,g}^{SO} \quad \forall t \in \Gamma, \forall g \in G \tag{62}
\]

\[
\sum_{t \in T} \sum_{g \in G} r_{t,g} (n_{g}^{SO} - k_{t,g}^{SO}) = 0 \tag{63}
\]

then \( (\rho^{SO}, n_{g}^{SO}, k_{t,g}^{SO}) \) are the SO credit price, and credit charging and allocation schemes.

**Proof.** It is sufficient to demonstrate that \( (\rho^{SO}, n_{g}^{SO}, k_{t,g}^{SO}) \) satisfy the model (10)-(16) where queueing delay is equal to zero. Using the duality theorem and principle of complementarity slackness, \( r_{t,g}^{SO} > 0 \) holds if \( C_g - \phi_{t,g} = u_{t,g} \) which is equivalent to constraint (10). Since queuing delay is zero for the SO condition of the single bottleneck model, constraints (11) and (12) are also satisfied. Constraint (13) is implicitly accounted for by the cost of early arrival in \( u_{t,g} \). Constraints (62) and (63) ensure that constraints (14) and (16) are satisfied. SO departure rates satisfy the travel demand conservation constraint (15). Therefore, the optimal solution of the primal-dual problem \( (r_{t,g}^{SO}, \phi_{t,g}^{SO}, C_{g}^{SO}) \) is a feasible solution to UE problem (10)-(16). Then, if the central authority designs the TCS such that the equilibrium credit price, credit allocation scheme and credit charging schemes satisfy constraints (62) and (63), the outcome of TCS is the SO credit consumption disutility. This concludes the proof. \( \blacksquare \)
The system of equations (62) and (63) consist of \(|T| \times |G| + 1\) equations and \(|T| \times |G| + |G| + 1\) variables. Because the SO credit price \(\rho^S\) can be set to be a constant (e.g. equal to one), the number of variables is reduced to \(|T| \times |G| + |G|\) variables in the system of equations (62) and (63). Since there are fewer equations than number of variables, the system of equations (62) and (63) is underdetermined. It either has no solution or has infinitely many solutions in terms of credit allocation and credit charging schemes. If the central authority implements the uniform credit allocation scheme, the number of variables reduce to \(|T| \times |G| + 1\) which is equal to the number of equations. Then, the system equations (62) and (63) has either one or no solution under a uniform credit allocation scheme and a group-specific time-varying charging scheme.

Note that a well-formulated group-specific TCS parameters can improve the equity aspect of TCS. For example, if the central authority plans to provide higher monetary benefits for low income commuters, it can provide more credits to this group. This leads to a group-specific credit allocation scheme where the central authority can factor value of time as an indicator for commuters’ income. The central authority can also charge more credits from high income commuters which leads to a group-specific credit charging scheme. To design the SO TCS parameters, the central authority can consider equity indices such as Gini coefficient to factor the income along with the monetary gains and losses of commuters.

As a simpler type of equity, Yang and Huang (2005) propose a method to develop static congestion pricing with equitable impact on commuters’ travel disutilities. This method prevents the percentage increase of travel disutility from exceeding a certain threshold in SO TCS design. Let \(C_{g}^{NOTCS}\) denote the equilibrium travel disutility of group \(g\) without TCS. Then, it is sufficient to derive credit consumption disutility by solving the following linear model:

\[
\text{max} - \sum_{g \in G} \sum_{t \in T} r_{t,g}^{SO} \phi_{t,g} + \sum_{g \in G} N_{g} C_{g} \\
C_g - \phi_{t,g} \leq u_{t,g} \quad \forall t \in \Gamma, \forall g \in G \tag{64}
\]

\[
\frac{C_{g}}{C_{g}^{NOTCS}} \leq \chi_{g} \tag{66}
\]
where $\chi_g$ denotes the equity index that dictates the pre-defined degree of inequity associated with the SO TCS design. Constraint (66) states that the ratio of travel disutility of group $g$ under SO TCS design to the one without TCS should be less than or equal to $\chi_g$. Yang and Huang (2005) suggest that $\chi_g$ is greater than or equal to one. They discuss that the higher value of $\chi_g$ leads to a greater inequity impact on commuters. If the central authority sets the value of $\chi_g$ to 1, then linear model (64)-(67) reduces to a Pareto-improving TCS design in which every commuter is better off in terms of travel disutility. The numerical experiments illustrate that a well-formulated group-specific credit allocation scheme enables achieving a Pareto-improving TCS design.

5. Numerical experiments

The numerical experiments seek to examine the effects of commuters’ schedule delay penalty, value of travel time, and desired arrival time on: (i) the UE departure rates, (ii) commuters’ travel disutilities, and (ii) total system travel cost under TCS. To do so, MLCP (10)-(16) is solved using GAMS (Rosenthal 2015). Further, we investigate the effect of commuters’ loss aversion on: (i) the UE departure rates and credit price under TCS, and (ii) the SO design of TCS. To do so, the dual problem (59)-(61) and system of equations (62)-(63) are solved. In the experiments, the time duration is 100 time units and the bottleneck capacity is 10 vehicles per time unit. The parameters used are shown in Table 1. There are three groups with different values of $\alpha_g$, $\beta_g$, $\gamma_g$, $t^*_g$. Group 1 has flexible work hours but higher value of time compared to groups 2 and 3. Group 2 has more flexible work hours and higher value of time compared to group 3.

Table 1. Input data.

<table>
<thead>
<tr>
<th>Group</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$t^*$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>35</td>
<td>10</td>
<td>26</td>
<td>70</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>27</td>
<td>11</td>
<td>28</td>
<td>65</td>
<td>200</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>13</td>
<td>29</td>
<td>60</td>
<td>500</td>
</tr>
</tbody>
</table>
Figure 1. The credit charging scheme

Section 5.1 investigates the effects of the credit allocation schemes of Table 2 on traffic and market equilibrium conditions. The TCS, adopted in Section 5.1, is characterized by the group-specific credit allocation schemes shown in Table 2 and the group-specific time-varying credit charging scheme shown in Figure 1. Under these credit allocation schemes, credits are uniformly allocated to only one group of commuters under each scheme. The total endowment of credits in all schemes is 500. Under the credit charging scheme, commuters of group 3 do not pay any credits to depart in any time interval. Section 5.2 focuses on the SO TCS, unlike the other one which is suboptimal.

Table 2. Credit allocation schemes

<table>
<thead>
<tr>
<th></th>
<th>Scheme 1</th>
<th>Scheme 2</th>
<th>Scheme 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group 1</td>
<td>5.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Group 2</td>
<td>0.0</td>
<td>2.5</td>
<td>0.0</td>
</tr>
<tr>
<td>Group 3</td>
<td>0.0</td>
<td>0.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

5.1. Impact of credit allocation scheme

In this section, we investigate the effects of credit allocation scheme and commuter’s loss aversion behavior on departure rates and credit price. Figure 2 shows the UE departure rates of commuters for the case in which the TCS is not implemented. The total system
travel cost under the UE condition is equal to 476,937. The travel disutilities of groups 1, 2 and 3 are 505.62, 605 and 610.75, respectively. Because of high schedule delay penalty and low value of time, commuters of group 3 depart early in order to arrive at the workplace close to the desired arrival time. On the other hand, commuters of group 1 depart later in the peak period to avoid queuing delay.

Figure 2. UE departure rates of commuters without TCS

Figure 3 shows the effect of commuters’ loss aversion on credit price under the three credit allocation schemes, illustrated in Table 2. Because commuters of groups 1 and 2 have zero initial credit endowment under credit allocation scheme 3, they have to purchase credits in the market for their own travel needs, and hence credit demand is higher compared to schemes 1 and 2. Further, as commuters of groups 1 and 2 consume a portion of their allocated credits under credit allocation schemes 1 and 2 for their own travel needs, credit supply in the market is less compared to credit allocation scheme 3. The credit demand in the market is also lesser under credit allocation schemes 1 and 2 compared to scheme 3 because commuters of groups 1 and 2 consume a portion of their allocated credits under schemes 1 and 2. Despite lesser credit demand in the market, lesser credit supply leads to higher credit price under schemes 1 and 2 compared to scheme 3. Figure 3 also validates that credit price approaches zero as commuters’ loss aversion increases. Further, it indicates that if commuters are equally sensitive to monetary losses and gains, then credit prices are identical under the three credit allocation schemes and equal to 22.41. This validates that if commuters are equally sensitive to
monetary losses and gains, the credit allocation method does not impact credit price.

Figure 3. Effect of commuters' loss aversion on credit price under the three credit allocation schemes

Figure 4. Effect of commuters' loss aversion and credit allocation schemes on the total value of traded credits in the market

Figure 4 shows the total value of traded credits in the market. It validates proposition 3 where total value of traded credits in the market approaches zero as the loss sensitivity of commuters increases. When commuters’ loss aversion coefficient is equal to one, credit price is identical under the three credit allocation schemes. Because group 3 receives and sells all credits in the market under the third credit allocation scheme, the total value of traded credits in the market is highest under credit allocation scheme 3 in which commuters’ loss aversion coefficient is equal to one. As commuters’ loss aversion coefficient increases, the total value of traded credits under the third credit allocation scheme becomes higher compared to those under the other schemes despite the higher
credit price under the second credit allocation scheme. This is because as the loss aversion coefficient increases, the difference between the credit price under schemes 1 and 2 reduces while the total number of traded credits is higher under the third credit allocation scheme in which group 3 receives and sells all credits in the market.

Figure 5 shows the effect of the three credit allocation schemes on travel disutility of groups 1 through 3 as commuters’ loss aversion increases from 1 to 25. It also compares the travel disutility for the cases with TCS and without TCS (which is labeled as NoTCS in the figure). Under the first scheme, group 1 is the sole recipient of credits. The group 1 commuters use the credit endowments to fulfill their travel needs and sell excess credits in the market. As illustrated by Figure 5(a), they experience a lesser travel disutility under the first credit allocation scheme compared to second and third credit allocation schemes, and the NoTCS case. Under the third credit allocation scheme, commuters of group 3 cannot reduce their travel disutility by changing their departure times. Commuters of groups 1 and 2 also cannot reduce their travel disutilities by changing their departure times. Consequently, the credit price reduces proportional to the loss aversion sensitivity of commuters (equation (32)), and travel disutilities remain unchanged despite the increase in loss sensitivity. Hence, under this specific case, the loss sensitivity only impacts the credit price and does not impact the travel disutility of commuters. Figure 5(b) shows that commuters of group 2 incur the least travel disutility under credit allocation scheme 2 where they are the sole recipient of credits. However, they experience the highest travel disutility under scheme 1 in which they need to purchase credits from group 1. Similar to Figure 5(a), commuters’ loss aversion does not affect the travel disutility of group 2 under the third credit allocation scheme. Figure 5(c) indicates that commuters of group 3 has lesser travel disutility under all credit allocation schemes compared to NoTCS since they are not required to pay credits to fulfill their travel needs, and groups 1 and 2 have to change their departure time to reduce their travel disutility. As commuters’ loss aversion coefficient increases, the credit price decreases, and hence the travel disutility of group 3 increases due to the higher credit consumption disutility. Figure 5 also demonstrates that the loss aversion behavior of commuters can significantly change the travel disutility and, consequently, the departure rates of commuters.
Figure 5. Effect of commuters' loss aversion on commuters’ travel disutility under different credit allocation schemes.

Figure 6 illustrates commuters’ departure rates under credit allocation schemes 1 to 3 when loss aversion coefficient is 25. Because of the high loss aversion in purchasing credits, commuters aim to reduce credit consumption in the market by shifting departure times. As can be seen in Figure 6(a), more commuters of group 2 depart earlier during
the peak period under the first credit allocation scheme to avoid paying credits compared to the NoTCS case. Further, commuters of group 1 depart closer to their desired arrival time under the first credit allocation scheme compared to the NoTCS case. Figure 6(b) illustrates that under the second credit allocation scheme, some commuters of group 3 start earlier to avoid paying credits. Figure 6(c) shows that under the third credit allocation scheme, some commuters of group 2 start earlier to avoid paying credits while some commuters of group 3 depart later to avoid queuing delay.

Figure 6. Effect of commuters' loss aversion on commuters’ departure rates under different credit allocation schemes.
5.2. System optimal TCS

In this section, we first illustrate the importance of considering the effect of commuters’ heterogeneity in three dimensions (value of time, schedule delay penalty and desired arrival time) to determine the SO credit allocation and charging schemes. Then, we illustrate the effect of loss sensitivity of commuters on the SO design of TCS. Figure 7 shows the prespecified SO departure rates of commuters obtained using the SO model of Doan, Ukkusuri, and Han. Although, in our study, the departure order is identical under SO and UE conditions (Group 2-Group 3-Group 2-Group 1), there are heterogeneity examples that lead to different departure orders under UE and SO. Under the SO condition, the total departure rate of commuters is equal to the bottleneck capacity, and hence commuters experience zero travel time. The total system travel cost under the SO condition is equal to 232,400. While some commuters of group 2 depart earlier during the peak period, all commuters of group 1 depart later in the peak period.

![Figure 7. Departure rate of commuters under the SO condition](image)

To illustrate the importance of considering the effect of commuters’ heterogeneity, it is assumed that the central authority designs the SO TCS such that commuters are homogeneous in these three dimensions. Commuters’ value of time, early arrival penalty, late arrival penalty and desired arrival time are assumed to be equal to 23.625, 12.125, 28.375, and 62.5, respectively. These values are the weighted average of parameters shown in Table 1. The loss aversion coefficient of commuters is assumed to be equal to 5. Figure 8 shows the SO credit charging scheme where commuters receive 15 credits and the credit price is set to 20. The SO total system cost is equal to 272,690 and travel
disutility of commuters is equal to 472. If the travel time and schedule delays are calculated based on the heterogeneity of commuters in terms of values of time, schedule delay penalties and desired arrival times, the total system travel cost increases to 278,570. It can also be seen in Figure 8 that the slopes of credit charging scheme change at time intervals 23 and 78 with the number of charged credits equal to 15. This is due to the loss sensitivity of commuters where they consider charged credits greater than 15 as monetary loss. Consequently, the credit charging scheme has lower slopes in intervals 23-78 because commuters’ loss sensitivity increases their credit consumption disutility. If the central authority implements this TCS design that does not factor the heterogeneity of commuters, the total system travel cost becomes equal to 476,937. It is significantly higher than the total system travel time under SO TCS design which is equal to 232,400. This demonstrates the importance of factoring the heterogeneity of commuters in SO TCS design.

Figure 9 illustrates the system optimal credit charging schemes under commuter heterogeneity. If the central authority implements the uniform credit allocation scheme, the SO credit charging scheme is presented in Figure 9(a) where commuters of each group receive 15 credits. The total system travel cost is equal to 232,400 and travel disutilities of groups 1, 2 and 3 are equal to 754, 616 and 327, respectively. Hence, if the central authority factors the heterogeneity of commuters (in terms of value of time, schedule delay penalty and desired arrival time) to design the TCS, the total system travel cost can be reduced compared to the case where commuters are assumed to be homogeneous. Given the credit charging scheme in Figure 9(b), the central authority can achieve the Pareto-improving SO TCS design that makes everyone better off by leveraging an appropriate group-specific credit allocation scheme. In this scheme, if the central authority allocates 16, 14 and 17 credits to each commuter of groups 1, 2 and 3, respectively, the travel disutilities of groups 1, 2 and 3 are equal to 480.7, 596, and 351, respectively.
Figure 8. System optimal credit charging scheme with homogeneous commuters

To explore the importance of factoring commuters’ loss aversion in SO TCS design, two cases are considered, with the commuters’ loss aversion coefficient equal to 1 and 20 in cases 1 and 2, respectively. Figure 10 shows the SO credit charging schemes for the two cases. In both cases, the credit price is set to 20. In case 1, the central authority allocates 16.40, 8.82 and 18.15 credits to groups 1, 2 and 3, respectively. The SO travel disutilities of the three groups are 140, 439 and 261, respectively. The credit revenue distribution in case 1 is shown in Figure 11. Group 1 is the only one with positive revenue from trading credits while group 2 has higher revenue loss compared to group 3. We assume that the central authority implements the SO TCS design that does not factor loss aversion behavior where commuters’ loss aversion coefficient is equal to 20. Then, the credit price becomes equal to 0 which is due to a reduction in the commuters’ credit demand because of significant loss aversion. This makes the TCS inactive, and consequently the total system travel cost increases to 476,937 which is significantly higher than the SO total system travel cost (232,400). Further, the travel disutilities of the three groups become equal to 505.62, 605 and 610.75.

If the central authority factors the loss aversion in SO TCS design, it leads to the SO credit charging scheme of case 2 (Figure 10(b)). Under this SO TCS design, the central authority allocates 8.31, 1.65 and 3.90 credits to groups 1, 2 and 3, respectively. The travel disutilities of the three groups are equal 302, 583 and 533, respectively. Further, the total system travel cost becomes equal to 232,400. It shows that if loss aversion behavior of commuters is not considered, the SO design of TCS can result in an ineffective scheme to achieve the minimum total system travel cost. The credit revenue
distribution of commuters under case 2 is illustrated in Figure 11. While the pattern of credit revenue distribution is identical to case 1, the revenues and losses of commuters are significantly lower compared to that case. It is primarily because the CA allocates fewer credits in case 2 since commuters are highly loss averse in purchasing credits. This leads to lower credit revenue distribution across travelers.

(a) System optimal credit charging scheme under the uniform credit allocation scheme

(b) System optimal credit charging scheme under the group-specific credit allocation scheme

Figure 9. System optimal credit charging schemes with heterogeneous commuters

6. Concluding comments

This study develops an analytical formulation for the management of morning commute congestion using TCS under commuter heterogeneity. It contributes to the literature by considering commuter heterogeneity and the effect of commuters’ loss aversion. The existence and uniqueness of the equilibrium departure rates, travel disutility and credit
price are investigated. The effects of initial credit allocation scheme, total endowment of credits and method of credit allocation are analyzed while considering commuter loss aversion behaviors in trading credits. It is proved that as commuter sensitivity to loss increases, credit price and total value of traded credits approach zero. It is also demonstrated that if commuters are equally sensitive to loss and gain, the credit allocation method does not impact the equilibrium departure rates and credit price. Finally, a primal-dual formulation is developed to derive the SO credit allocation and charging schemes.

Figure 10. System optimal credit charging schemes under different commuter loss aversion behaviors.
Numerical experiments are performed for three groups of commuters with different values of travel time and schedule delay penalties. Numerical results validate that credit price and total value of traded credits approach zero as commuters’ loss sensitivity increases. It is demonstrated that while the total number of allocated credits remains constant, the credit allocation method impacts the credit price and travel disutility of commuters, and consequently it affects the departure rates of commuters. The SO design of TCS, in terms of credit allocation and charging schemes, is computed under different commuter loss aversion coefficients. It is observed that if loss aversion behavior is not considered, the SO TCS design results in less effective scheme to minimize the total system travel cost.

The study insights are twofold. First, if the central authority considers the loss aversion behavior of commuters in trading credits, then the SO credit allocation method impacts the equilibrium credit price and departure rates. Hence, it provides another control parameter for the central authority to achieve system-level objectives beyond credit charging scheme and therefore, it provides greater flexibility for the central authority in designing the TCS. Second, the central authority must factor the loss aversion behavior in SO TCS design because it impacts the departure rates, and consequently credit price in the market. Otherwise, it results in a less effective scheme to minimize the total system travel time.
This study can be extended in several directions in the future. First, the proposed model can be extended to determine equitable design of TCS. A central authority can address the equity issues associated with implementing the TCS by designing group-specific credit allocation schemes and time-varying group-specific credit charging schemes. Second, a continuous time-varying group-specific credit charging scheme can be difficult to implement for managing morning commute congestion. Hence, a “step credit charging scheme” can be determined in which the credit charging scheme is constant over a few time intervals within the peak period. The discontinuity of the number of charged credits at the end of each time interval leads to the discontinuity of departure patterns of commuters. Different behavioral assumptions are proposed to model the discontinuity of departure patterns such as mass arrival (Arnott, de Palma, and Lindsey 1990), separated waiting (Laih 1994; Laih 2004), and braking-induced idling (Lindsey, Van den Berg, and Verhoef 2012). A future research direction is to analyze the effect of the TCS with a step credit charging scheme in a discrete time setting under the aforementioned behavioral assumptions. Third, this study only considers the loss sensitivity of commuters toward monetary gains and losses of trading credits in the market. Another future research direction is to consider the loss sensitivity of commuters related to desired arrival time. Fourth, several studies (Arnott, de Palma, and Lindsey 1988; Arnott, Palma, and Lindsey 1994; Doan, Ukkusuri, and Han 2011; Van den Berg and Verhoef 2011; Xiao, Qian, and Zhang 2011) assume the total system travel cost includes the costs of schedule delay and travel time cost. Another interesting future research direction is to minimize the total system travel disutility which also includes the credit consumption disutility. Fifth, this study assumes deterministic travel demand over the morning peak period. However, the central authority cannot accurately forecast travel demand and hence, TCS needs to design robustly to reduce the impact of forecast inaccuracy on TCS efficiency (Miralinaghi and Peeta 2019; Miralinaghi 2018). Finally, the central authority is assumed to provide initial credit endowments for travelers under TCS. A future research direction is to investigate the effect of TCS on managing morning commute congestion when the central authority sells all credits to travelers through a competitive bidding process in an auction market.
Acknowledgments

This study is supported by the Center for Connected and Automated Transportation (CCAT), Region V University Transportation Center funded by the U.S. Department of Transportation, Award #69A3551747105. The authors are grateful to the constructive comments provided by three anonymous reviewers. The remaining errors are those of the authors alone.

Appendix A. Proof of Theorem 2

This appendix presents a proof for Theorem 2. In this theorem, the assumption of \((n, k) \in \Psi\) ensures that total supply of credit is sufficient to address the travel need of commuters. It is sufficient to show that \(M\) is both \(R_0\)-matrix and copositive. First, it is shown that \(M\) is a copositive matrix. Since \(S\) is a positive definite matrix, the matrix \(M\) can be decomposed as the sum of a positive semi-definite matrix \(P\) and a non-negative matrix \(Q\), where

\[
P = \begin{bmatrix}
0 & M_1 & 0 & (\eta - 1)M_5 & -M_3^T & -\eta M_4 \\
-M_1^T & S & 0 & 0 & 0 & 0 \\
0 & 0 & I & 0 & 0 & 0 \\
0 & 0 & 0 & M_5 & 0 & -M_4 \\
M_3 & 0 & 0 & 0 & 0 & 0 \\
M_4^T & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\quad \text{and} \quad
Q = \begin{bmatrix}
0 & 0 & M_2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & M_1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

For any given positive vector \(x^T = (r^T \quad TT^T \quad e^T \quad z^T \quad RC^T \quad \rho)\), it follows that:

\[
x^TPx = [v_1 \quad v_2 \quad v_3 \quad v_4 \quad v_5 \quad v_6]
\]

where

\[
\begin{align*}
v_1 &= -TT^T M_1^T + RC^T M_3 + \rho M_4^T \\
v_2 &= r^T M_1 + TT^T S \\
v_3 &= e^T I \\
v_4 &= (\eta - 1)r^T M_5 + z^T M_5 \\
v_5 &= -r^T M_3^T \\
v_6 &= -\eta r^T M_4 - z^T M_4
\end{align*}
\]

By using equation (68), \(x^TPx\) is written as follows:
\[ x^T P x = [v_1 \ v_2 \ v_3 \ v_4 \ v_5 \ v_6] \cdot \begin{bmatrix} r \\ T T \\ e \\ z \\ R C \\ \rho \end{bmatrix} \]

\[ = -(T T)^T M_1^r r + (R C^T) M_3 r + \rho M_4^T r + r^T M_1 (T T) + (T T)^T S (T T) + e^T I e \]

\[ + (\eta - 1) r^T M_5 z + z^T M_5 z - r^T M_3^T (R C) - \rho \eta r^T M_4 \]

\[ - \rho z^T M_4 \]

Equation (69) can be simplified using the following equations:

\[ z^T M_5 z - \rho z^T M_4 = \sum_{(t,g)} \frac{z_{t,g} \cdot (z_{t,g} - \rho(n_{g} - k_{t,g}))}{\alpha_{g} + \gamma_{g}} = 0 \]  

\[ \rho M_4^T r + (\eta - 1) r^T M_5 z - \rho \eta r^T M_4 = (\eta - 1) \sum_{(t,g)} \frac{r_{t,g} \cdot [z_{t,g} - \rho(n_{g} - k_{t,g})]}{\alpha_{g} + \gamma_{g}} \]

Note that \((T T)^T M_1^r r\) and \((R C^T) M_3 r\) are scalar. Therefore, \((T T)^T M_1^r r = [(T T)^T M_1^r r]^T = r^T M_1 (T T)\), and \((R C^T) M_3 r = [(R C^T) M_3 r]^T = r^T M_3^T (R C)\). Then, \(x^T P x\) can be rewritten as follows:

\[ x^T P x = (T T)^T S (T T) + e^T I e + (\eta - 1) \sum_{t \in I} \sum_{g \in G} \frac{z_{t,g} \cdot (z_{t,g} - \rho(n_{g} - k_{t,g}))}{\alpha_{g} + \gamma_{g}} \]

Applying equation (72) to \(x^T M x\) yields

\[ x^T M x = x^T P x + x^T Q x \]

\[ = (T T)^T S (T T) + e^T I e + (\eta - 1) \sum_{t \in I} \sum_{g \in G} \frac{z_{t,g} \cdot (z_{t,g} - \rho(n_{g} - k_{t,g}))}{\alpha_{g} + \gamma_{g}} + e^T M_1 (T T) + r^T M_2 e \]

In equation (73), \(e^T I e > 0\) and
\[(TT^T)S(TT) = \begin{bmatrix} TT_0 \\ \vdots \\ TT_{T-1} \end{bmatrix}^T S \begin{bmatrix} TT_0 \\ \vdots \\ TT_{T-1} \end{bmatrix} = \begin{bmatrix} s(TT_0 - TT_1) \\ s(TT_1 - TT_2) \\ \vdots \\ s(TT_{T-1}) \end{bmatrix}^T (TT) \]

\[= s(TT_0 - TT_1)TT_0 + s(TT_1 - TT_2)TT_1 + \cdots + s(TT_{T-1})TT_{T-1} \]

\[= \frac{1}{2} s\left( 2TT_0^2 - 2TT_0TT_1 + 2TT_1^2 - 2TT_1TT_2 + \cdots + 2TT_{t-2}^2 \right) \]

\[= \frac{1}{2} s\left( TT_0^2 + (TT_0 - TT_1)^2 + (TT_1 - TT_2)^2 + \cdots + (TT_{t-2} - TT_{t-1})^2 \right) + TT_{t-1}^2 \]

The third term in equation (73) can be written as follows:

\[
(\eta - 1) \left( \sum_{t \in T} \sum_{g \in G} z_{t,g} \cdot \left( \frac{z_{t,g} - \rho(n_g - k_{t,g})}{\alpha_g + \gamma_g} \right) \right) \]

\[= (\eta - 1) \left( \sum_{t \in T} \sum_{g \in G} \left( \frac{\rho \left( [n_g - k_{t,g}]_+ - (n_g - k_{t,g}) \right)}{\alpha_g + \gamma_g} \right) \right) > 0 \quad (75) \]

In equation (73), \(e^T M_1(TT)\) and \(r^T M_2 e\) are also positive because the elements of \(e\), \(M_1\), \(TT\), \(M_2\) and \(r\) are positive. Consequently, \(x^T M x > 0\). Hence, \(M\) is a copositive matrix. Next, we need to prove that \(M\) is \(R_0\)-matrix. To do so, we must show that zero vector is the only solution to the \(LCP(0,M)\). In other words, \(x = 0\) if and only if:

\[0 \leq x \perp Mx \geq 0 \quad (76)\]

If \(x = 0\), it yields that \(0 \leq x^T, Mx \geq 0\) and \(x^T \cdot Mx = 0\). To prove the “only if” condition, assume that \(0 \leq x^T, Mx \geq 0\) and \(x^T \cdot Mx = 0\). Then,

\[x^T Mx = x^T Px + x^T Qx \]

\[= (TT^T)S(TT) + e^T I e + (\eta - 1) \left( \sum_{t \in T} \sum_{g \in G} z_{t,g} \cdot \left( \frac{z_{t,g} - \rho(n_g - k_{t,g})}{\alpha_g + \gamma_g} \right) \right) + e^T M_1(TT) \]

\[+ r^T M_2 e = 0 \quad (77)\]

According to equation (77), if \(x > 0\), then \(x^T Mx > 0\). Therefore, \(x^T Mx = 0\) only if \(x = 0\). So, \(M\) is both a \(R_0\)-matrix and copositive. Hence, \(MLCP(q,M)\) has a solution.
References


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