Design of a Multiperiod Tradable Credit Scheme under Vehicular Emissions Caps and Traveler Heterogeneity in Future Credit Price Perception

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Abstract: The transportation sector is a major source of traffic congestion and greenhouse gas (GHG) emissions in urban areas. This study aims to design the system-optimal (SO) traffic management policy, i.e., multiperiod tradable credit scheme (MPTCS), for urban transportation infrastructure. Under this scheme, the central authority (CA) seeks to minimize the total system travel time while achieving a predetermined vehicular emissions standard in each period of a long-term horizon with a duration of multiple years. Because travel demand and supply are uncertain in transportation infrastructure over a long-term horizon, the CA cannot provide accurate forecasts of future credit prices (CPs) to travelers a priori, leading to traveler heterogeneity in perceiving these future CPs. It impacts travelers’ decisions about using credits or transferring them to the next period. Further, travelers perceive credit purchases in the market as monetary losses and selling credits as monetary gain. We formulate the SO MPTCS design as a bilevel model. The CA determines the credit charging and allocation schemes in the upper-level model to minimize total system travel time over a horizon constrained by the emissions standard for each period. The MPTCS equilibrium condition is formulated in the lower level where travelers decide about credit consumption and path choice to minimize their travel costs. Numerical experiments suggest that as the difference between travelers’ perception of future CPs and the actual CPs (set by the CA for various periods) increases, the effectiveness of the SO MPTCS design in minimizing total system travel time decreases unless this difference is explicitly factored in the design. Also, if the CA increases emissions standards under the SO MPTCS design, the travel costs increase. DOI: 10.1061/(ASCE)IS.1943-555X.0000570. © 2020 American Society of Civil Engineers.

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Introduction

The concept of smart cities continues to receive attention among researchers and metropolitan authorities (Albino et al. 2015; Khan et al. 2019). It aims to enhance the quality and efficiency of urban service systems (e.g., transportation and energy) to reduce consumption and overall costs (Martin et al. 2019). Sustainability has been recognized as one of the main drivers of smart cities that curb resource consumption in the face of climate pressures in the era of massive urban development (Labi 2014). The three pillars of sustainability are environmental, social (Guo et al. 2018, 2020; Guo and Peeta 2020), and economic, with emissions being a key criterion in the environmental pillar.

A sustainable transportation system is an essential component of sustainable urban infrastructure development. Rapid urbanization has caused tremendous stress on urban transportation systems, leading to increasing environmental concerns (Labi 2014). High levels of highway traffic congestion contribute significantly to greenhouse gas (GHG) emissions, which include carbon monoxide (CO), carbon dioxide (CO₂), nitrogen oxide (NOₓ), and nonmethane volatile organic compounds (NMVOCs) in metropolitan areas. For example, road transportation contributed to 39% of NOₓ, 22% of CO, and 12% of NMVOC emissions in the European Union (EU) in 2013 (European Environment Agency 2015). Vehicular emissions also affect the health of residents in metropolitan areas because they increase the risk of respiratory infection, heart disease, and lung cancer. More than 3 million residents of cities worldwide die every year because of air pollution, and this number is expected to double by 2050 (Lelieveld et al. 2015). In the literature, the need to assess and monitor vehicular emissions in response to transportation projects or policies has been highlighted (Sinha and Labi 2007). To address the problem of emissions, agencies need to develop environmentally sustainable solutions that manage traffic congestion and reduce excessive emissions in urban transportation systems.

Quantity-based market instruments have been suggested in different sectors to regulate the use of natural resources and system performance (e.g., in terms of air quality, water resources, and traffic congestion) (Tietenberg 2004). The EU Emissions Trading System (EU ETS) is a prominent example of the practical deployment of quantity-based instruments. Through the EU ETS, the EU aims to reduce CO₂ emissions by at least 21% in 2020 and 80% in 2050 (Ellerman and Joskow 2008). In the context of managing traffic congestion, quantity-based market instruments have been proposed as an alternative to congestion pricing strategies to address public opposition to pricing. License plate rationing is a well-known quantity-based instrument that has been implemented in several cities such as Mexico City and Tehran, Iran. While it can alleviate congestion and improve air quality in the short term, this initiative could increase vehicle ownership in the long term because

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some travelers purchase more vehicles with different plate numbers to circumvent travel restrictions. Often, such additional vehicles tend to be older and therefore have higher emissions. For this reason, license plate rationing could result in higher emissions in the long term and may therefore not be an effective emissions reduction strategy (Nie 2017). A tradable credit scheme (TCS) was proposed by Yang and Wang (2011) as a quantity-based instrument, which is characterized by credit charging and allocation schemes. The credit allocation scheme is predetermined by a central authority (CA) in terms of the total number of allocated credits and the method for allocation. The CA also predetermines the link-based credit charging scheme in terms of the number of charged credits for traveling on each link. Yang and Wang (2011) also showed that the equilibrium credit price (CP) and link flow pattern under a TCS are independent of the credit allocation method if travelers do not view credits purchased in the market as losses.

Based on the planning horizon length, TCS models are classified into two types in the literature (Miralinaghi and Peeta 2016). The first type, single-period TCS (SPTCS), entails the implementation of a TCS over a short planning horizon in which traffic network demand and supply (e.g., link travel cost functions) are assumed constant during the planning horizon (e.g., He et al. 2013; Nie 2012; Ye and Yang 2013). A comprehensive review of single-period TCS literature is provided by Dogterom et al. (2017), Grant-Muller and Xu (2014), Wang et al. (2018), and Miralinaghi (2018). Hence, the SPTCS is not suitable for achieving the long-term system-level goals related to traffic emissions and congestion externalities. Planners have used quantity-based instruments in other sectors to achieve steady progress toward environmental objectives over a long-term horizon. An example of a TCS system, the Regional Greenhouse Gas Initiative (RGGI) is the first mandatory system in the US that seeks to reduce GHG emissions of power plants. It aims to achieve a 2.5% annual reduction in GHG emissions between 2009 and 2020 (Bifera 2013). To achieve such goals, the second TCS type, multiperiod TCS (MPTCS), deals with the implementation of a TCS over a long-term horizon.

In the literature, researchers have proposed an MPTCS framework for a planning context in which a CA designs a TCS by factoring in the long-term fluctuation in network supply and demand (Miralinaghi and Peeta 2016, 2018, 2019). The researchers divided the planning horizon into multiple periods of equal length, in the order of years, in which the traffic network demand and/or supply were constant within each period but could change across periods. The CA predetermines the credit allocation scheme to distribute credits among travelers in each period. The CA also predetermines the link-based credit charging scheme to charge travelers for link usage in each period. The credit charging and allocation schemes, called the TCS parameters, can vary across periods. Under the MPTCS, travelers decide to store credits to consume or sell them in next period based on various parameters such as the TCS parameters, the network supply and demand, and perfect information on future CPs provided by the CA. The MPTCS framework allows the CA to achieve steady progress toward long-term goals (e.g., reducing vehicular emissions or traffic congestion) by predetermining the TCS parameters. It also allows the CA to manage the progress toward predetermined planner objectives in any period through adjusting the TCS parameters for the next periods based on the current conditions related to planner objectives and current forecasts of future network supply and demand.

This paper seeks to develop the system-optimal (SO) MPTCS design to minimize the total system travel time by factoring in the predetermined emissions standard for each period. Thereby, it determines the SO credit charging and allocation schemes that minimize the total system travel time over the planning horizon subject to the emissions standard for each period. Thereby, the SO MPTCS design is constrained by a vehicular emissions rate for each period that does not exceed a predetermined emissions cap for that period; a higher emissions standard implies a lower emissions cap. These emissions caps are predetermined by a government entity (such as the EU) and need to be factored in by transportation planners in minimizing the total system travel time. We formulate the SO MPTCS design as a bilevel model. The CA determines the MPTCS parameters in the upper level that induce travelers to choose paths that minimize total system travel time constrained by the emissions cap for each period. In the lower level, travelers select the paths with minimum travel costs (including credit consumption and travel time costs) under the MPTCS design determined in the upper level. Later, numerical experiments illustrate that travel costs increase as emissions caps reduce. This implies that the CA needs to trade off reduction in emissions caps with increase in travel costs so that the TCS is sustainable in practice.

The SO MPTCS design is affected by the CA-forecasted CPs for future periods. This is because these future CP forecasts can be inaccurate because the TCS parameters of future periods are difficult to accurately predict a priori because of demand and supply uncertainty. Consequently, heterogeneity exists in travelers’ perception of future CPs. Hence, this paper also examines the effect of travelers’ heterogeneity in terms of perceived future CPs on the SO MPTCS design. Miralinaghi and Peeta (2016) also explore the evolution of equilibrium CPs for a known MPTCS, that is, a TCS with known credit allocation and charging schemes. They showed that CP volatility is reduced under a MPTCS. However, they assume that travelers have perfect knowledge of future CPs a priori. If travelers have imperfect knowledge of future CPs and perceive them differently, some travelers may purchase credits from others who perceive the future CPs as being lower. Then they can hoard these purchased credits to increase CPs and make profit. To mitigate this type of credit hoarding behavior under the MPTCS, this paper assumes that the CA does not permit the transfer of credits purchased in a period to future periods. However, a traveler can transfer their CA-allocated credit endowments to future periods, which can also cause hoarding.

Additionally, the SO MPTCS design in this paper incorporates the effect of interactions between perceived future CPs and loss aversion behavior of travelers toward purchasing credits. Under a TCS, travelers perceive purchasing credits as monetary loss and selling credits as monetary gain. Under loss aversion behavior, monetary losses loom larger than gains for travelers. Bao et al. (2014) proposed a reference-dependent user equilibrium (UE) condition to minimize the total system travel time using a SPTCS. However, it does not consider the impact of the credit allocation method on the equilibrium condition, nor the relationship between CP and market loss aversion behavior. In the context of traffic congestion mitigation at a single bottleneck, Miralinaghi et al. (2019) investigated the effect of travelers’ loss aversion behavior on departure time choices during the morning rush hour using a SPTCS. They showed that as the loss aversion of travelers increases, the CP approaches zero. Then the TCS becomes inactive and cannot be used to reduce vehicular emissions and congestion. In contrast, this paper shows that when travelers’ heterogeneity in perceiving future CPs is factored, the equilibrium CPs may not approach zero because they also depend on travelers’ perceptions of future CPs. This implies that if CPs approach zero over time owing to travelers’ loss aversion behavior, the CA can influence the perceived future CPs to preclude the TCS from becoming inactive. This is another advantage of an MPTCS over an SPTCS. The structure of MPTCS with perceived future CP and loss aversion of travelers is shown in...
Fig. 1. Structure of MPTCS with travelers’ perceived future CPs and loss aversion behavior.

In this paper, the lower-level model is first formulated, in which travelers select the path with minimum costs of travel under given MPTCS parameters. It can be classified as a traffic assignment problem. It addresses the equilibrium condition by factoring in market loss aversion behavior and perceived future CP heterogeneity under the MPTCS, which is formulated using complementarity constraints. Next, the solution uniqueness and existence of the equilibrium CPs and link flows are investigated under an MPTCS when the perceived future CP heterogeneity is factored. Finally, the SO MPTCS design is formulated as a bilevel model. In the upper level, the CA minimizes the total system travel time over the planning horizon constrained by the predetermined vehicular emissions cap for each period. The lower-level model is formulated as the equilibrium condition under the MPTCS parameters determined in the upper level.

The contributions of this paper are threefold. First, the proposed SO TCS design under emissions caps serves as a guideline for transportation planners to manage traffic congestion. It also allows the CA to achieve a predetermined emissions standard for metropolitan transportation infrastructure in each period. Numerical experiments suggest that the effectiveness of the SO TCS design reduces as the difference between travelers’ perception of future CPs and the actual CPs set by the CA for various periods increases. Second, the equilibrium condition that factors travelers’ perceived future CPs improves practical realism by enabling the CA to understand the impact of MPTCS designs in practice. Third, the study illustrates the effect of interactions involving travelers’ perceived future CPs and loss aversion behavior on CPs. It is shown that as the market loss aversion of travelers increases, the equilibrium CP of a period approaches its lower bound, which is the maximum value of the perceived future CPs by travelers selling credits in the market. This implies that if the equilibrium CPs approach zero as market loss aversion of travelers increases, the CA can influence the perceived future CPs to preclude the TCS from becoming inactive.

The remainder of this paper is organized as follows. The next section introduces the key notations and the TCS types. Then the equilibrium condition under the MPTCS is formulated by factoring in the perceived future CP heterogeneity and loss aversion behavior of travelers. Next, the solution existence and uniqueness of the equilibrium CPs and link flow patterns are proven. Then the SO design of MPTCS is formulated to minimize the total system travel time subject to vehicular emissions caps. Finally, the results and insights from numerical experiments are discussed, followed by a few concluding comments.

**Preliminaries**

**Network**

Consider an urban transportation network $G(N, A)$, where $A$ and $N$ represent a set of links and nodes, respectively. Let $R_W$ represent the set of all paths of origin–destination (O-D) pair $w \in W$, where $W$ represents the set of O-D pairs. Let $\Gamma$ denote the set of time periods. Let $c_w^t$ denote the link travel time function, which is assumed to be monotonically increasing with the amount of aggregate link flow $\nu_a^t$ on link $a \in A$ in period $t \in \Gamma$. The traffic network characteristics and travel demands are assumed to be unchanged in each period. The aggregate traffic network demand rate of each O-D pair $w \in W$ in period $t$ is denoted by $q_w^t$. Travelers are assumed to be homogeneous in terms of valuation of travel time, and interest rate is assumed to be zero.

**Types of TCSs**

As discussed previously, TCS models are classified into two types. The first type is SPTCS, which focuses on the traffic network equilibrium state when traffic network demand and supply rates are constant. Single-period CP and single-period equilibrium CP denote the CP and its equilibrium condition under SPTCS, respectively. In MPTCS, the planning horizon is divided into a number of time periods. Multi-period CP and multiperiod equilibrium CP denote the CP and its equilibrium condition under MPTCS, respectively.

Previous studies investigated another scheme, multi-SPTCS,
in which the unused credits are used or traded within each period and voided at the end of each period without any monetary value. Because the equilibrium condition of each period can be investigated separately without factoring in interactions between periods, the CP under multi-SPTCS is equal to its single-period CP in each period.

### Equilibrium Condition under MPTCS

In this section, we first formulate the equilibrium condition for an MPTCS under travelers’ perceived future CP heterogeneity and loss aversion behavior. It serves as the lower-level model of the bilevel SO MPTCS framework.

The TCS is characterized by a link-based credit charging scheme and O-D-specific credit allocation scheme. The credits are issued by the CA to travelers of O-D pair \(w\) at the average rate of \(n^w_t\) per unit of time, where \(N^t = \sum_w n^w_t q^w_t\) is the issued credit rate in period \(t\). The credit allocation scheme is denoted by \(\mathbf{n} = \{n^w_t, w \in W, t \in \Gamma\}\). The CA predetermines the user-agnostic link-based credit charging scheme \(k = (k_t^r, a \in A, t \in \Gamma)\), where \(k_t^r\) is the number of charged credits for using link \(r\) in period \(t\).

Based on the credit charging scheme, travelers are charged \(d^w_{t,S} = \sum_{r \in A} k_t^r \delta_{a;r,w} \) credits to use path \(r\) of O-D pair \(w\), where \(\delta_{a;r,w} = 1\) if link \(a\) is a component of path \(r\) of O-D pair \(w\), and \(0\) otherwise. It is assumed that the CA enforces regulation that prohibits travelers from transferring purchased credits to future periods. Without this regulation, credit hoarding behavior could happen. Travelers are assumed to be homogeneous in terms of their sensitivity to monetary loss associated with credit purchasing behavior. Let \(\eta\) denote the market loss aversion coefficient of travelers. The credit supply of travelers includes the transferred credits from the previous period and issued credits in the current period. The credit supply is assumed to be sufficient for addressing travelers’ credit demand. The credit transfer rate of a traveler of each O-D pair \(w\) from period \(t-1\) to period \(t\) is \(\alpha^w_{t-1}\), and is defined as the average rate of credits transferred by travelers of each O-D pair \(w\) to period \(t\). Let \(\alpha^w\) be the set of multipeiod equilibrium average credit transfer rates of travelers from period \(t\) to period \(t+1\).

Travelers decide to store their credit endowments and transfer them to the next period to consume or sell based on their travel needs, credit supply, market loss aversion behavior, and perception of future CP in each period. The perception of future CP affects the valuation of credits by travelers because they can consume these credits in future periods rather than the current period. Hence, they consider the perceived future CP as the lowest price to trade credits in the current period. If they have higher perceived future CPs compared to the price in the current period, they transfer the unused credits to use in future periods rather than in the current period. For each O-D pair \(w \in W\), travelers are divided into a discrete set of user classes corresponding to their perceptions of future CP. Let \(G\) denote the set of such user classes, and \(\tau^G\) be the set of perceived future CPs for all user classes in period \(t\). In other words, \(\tau^G\) denotes the average perceived CP in period \(t+1\) by travelers of user class \(g\) in period \(t\). Because credits are voided at the end of horizon without gainful value, the perceived future CP in period \(T, \tau^G\), is zero. The traffic network demand rate of each user class \(g \in G\) for each O-D pair \(w \in W\) in period \(t\) is represented by \(d^g_{p,w}\) which is assumed to be known and constant. The aggregate traffic network demand rate is \(d^G_{p,w}\). Let \(x^G_{t,w}\) denote the flow of class \(g\) on path \(r\) of O-D pair \(w\) that transfers credits from time period \(t\) to the next period \(t+1\). Let \(x^G_{t,w}\) denote the vector of path flows of travelers who transfer unused credits to period \(t+1\). Let \(m^G_{t,w}\) represent the flow of class \(g\) on path \(r\) of O-D pair \(w\) that sells credits in the market in period \(t\). Let \(m^G_t = \{m^G_{t,r,w}, w \in W, g \in G, r \in R_w\}\) denote the vector of path flows of travelers who sell credits in period \(t\). To describe the equilibrium condition, paths are divided into two sets. In the first set of paths, travelers have a higher credit supply rate than credit consumption rate, where the difference between the former and latter is referred as the excess credit rate. Let \(z^G_{t,w}\) denote the excess credit rate of travelers using path \(r\) of O-D pair \(w\); it is specified as

\[
z^G_{t,w} = [n^w_t + \alpha^w_{t-1} - k_t^w]_+ \quad \forall \ t, r, w
\]

where \([n^w_t + \alpha^w_{t-1} - k_t^w]_+ = (n^w_t + \alpha^w_{t-1} - k_t^w)\) if \((n^w_t + \alpha^w_{t-1} - k_t^w) \geq 0\), and \(0\) otherwise. In the second set of paths, travelers have a credit supply rate that is lower than their credit consumption rate. Let \(J^G_t\) denote the set of paths of O-D pair \(w\) in period \(t\) for which \(z^G_{t,w}\) is positive. Let \(y^G_{t,w}\) denote the credit demand rate for using path \(r\) of O-D pair \(w\). It is derived as

\[
y^G_{t,w} = [k_t^w - n^w_t - \alpha^w_{t-1}]_+ \quad \forall \ t, r, w
\]

Let \(P_w\) denote the set of paths of O-D pair \(w\) in period \(t\) for which \(y^G_{t,w}\) is positive. Let \(\nu^a\) denote the flow of user class \(g\) on link \(a\) in period \(t\), and \(\nu^a\) denote the aggregate flow of link \(a\) in period \(t\). The flow of user class \(g\) on link \(a\) is represented by \(f^G_{t,w}\), and the aggregate flow of path \(r\) in period \(t\) is denoted by \(f^G_{t,w}\). The path and link flow vectors are denoted by \(f^G_t = \{f^G_{t,w}, w \in W, g \in G\}\), \(f^a_t = (\nu^a, a \in A)^T\), and \(f^G_t = (\nu^a, a \in A, g \in G)\). Let \(\Omega(f_{J_G,w})\) be the feasible set of link and path flows, defined by

\[
\Omega(f_{J_G,w}) = \left\{ (f^G_t, \nu^a, f^G_{t,w}), \nu^a = \sum_{g \in G} \nu^G_{t,w}, \sum_{w \in W} \sum_{r \in R_w} f^G_{t,w} \delta_{a;r,w} = \nu^G_{t,w}, \quad d^G_{p,w} = \sum_{r \in R_w} f^G_{t,w} \sum_{r \in R_w} f^G_{t,w} \geq 0 \right\}
\]

Let \((f^G_t, \nu^a, m^G_t, k^G_t, y^G_t)\) denote the vector of equilibrium path flows, link flows, path flows of travelers selling excess credits in the current period, path flows of travelers transferring excess credits to the next period, and aggregate link flows. Let \(\lambda^G_{t,r,w}\) denote the value of unused credits for a traveler of class \(g\) using path \(r\) of O-D pair \(w\) in time period \(t\); its equilibrium value is represented by \(\lambda^G_{t,r,w}\). It can be expressed as follows:

\[
\lambda^G_{t,r,w} = \left\{ \begin{array}{ll}
p^t_{z^G_{t,r,w}}, & p^t \geq r^t, \ \forall \ g, w, t < T, \ \forall \ r \in J^G_t \\
p^t_{z^G_{t,r,w}}, & p^t \geq r^t, \ \forall \ g, w, t < T, \ \forall \ r \in J^G_t \\
p^t_{z^G_{t,r,w}}, & \forall \ g, w, t = T, \ \forall \ r \in R_w
\end{array} \right.
\]
By using this notation, the credit consumption cost can be reformulated as follows:

\[ \phi_{g,r,t} = \begin{cases} \eta p^t y^t_{g,w} & \forall g,w,t,r \in I_w \ni J_w, \\ -\lambda^t_{g,r,w} & \forall g,w,t,r \in J_w \end{cases} \]  

where \( \eta \) is greater than or equal to 1 because of the loss aversion behavior of traveler. Let \( u^t_{g,r,w} \) denote the travel cost of a traveler of class \( g \) on path \( r \) of O-D pair \( w \), which includes the costs associated with travel time and credit consumption. Here, the unit of travel time and CP are identical. Hence, the unit of travel cost is equal to the time units, and can be expressed as follows:

\[ u^t_{g,r,w} = \phi_{g,r,t} + \sum_{a \in A} c^t_a (v^t_{g,r}) \delta_{a,r,w} \quad \forall g,w,t,r \in R_w \]  

Let \( u^t_{g,r,w} \) denote the multiperiod equilibrium travel cost of a traveler of class \( g \) using path \( r \) of O-D pair \( w \). The \( u^t_{g,r,w} \) is the set of multiperiod equilibrium travel costs of all user classes in period \( t \). Given TCS(\( n,k \)), the multiperiod equilibrium condition is expressed as follows:

\[ 0 \leq f^t_{g,r,w} \leq (u^t_{g,r,w} - m^t_{g,r,w}) \geq 0 \quad \forall g,w,t,r \in R_w \]  

\[ m^t_{g,r,w} + x^t_{g,r,w} = f^t_{g,r,w} \quad \forall g,w,t,r \in J_w \]  

\[ 0 \leq m^t_{g,r,w} - \left( -p^r y^t_{g,r} + \lambda^t_{g,r,w} \right) \geq 0 \quad \forall g,w,t,r \in J_w \]  

\[ 0 \leq x^t_{g,r,w} - \left( -p^r y^t_{g,r} + \lambda^t_{g,r,w} \right) \geq 0 \quad \forall g,w,t,r \in J_w \]  

\[ 0 \leq \lambda^t_{g,r,w} \leq (f^t_{g,r,w} - m^t_{g,r,w}) \geq 0 \quad \forall g,w \in J_w \]  

\[ 0 \leq p^r \leq \sum_{r \in R_w} \sum_{d_g} (m^t_{g,r,z,t-r} - f^t_{g,r,z} y^t_{g,z}) \geq 0 \quad \forall t < T \]  

\[ 0 \leq p^T \leq \sum_{r \in R_w} \sum_{d_g} (f^T_{g,r,z} y^T_{g,z}) \geq 0 \quad \forall t < T \]  

\[ \alpha^t_{g} = \sum_{r \in R_w} \sum_{d_g} (f^t_{g,r,z} - m^t_{g,r,z}) \geq 0 \quad \forall t < T \]  

where \( m^t_{g,r,w} \) is minimum cost of traveler of class \( g \) between O-D pair \( w \) in period \( t \). The \( \perp \) operator implies that vectors \( a \perp b \) if and only if \( a^T b = 0 \). The complementarity constraint in Eq. (7) is the UE condition. In this period, the traffic equilibrium condition, the paths utilized by class \( g \) of O-D pair \( w \) have equal and minimum travel cost \( m^t_{g,r,w} \) in period \( t \). The constraints in Eqs. (8)–(11) describe the decision-making process of travelers about the unused credits. Eq. (8) ensures that the multiperiod equilibrium flow of class \( g \) on path \( r \) of O-D pair \( w \) is equal to or greater than the monetary gain of selling them in that period. The complementarity constraint in Eq. (9) ensures that the multiperiod equilibrium value of unused credits for class \( g \) is positive, then its multiperiod equilibrium value of unused credits is equal to the monetary gain of selling these unused credits in that period. The complementarity constraint in Eq. (10) ensures that the multiperiod equilibrium value of unused credits for class \( g \) on path \( r \) of O-D pair \( w \) is equal to or greater than the monetary gain of transferring credits to the next period \( t + 1 \). Eq. (10) also ensures that if the equilibrium path flow \( x^t_{g,r,w} \) of class \( g \) on path \( r \) of O-D pair \( w \) in time period \( t < T \), which transfers unused credits to period \( t + 1 \), is positive, then its equilibrium value of unused credits is equal to the monetary gain of transferring them.

The constraint in Eq. (11) ensures that the multiperiod equilibrium flow of class \( g \) on path \( r \) of O-D pair \( w \) in the last time period \( T \) is equal to or greater than the travel cost of using path and selling the unused credits in that period. The remaining travelers discard unused credits in the market. Eq. (11) also ensures that if the multiperiod equilibrium value of unused credits for class \( g \) on path \( r \) of O-D pair \( w \) in the last time period \( T \) is positive, travelers consume the entire credit supply in that period. The constraints in Eqs. (12) and (13) are the market equilibrium conditions. Eq. (12) states that the multiperiod equilibrium CP of period \( t < T \) is positive only if travelers purchase the entire market credit supply, i.e., the credits available to sell in the market. Further, it indicates that the market credit supply is greater than or equal to the credit demand of travelers. Eq. (13) states that the multiperiod equilibrium CP of period \( t < T \) is positive only if travelers consume the entire credit supply in that period. Further, it states that the credit supply of period \( T \) is equal to or greater than the credit consumption of that period. The constraint in Eq. (14) ensures the feasibility of multiperiod equilibrium link and path flow patterns in traffic network. The constraint in Eq. (15) determines the average rate of transferred credits for travelers of O-D pair \( w \) from period \( t \) to period \( t + 1 \). Travelers may have different credit transfer rates from previous periods based on the chosen paths. If the charged credits are different on every path for each O-D pair, there are \( |R_w| \) classes of travelers based on the credit transfer rate in each period, where \( |R_w| \) is the cardinality of \( R_w \). Then there are \( |R_w| \times |W| \) classes of travelers based on credit supply in the next period. Eq. (15) is included to avoid significant clustering of travelers based on the credit supply in each period. There are only \( |W| \) classes of travelers based on credit supply rate.

The multiperiod equilibrium conditions in Eqs. (7)–(14) can be divided into \( T \) subproblems, where each subproblem solves the equilibrium condition in a period given the TCS parameters and credit transfer rate from the previous period. Thereby, the equilibrium condition is first analyzed for the first period, and then the multiperiod equilibrium credit transfer rate for the second period is determined for each O-D pair. Similarly, the multiperiod equilibrium credit transfer rates for other periods are determined using the subproblems for the corresponding equilibrium conditions.

**Existence of Equilibrium Solution**

Given the TCS (\( n,k \)) and multiperiod equilibrium average credit transfer rates for each period \( t < T \), the equilibrium condition for that period can be formulated as the following variational inequality (VI) to determine \( f^t_{G}, \nu^t, x^t_{G} \in \Omega_{(f^T_{G},V)} \) and \( m^t_{G}, \alpha^t_{G}, p^T \in \mathbb{R}^+ \):

\[ u^t_{G} + \sum_{(g,w) \in R_w} (m^t_{g,r,w} - f^t_{g,r,w}) (p^T - p^r) + 0.5 \sum_{(g,w) \in R_w} \sum_{r \in J_w} (m^t_{g,r,w} - m^t_{g,r,w}) (p^T - p^r) + x^t_{G} (x^t_{G} - x^t_{G}) \geq 0 \]  

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\[ m'_{g,r,w} + x'_{g,r,w} = f'_{g,r,w} \quad \forall \ g,w, \forall \ r \in J'_W \] (17)
\[ (m'_G,x'_G) \in \mathcal{R}^+ \] (18)
\[ (f'_G,\nu',\nu'_G) \in \Omega_{(f_G,x_G)} \] (19)

where \( \mathcal{R}^+ \) is the nonnegative orthant of \( \mathcal{R} \).

**Proposition 1:** The VI problem in Eqs. (16)-(20) is equivalent to the multiperiod equilibrium condition in Eqs. (7)-(14) of period \( t \).

**Proof:** The proof of Proposition 1 can be found in Appendix I.

**Proposition 2:** The solution set of the VI problem in Eqs. (16)-(19) is nonempty.

**Proof:** Because of space constraints, the proof is omitted.

### Uniqueness of Equilibrium Solution

**Proposition 3:** The multiperiod equilibrium aggregate link flow pattern of period \( t \) is unique if the multiperiod equilibrium CP of that period is unique.

**Proof:** The proof of Proposition 3 can be found in Appendix II. The uniqueness of multiperiod equilibrium aggregate link flows of each period leads to the uniqueness of credit consumption rate because the credit charging scheme \( k \) is constant. If the credit consumption rates are unique, the credit transfer rate from the first period to the second period is also unique under the equilibrium condition because travelers transfer unused credits to the second period. Thereafter, the credit supply and transfer rates in each period are also unique under the uniqueness of multiperiod equilibrium CP.

Under MPTCS, if the perceived future CPs are equal to zero, then the equilibrium condition is identical to the one under SPTCS. Consider the following two cases for multiperiod equilibrium CP. First, if the multiperiod equilibrium CP is positive, then it follows from Eqs. (8)-(10) that the equilibrium path flow \( x'_{g,r,w} \) of class \( g \) using path \( r \) of O-D pair \( w \), which transfers unused credits to period \( t + 1 \), is equal to zero. Then the credit transfer rate from period \( t \) to period \( t + 1 \) is equal to zero. Hence, the multiperiod equilibrium link flow pattern and CP are equal to those under SPTCS. Second, if the multiperiod equilibrium CP is also equal to zero, it implies that the credit supply is strictly higher than the credit consumption of travelers according to Eq. (12) under MPTCS. Then, MPTCS in period \( t \) becomes inactive, and the multiperiod equilibrium link flow pattern and CP are identical to those under SPTCS.

As discussed by Yang and Wang (2011), the equilibrium CP uniqueness is an indicator for a healthy market. If the equilibrium CP is nonunique, travelers purchase the required credits to fulfill their travel needs at uncertain prices. Hence, it can decrease their travel choices as CP increases. The next proposition provides sufficient conditions for the uniqueness of the multiperiod equilibrium CP that factor in the effect of perceived future CPs under MPTCS.

**Proposition 4:** Given a TCS \((n,k)\), the multiperiod equilibrium CP of each period \( t \) is unique if the following conditions hold:
1. The multiperiod equilibrium link flows are unique.
2. There exists at least one class of O-D pair \( w \) whose equilibrium path set contains more than one equilibrium path with a different number of charged credits where either (a) this class credit supply rate is greater than the credit charges of at least one of the used paths, or (b) this class has a lower perceived future CP than the multiperiod equilibrium CP in period \( t \).

**Proof:** The proof of Proposition 4 can be found in Appendix III.

Based on the proof of Proposition 4, the future CP perception of travelers with unused credits has a significant impact on the uniqueness of multiperiod equilibrium CP. Next, Proposition 4 analyzes the effect of perceived future CPs of travelers with unused credits on the multiperiod equilibrium CP in the market.

**Proposition 5:** The multiperiod equilibrium CP in period \( t \) is greater than or equal to the maximum perceived future CP of travelers selling their unused credits in that period under the equilibrium condition.

**Proof:** Because of space constraints, this proof is omitted.

Proposition 5 has an intuitive interpretation. Travelers decide either to sell the credits in the current period or transfer them to the next period based on the multiperiod CP in the current period and their perceived future CPs. If their perceived future CPs are higher than the multiperiod CP in the market, they transfer the unused credits to consume or sell in future periods. Hence, market credit supply decreases, and if some travelers need to purchase extra credits to address their travel needs, they must purchase credits at a higher price. Then the multiperiod CP increases until the market credit supply is sufficient to address the travel needs. Proposition 5 proves that the multiperiod equilibrium CP is greater than or equal to the maximum perceived future CP of travelers selling their excess credits.

In the proof of Proposition 4, the equilibrium CPs in Eqs. (60), (62), and (63) depend on travelers’ loss aversion behavior. However, they are independent of the market loss aversion coefficient of travelers, \( \eta \), in Eq. (64) if the credit supply rate of a traveler of class \( g \) is higher than or equal to the charged credits for used paths. Consequently, if the equilibrium CP is solely derived by the path choice of travelers with higher credit supply than charged credits, then the multiperiod CP is independent of travelers’ loss aversion behavior. This special case is unlikely to occur in practice because it is probable that there exists a class with an equilibrium path set containing more than one equilibrium path for one O-D pair, which chooses the path with higher credit charges compared to its credit supply. Hereafter, we ignore this special case in analyzing the impact of loss aversion behavior on the multiperiod equilibrium CPs. Next, Proposition 6 investigates the impact of the loss aversion of travelers on the multiperiod equilibrium CP.

**Proposition 6:** If market loss aversion of travelers increases in a period, then the multiperiod equilibrium CP of that period approaches the maximum value of perceived future CPs of travelers selling unused credits.

**Proof:** The multiperiod equilibrium CP can be derived using any of Eqs. (60), (62) or (63). Using Eq. (60), it follows that
\[
\lim_\eta \rho^* = \lim_\eta \left( \frac{\sum_{w \in \bar{A}} \sum_{a \in A} c_w'(y'_{1,w} - \delta_{a,r_{1,w}} + \lambda'_{y_{1,w} - y'_{2,w}})}{\eta(y'_{1,w} - y'_{2,w})} \right) = 0
\] (20)

Similarly, it can be proven that the multiperiod equilibrium CP, obtained using Eq. (62) or (63), also approaches zero. However, Proposition 5 shows that the maximum perceived future CP of travelers selling their credits in period \( t \) is the lower bound for the multiperiod equilibrium CP in that period. Hence, the multiperiod equilibrium CP approaches its lower bound as the market loss aversion of travelers increases. This concludes the proof.

Proposition 6 has an important implication. If the loss aversion of travelers increases, then travelers use paths with fewer charged credits. As the market loss aversion increases, travelers purchase fewer credits, and hence the multiperiod equilibrium CP decreases. However, travelers with unused credits consider their perceived future CPs as the lowest price to trade the unused credits in the market. Hence, the multiperiod equilibrium CP approaches the maximum perceived future CP of travelers selling credits in the market. This implies that if CPs approach zero over time because of travelers’ loss aversion behavior, the CA can influence the perceived future CPs to preclude the TCS from becoming inactive. This is another advantage of an MPTCS over an SPTCS.
System-Optimal Design of MPTCS

This section formulates a mathematical program to obtain the SO MPTCS design to minimize the total system travel time while satisfying the predetermined vehicular emissions cap in each period. Let \( E^t \) denote the predetermined emissions cap of period \( t \). It is obtained by assuming that the total vehicular emissions rate in period \( t \), \( E^t \), is proportional to the aggregate link flows (Yang and Huang 2005)

\[
E^t = \sum_{a \in A} \theta_a v^t_a \quad \forall \ a \in A
\]

where \( \theta_a \) = emissions factor of link \( a \) in period \( t \), and depends on several factors such as link length, the vehicle type, and speed. We formulate the SO design of the MPTCS as a mathematical program with equilibrium constraints (MPEC)

\[
\min_{\mu^t} \sum_{r \in R} \sum_{g \in G} c^t_g(v^t_g) v^t_g
\]

\[
E^t \leq \bar{E}^t \quad \forall \ t
\]

\[
0 \leq f^t_{g,r,w} \perp (a^t_{g,r,w} - v^t_{g,w}) \geq 0 \quad \forall \ g,w, t, \forall \ r \in R_w
\]

\[
m^t_{g,r,w} + x^t_{g,r,w} = f^t_{g,r,w} \quad \forall \ t < T, g, w, \forall \ r \in J^t_w
\]

\[
0 \leq m^t_{g,r,w} \perp (-p^t z^t_{g,r,w} + \lambda^t_{g,r,w}) \geq 0 \quad \forall \ r \in J^t_w, \quad \forall \ t, g, w
\]

\[
0 \leq n^t_{g,r,w} \perp (-r^t g_{r,w} + \lambda^t_{g,r,w}) \geq 0 \quad \forall \ g, w, t < T, \forall \ r \in J^t_w
\]

\[
\lambda^t_{g,r,w} \perp (f^t_{g,r,w} - m^t_{g,r,w}) \geq 0 \quad \forall \ g, w, r \in J^t_w
\]

\[
0 \leq p^t \perp \sum_{r \in R} \sum_{g \in G} (m^t_{g,r,w} + x^t_{g,r,w} - f^t_{g,r,w} y^t_{r,w}) \geq 0 \quad \forall \ t < T
\]

\[
0 \leq p^t \perp \sum_{r \in R} \sum_{g \in G} (f^t_{g,r,w} y^t_{r,w} - f^t_{g,r,w} y^t_{r,w}) \geq 0
\]

\[
\alpha^t_w = \sum_{r \in R} \sum_{g \in G} x^t_{g,r,w} y^t_{r,w} \quad \forall \ t < T
\]

The MPEC in Eqs. (22)–(32) is a bilevel model. The upper-level model consists of the objective function in Eq. (22) and the constraint in Eq. (23). The CA solves the bilevel model to determine the SO MPTCS design that minimizes the total system travel time constrained by the predetermined emissions cap for each period. Eq. (22) minimizes the total system travel time over the planning horizon. Eq. (23) states that the total vehicular emissions rate of each period \( t \) should not exceed the emissions cap for that period. In the lower-level model [the constraints in Eqs. (24)–(32)], travelers minimize their travel costs under the SO MPTCS design derived in the upper level. Eqs. (24)–(32) are identical to the MPTCS model in Eqs. (7)–(15).

Numerical Experiments

This section discusses the numerical experiments to analyze (1) the impacts of travelers’ heterogeneity in terms of perceived future CP on the equilibrium condition under the MPTCS, and (2) the SO MPTCS design. Fig. 2 shows a network with eight nodes and 14 links. The travel time function for each link is assumed to mimic the US Bureau of Public Roads (BPR) function.

It is assumed that the CA aims to reduce CO emissions using TCS compared to the UE solution without TCS. Using the model proposed by Yin and Lawphongpanich (2006), the CO emissions function \( e_a^t(v^t_a) \) for each link \( a \) in period \( t \) is

\[
e_a^t(v^t_a) = 0.2038 \cdot c_a^t(v^t_a) \cdot \exp\left(0.7962 \cdot \frac{p^t_a}{c_a^t(v^t_a)}\right) \quad \forall \ a, t
\]

where \( c_a^t(v^t_a) \) = travel time (min); and \( p^t_a \) = length (km) of link \( a \) in period \( t \). The emissions factor \( \theta_a \) is assumed to be derived by the CO emissions function under the equilibrium condition without TCS.

Table 1 gives the parameters of the link travel time functions including capacity, length, free flow travel time, credit charging scheme, and emissions factors. The aggregate O-D demand and total issued credit rates during the planning horizon are given in Table 2. We divide the planning horizon into 10 periods. There are two classes in terms of perceived future CP in the traffic network; the first has a higher perceived future CP compared to the second. It is assumed that credits are uniformly allocated to travelers unless stated otherwise. The models are solved using CONOPT and GAMS.

Sensitivity of Equilibrium Condition under MPTCS to the Perceived Future CP

Figs. 3 and 4 illustrate the effect of future CP perceived by travelers on the evolution of the equilibrium CPs under MPTCS. The market loss aversion coefficient of travelers is assumed to be 3, i.e., \( \eta = 3 \). Let \( p^t \) denote the multiperiod equilibrium CP under MPTCS with perfect knowledge of travelers about future CP in period \( t \), referred to as TCSP, depicted in Fig. 3. Under TCSP, travelers have perfect knowledge of future CP a priori. The traffic network demand rates of the classes are given by

\[
d^1_{w,t} = (\sigma) q^w, d^2_{w,t} = (1 - \sigma) q^w \quad \forall \ w, t
\]

where \( \sigma = \) parameter that determines the percentage of each class in the traffic network.

Three cases were analyzed for the demand rates of the two classes. In the first case, the demand consisted only of travelers of the first class, i.e., \( \sigma = 1 \). Fig. 3 shows that the multiperiod equilibrium CP in Periods 1–6 for this case is higher than the multiperiod equilibrium CP of those periods under TCSP. This is because the higher perceived future CP under the first case compared to the equilibrium CPs under TCSP leads travelers to store more credits in those periods, which increases the multiperiod equilibrium CPs. Consequently, travelers have more credits to consume in Periods 7 and 8 compared to TCSP. Hence, the multiperiod equilibrium CP of that period is less than the equilibrium CP under TCSP. Because of the low multiperiod equilibrium CPs in Periods

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8–10, the perceived future CPs of travelers have a negligible effect on the multiperiod equilibrium CPs.

In the second case, the demand rates of the two classes are $d_{1,w} = (0.5)q_{tw}$ and $d_{2,w} = (0.5)q_{tw}$, i.e., $\sigma = 0.5$. Because of the lower perceived future CP by the second class compared to the equilibrium CPs under TCSP, the credit transfer rates from previous periods, travelers have a lower credit supply in Periods 3, 4, 6, and 7, and the multiperiod equilibrium CP is higher than the equilibrium CP under TCSP. Hence, the fluctuation of the multiperiod equilibrium CPs is higher through the planning horizon compared to the first case and TCSP because of the lower credit transfer rates across periods. In the third case, the demand consists only of travelers of the second class, i.e., $\sigma = 0$. Similar to the previous case, the credit transfer rate reduces through the planning horizon because of the lower perceived future CP. Hence, the fluctuation of multiperiod equilibrium CP increases compared to the previous cases and TCSP.

Previous studies demonstrated that if travelers have perfect knowledge of future CPs a priori, the multiperiod equilibrium CP decreases through the planning horizon. However, as illustrated in Fig. 3, the multiperiod equilibrium CP can increase or decrease because of travelers’ heterogeneity of future perceived CP.

Fig. 4 illustrates the effect of perceived future CPs and loss aversion of travelers on the evolution of the multiperiod equilibrium CPs under MPTCS and TCSP. The perceived future CPs of the first and second classes are equal to 1.3 and 0.7, respectively. The demand rates of these classes are $d_{1,w} = (0.5)q_{tw}$ and $d_{2,w} = (0.5)q_{tw}$. When the loss aversion coefficient is equal to 5, travelers of Class 2 need to purchase credits from Class 1 during Periods 1, 3, and 7. Hence, the perceived future CPs of Class 1 are the lower bounds for trading credits in the market (Proposition 5). However, the credit supply of Class 2 is sufficient to address its travel needs. Hence, these travelers do not purchase credits from travelers of Class 1. As the loss aversion increases to 30, the multiperiod equilibrium CPs decrease (Proposition 6). Further, the multiperiod equilibrium CPs decrease monotonically through the planning horizon under TCSP with travelers’ market loss aversion. They approach zero as the market loss aversion increases, consistent with Proposition 6. If CPs become equal to zero, then the TC becomes inactive and loses its effectiveness in reducing total system travel time. However, when travelers have higher perceived future CPs than equilibrium CPs under TCSP, the equilibrium CPs do not approach zero except in the last period. This is because the credit

### Table 1. Link characteristics for the eight-node network

<table>
<thead>
<tr>
<th>Link</th>
<th>Start node</th>
<th>End node</th>
<th>Capacity</th>
<th>Free flow travel time (min)</th>
<th>Charged credits (number of credits)</th>
<th>Length (km)</th>
<th>Emissions factor (g/vechile)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>4</td>
<td>12</td>
<td>5</td>
<td>5</td>
<td>10</td>
<td>5.06</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>5</td>
<td>18</td>
<td>6</td>
<td>6</td>
<td>12</td>
<td>5.88</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>5</td>
<td>20</td>
<td>9</td>
<td>9</td>
<td>18</td>
<td>9.36</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>6</td>
<td>11</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>8.36</td>
</tr>
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<td>4</td>
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<td>26</td>
<td>4</td>
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<td>8</td>
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<td>8</td>
<td>8</td>
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<td>33</td>
<td>6</td>
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<td>5.89</td>
</tr>
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<td>3</td>
<td>43</td>
<td>6</td>
<td>6</td>
<td>12</td>
<td>6.17</td>
</tr>
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</table>

### Table 2. Aggregate traffic network O-D demand and issued credit rates

<table>
<thead>
<tr>
<th>Time period</th>
<th>Total issued credit rate ($N^i$)</th>
<th>Aggregate traffic network demand ($q_{tw}$) (1, 2)</th>
<th>Aggregate traffic network demand ($q_{tw}$) (1, 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>970</td>
<td>25</td>
<td>28</td>
</tr>
<tr>
<td>2</td>
<td>900</td>
<td>28</td>
<td>23</td>
</tr>
<tr>
<td>3</td>
<td>850</td>
<td>26</td>
<td>22</td>
</tr>
<tr>
<td>4</td>
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</tr>
<tr>
<td>5</td>
<td>900</td>
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<td>25</td>
</tr>
<tr>
<td>6</td>
<td>1,030</td>
<td>32</td>
<td>26</td>
</tr>
<tr>
<td>7</td>
<td>1,050</td>
<td>32</td>
<td>28</td>
</tr>
<tr>
<td>8</td>
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<td>25</td>
<td>26</td>
</tr>
<tr>
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<td>880</td>
<td>25</td>
<td>23</td>
</tr>
<tr>
<td>10</td>
<td>950</td>
<td>29</td>
<td>24</td>
</tr>
</tbody>
</table>
supply becomes higher than credit demand in the last period because of the high credit transfer rates from the previous period. This suggests that the CA can leverage information provision on future CPs under MPTCS to prevent significant reductions in equilibrium CPs as travelers’ loss aversion increases.

Fig. 5 illustrates the effect of heterogeneity in perceived future CPs on the market income of travelers selling credits in the market. The perceived future CPs of the first and second classes are equal to 1.3 and 0.7, respectively. The demand rates are $d_{1,w}^f = (0.5)q_{tw}$ and $d_{2,w}^f = (0.5)q_{tw}$. The loss aversion coefficient is 5. Because the first class has a higher perceived future CP than the equilibrium CP in the market, it stores credits to sell in future periods. Consequently, as seen in Fig. 5(a), this decision leads to lower market income than the one under TCSP. Because travelers of Class 2 only trade credits among themselves under CP heterogeneity, they have higher market income compared to that under TCSP [Fig. 5(b)]. This implies that if travelers have more accurate information about future CPs, especially if their perceived future CPs are higher than the equilibrium CPs, they can avoid potential monetary losses in the market.

**SO Design of TCS**

This section addresses the SO design of MPTCS to minimize the total system travel time subject to emissions caps. Here, the CA aims to reduce CO emissions by $\zeta \%$ compared to the equilibrium condition without TCS. The market loss aversion coefficient is assumed to be 2, and demand rates are $d_{1,w}^f = (0.5)q_{tw}$ and $d_{2,w}^f = (0.5)q_{tw}$. The multiperiod equilibrium CP during the planning horizon is set to 1. The perceived future CPs of the first and second classes are set to 1.5 and 0.5, respectively. The evolution of vehicular CO emissions rate and total system travel time under the equilibrium condition, SO design without emissions caps, and SO design with 2% and 4% emissions reductions are shown in Fig. 6.

Fig. 6(a) shows that the system CO emissions are lower under the UE solution without TCS compared to the SO design without emissions caps. This is consistent with the findings of previous studies (e.g., Yin and Lawphongpanich 2006), which shows that the CO emissions under SO are higher than those of UE without congestion pricing. Because the CA seeks to reduce emissions by 2% compared to the UE solution without TCS, the SO design leads to slightly higher total travel time. If the CA seeks to reduce CO emissions by 4%, the total travel time increases even more compared to the SO design with 2% emissions reduction. This illustrates that emissions reduction and congestion mitigation objectives can be conflicting.

Fig. 8 illustrates the impacts of factoring travelers’ heterogeneity in perceived future CPs on the TCS design by considering two scenarios. The first scenario assumes perceived future CPs of Classes 1 and 2 as 4 and 2, respectively. The second scenario assumes perceived future CPs of Classes 1 and 2 as 6 and 1, respectively. The actual CPs set by the CA are equal to 3. The loss aversion coefficient of travelers for both CP heterogeneity scenarios is set as 1.5. For the CP homogeneity scenario, the CA assumes that travelers have a perceived future CP of 3.

Fig. 8 indicates that the total system travel time in both scenarios under that the SO TCS design, which does not factor in the heterogeneity of travelers and assumes that travelers are homogeneous, achieves the emission reduction and congestion mitigation objectives.

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future CPs. Travelers are heterogeneous in terms of perception of future CPs because the CA-provided forecasts of future CPs can be inaccurate because the TCS parameters of future periods are difficult to predict a priori owing to demand and supply uncertainty. Travelers are divided into discrete classes based on their perceived future CPs.

The SO MPTCS design was formulated as a bilevel model. In the upper level, the CA determines the MPTCS design that minimizes the total system travel time subject to a predetermined vehicular emissions cap for each period. The lower-level problem is a traffic assignment model that has been used in several studies (Miralinaghi et al. 2015, 2017; Zhu et al. 2017). In the lower level, travelers minimize their own travel costs under the MPTCS design determined in the upper level. The lower-level model is the equilibrium condition under MPTCS that factors travelers’ loss aversion and perceived future CPs. The numerical experiments show that if travelers have perfect knowledge of future CPs a priori, the fluctuation in equilibrium CPs in the market reduces through the planning horizon. This helps travelers avoid potential monetary losses.

The management implications of this research are fourfold. First, the CA needs to factor travelers’ perceived future CP heterogeneity in the optimal design of MPTCS. Otherwise, the predetermined emissions standards and minimum total system travel time cannot be achieved over the long-term planning horizon. Second, regulatory instruments are necessary to enable the CA to address market manipulation by travelers such as credit hoarding behavior. If the perceived future CPs are high, they can motivate travelers to hoard credits, which artificially increases the equilibrium CPs. For example, Miralinaghi and Peeta (2016) suggested using a transfer fee as a regulatory instrument to prevent credit hoarding behavior of travelers. Third, public acceptance of TCS is important for its successful implementation. In practice, travelers may tolerate monetary losses because of the lower perceived future CPs compared to the actual equilibrium CPs in future periods. This significantly impacts the TCS public acceptance. Hence, the CA needs to increase the forecast accuracy of future equilibrium CPs to make the TCS sustainable in practice. Further, the CA can increase or decrease the credit supply to adjust CPs in market. Fourth, the CA needs to consider the possibility of conflicting objectives of emissions reduction and congestion mitigation under the SO MPTCS design, and trade off these goals based on real-world needs.

This paper can be extended in various directions. First, the uncertainty in traffic network demand/supply can be factored in determining the SO MPTCS design over the planning horizon. Another direction is to investigate the MPTCS design under a class-specific credit charging scheme in which travelers are classified and charged according to the emissions levels of their vehicles. This could motivate travelers to shift to alternative travel modes such as public transit or vehicle types with fewer emissions.

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Appendix I. Proof of Proposition 1

Let \((f_G^t, \nu^t, \nu^*_G, m_G^t, x_G^t, \lambda_G^t, p^r)\) be a solution of the VI problem in Eqs. (16)–(19). It solves the VI problem in Eqs. (16)–(19) if and only if it solves the following linear optimization problem:

\[
\min_{t} u^r \left( f_G^t, \nu^t, \nu^*_G, m_G^t \right) = \min_{t} \left( \sum_{w \in A} \left( \sum_{r \in R_w} \left( p^r m_{g,r,w}^t + \tau_{g,r,w}^t \right) \right) + \sum_{r \in R_w} p^r (m_{g,r,w}^t - f_{g,r,w}^t \nu_w^r) \right) \tag{35}
\]

\[
m_{g,r,w}^t + x_{g,r,w}^t = f_{g,r,w}^t \quad \forall g, w, r \in J_G^t \tag{36}
\]

\[
\lambda_G^t \in \mathbf{R}^+ \tag{37}
\]

\[
(f_G^t, \nu^t, \nu^*_G) \in \Omega_{(f, \nu, \nu^*)} \tag{38}
\]

The first-order conditions [Karush-Kuhn-Tucker (KKT) conditions] of the optimization problem in Eqs. (35)–(38) are

\[
0 \leq f_{g,r,w}^t - (m_{g,r,w}^t - \mu_{g,r,w}^t) \geq 0 \quad \forall r \in R_w, \forall t, g, w \tag{39}
\]

\[
m_{g,r,w}^t + x_{g,r,w}^t = f_{g,r,w}^t \quad \forall t, w, g, \forall r \in J_G^t \tag{40}
\]

\[
0 \leq m_{g,r,w}^t - (\lambda^t_{g,r,w} - p^r z^r_{g,r,w}) \geq 0 \quad \forall g, w, t, \forall r \in J_G^t \tag{41}
\]

\[
0 \leq m_{g,r,w}^t - \lambda_{g,r,w}^t \geq 0 \quad \forall g, w, \forall r \in J_G^t \tag{42}
\]

\[
0 \leq \lambda_{g,r,w}^t \geq 0 \quad \forall g, w, \forall r \in J_G^t \tag{43}
\]

\[
(f_G^t, \nu^t, \nu^*_G) \in \Omega_{(f, \nu, \nu^*)} \tag{44}
\]

where \(\lambda_{g,r,w}^t\) and \(\mu_{g,r,w}^t\) are Lagrange multipliers associated with the constraints in Eqs. (36) and (38), respectively. The KKT conditions in Eqs. (39)–(44) are equivalent to the multiperiod equilibrium conditions in Eqs. (7)–(14) of period \(t\), hence the optimal solution of multiperiod equilibrium conditions in Eqs. (7)–(14) of period \(t\) are identical to the solution of the VI problem in Eqs. (16)–(19). This completes the proof.

Appendix II. Proof of Proposition 3

Let \(p^r\) denote the multiperiod equilibrium CP of period \(t\). Given the TCS\((n, k)\) and multiperiod equilibrium credit transfer rates to period \(t\), the multiperiod equilibrium conditions in Eqs. (7)–(14) for period \(t\) can be formulated as the nonlinear optimization problem

\[
\begin{align*}
\min z &= \sum_{a \in A} \int_0^{\nu_{a}} c_a'(w) dw - \sum_{r \in R_w} \sum_{g \in G} \tau_{g,r,w}^t x_{g,r,w}^t \\
&\quad + \sum_{r \in R_w} \sum_{g \in G} (1 - p^r) f_{g,r,w}^t \nu_w^r \\
&\quad + \sum_{r \in R_w} \sum_{g \in G} (1 - p^r) m_{g,r,w}^t \\
&\quad + \sum_{r \in R_w} \sum_{g \in G} (1 - p^r) x_{g,r,w}^t \\
&\quad \leq \sum_{r \in R_w} \sum_{g \in G} (1 - p^r) (m_{g,r,w}^t - f_{g,r,w}^t) \nu_w^r \\
&\quad \in \mathbf{R}^+ \tag{45}
\end{align*}
\]

\[
(m_G^t, x_G^t) \in R^+ \tag{48}
\]

\[
(f_G^t, \nu^t, \nu^*_G) \in \Omega_{(f, \nu, \nu^*)} \tag{49}
\]

Because Eqs. (45)–(49) are a convex nonlinear program with linear constraints owing to the assumed properties of link travel time functions, there exist Lagrange multipliers \(\lambda_G^t\) and \(p^r\) associated with the constraints in Eqs. (46) and (47), respectively. The first-order conditions of nonlinear optimization problem in Eqs. (45)–(49) are

\[
0 \leq f_{g,r,w}^t - (c_a'(\nu_{a}) \delta_{a,r,w}^t + \eta p^r y_{r,w}^t - \lambda_{g,r,w}^t - \mu_{g,r,w}^t) \geq 0 \quad \forall g, w, t, \forall r \in J_G^t \tag{50}
\]

\[
0 \leq m_{g,r,w}^t - (c_a'(\nu_{a}) \delta_{a,r,w}^t + \eta p^r y_{r,w}^t - \mu_{g,r,w}^t) \geq 0 \quad \forall g, w, t, \forall r \in J_G^t \tag{51}
\]

\[
0 \leq x_{g,r,w}^t - (c_a'(\nu_{a}) \delta_{a,r,w}^t + \lambda_{g,r,w}^t) \geq 0 \quad \forall g, w, t, \forall r \in J_G^t \tag{52}
\]

\[
0 \leq \lambda_{g,r,w}^t \geq 0 \quad \forall g, w, \forall r \in J_G^t \tag{53}
\]

\[
(f_G^t, \nu^t, \nu^*_G) \in \Omega_{(f, \nu, \nu^*)} \tag{56}
\]

The first-order conditions in Eqs. (50)–(56) are equivalent to the multiperiod equilibrium conditions in Eqs. (7)–(14) of period \(t\), where the multiperiod equilibrium aggregate link flows are unique. This concludes the proof.

Appendix III. Proof of Proposition 4

Suppose two paths \(r_1, r_2 \in R_w\) with different path credit charges, \(k_{r_1,w} \neq k_{r_2,w}\), for class \(g \in G\) exist for O-D pair \(w \in W\) in period \(t\). The travel cost can be written as

\[
\sum_{a \in A} c_a'(\nu_{a}) \delta_{a,r_1,w}^t + \eta p^r y_{r_1,w}^t - \lambda_{g,r_1,w}^t = \mu_{g,w}^t \tag{57}
\]

\[
\sum_{a \in A} c_a'(\nu_{a}) \delta_{a,r_2,w}^t + \eta p^r y_{r_2,w}^t - \lambda_{g,r_2,w}^t = \mu_{g,w}^t \tag{58}
\]

Subtracting Eq. (58) from Eq. (57) yields

\[
\sum_{a \in A} c_a'(\nu_{a}) (\delta_{a,r_1,w}^t - \delta_{a,r_2,w}^t) + \eta p^r (y_{r_1,w}^t - y_{r_2,w}^t) - (\lambda_{g,r_1,w}^t - \lambda_{g,r_2,w}^t) = 0 \tag{59}
\]

Three possible cases exist related to the credit supply \(\nu^*_{a}\) of travelers of class \(g\) of O-D pair \(w\) and the number of charged credits for paths \(r_1\) and \(r_2\) in period \(t\). In the first case, travelers need to purchase extra credits to travel on either path \(r_1\) and \(r_2\), that is, \(k_{r_1,w} \neq k_{r_2,w}\). Then credit demand rates for using paths \(r_1\) and \(r_2\) are positive, that is, \(y_{r_1,w}^t, y_{r_2,w}^t \geq 0\). Because \(y_{r_1,w}^t \neq y_{r_2,w}^t\), Eq. (59) can be rewritten as
Given the uniqueness of multi-period equilibrium link flows $\nu^{r}$, the multi-period equilibrium CP in period $t$ is unique under the first case. In the second case, travelers need to purchase extra credits to travel on one of the paths, i.e., $r_{1}$, while they have unused credits on the other path, i.e., $r_{2}$, that is, $k_{t,r_{1}} > \nu^{r}_{t,r_{1}} > k_{t,r_{2}}$. Then travelers need to purchase credits to use path $r_{1}$ that is, $\gamma_{t,r_{1}} \geq 0$, while travelers of path $r_{2}$ have excess credits, that is, $\zeta_{t,r_{2}} > 0$. Then Eq. (59) can be reformulated as

$$\nu^{r}_{t,r_{1}} = \frac{\sum_{a \in A} c_{a}(\nu^{r}_{a}) (\delta_{a,r_{1},w} - \delta_{a,r_{2},w})}{\eta (\gamma_{t,r_{1}} - \zeta_{t,r_{2}})}$$

(60)

Because the multi-period equilibrium value of unused credits for class $g$ on path $r_{2}$ of O-D pair $w$, $\lambda^{r}_{t,r_{2},w}$, depends on the perceived future CP of class $g$ and multi-period equilibrium CP of period $t$, the second case is divided into two subcases. In the first subcase, the perceived future CP of user class $g$ is greater than the multi-period equilibrium CP in period $t$. Then the multi-period equilibrium CP of period $t$ can be obtained as

$$p^{t} = \frac{\sum_{a \in A} c_{a}(\nu^{r}_{a}) (\delta_{a,r_{1},w} - \delta_{a,r_{2},w})}{\eta (\gamma_{t,r_{1}} + \zeta_{t,r_{2}})}$$

(61)

Then the multi-period equilibrium CP of period $t$ is also unique under the second subcase. Hence, the multi-period equilibrium CP of period $t$ is unique under the second case. The first and second cases are satisfied under Condition 1. In the third case, the credit supply of a traveler of class $g$ on path $r_{2}$ of O-D pair $w$, $\lambda^{r}_{t,r_{2},w}$, depends on the perceived future CP of class $g$ and multi-period equilibrium CP of period $t$, and travelers need to purchase credits to use path $r_{1}$ that is, $\gamma_{t,r_{1}} \geq 0$, while travelers of path $r_{2}$ have excess credits, that is, $\zeta_{t,r_{2}} > 0$. Then Eq. (59) can be re-written as

$$\nu^{r}_{t,r_{1}} = \frac{\sum_{a \in A} c_{a}(\nu^{r}_{a}) (\delta_{a,r_{1},w} - \delta_{a,r_{2},w})}{\eta (\gamma_{t,r_{1}} - \zeta_{t,r_{2}})}$$

(62)

Then the multi-period equilibrium CP of period $t$ is also unique under the second subcase. Hence, the multi-period equilibrium CP of period $t$ is unique under the second case. The first and second cases are satisfied under Condition 1. In the third case, the credit supply of a traveler of class $g$ is greater or equal to or greater than the credits charged for both paths $r_{1}$ and $r_{2}$, that is, $k_{t,r_{1}} \geq \nu^{r}_{t,r_{1}} \geq \zeta_{t,r_{2}}$. Then travelers of class $g$ using either paths $r_{1}$ and $r_{2}$ have excess credits, that is, $\gamma_{t,r_{1}} < 0$. Similar to the second case, this case is also divided into two subcases. In the first subcase, the perceived future CP of class $g$ is less than or equal to the multi-period equilibrium CP in period $t$. Then the multi-period equilibrium CP of period $t$ can be derived as

$$p^{t} = \frac{\sum_{a \in A} c_{a}(\nu^{r}_{a}) (\delta_{a,r_{1},w} - \delta_{a,r_{2},w})}{\eta (\gamma_{t,r_{1}} + \zeta_{t,r_{2}})}$$

(63)

Hence, in the first subcase, the multi-period equilibrium CP of period $t$ is unique. This subcase is satisfied under Condition 2. In the second subcase, the perceived future CP of class $g$ is equal to or greater than the multi-period equilibrium CP in period $t$. Because $\gamma_{t,r_{1}} \geq 0$, Eq. (59) can be re-written as

$$\nu^{r}_{t,r_{1}} = \frac{\sum_{a \in A} c_{a}(\nu^{r}_{a}) (\delta_{a,r_{1},w} - \delta_{a,r_{2},w})}{\eta (\gamma_{t,r_{1}} - \zeta_{t,r_{2}})}$$

(64)

Hence, in the first subcase, the multi-period equilibrium CP of period $t$ is unique. This subcase is satisfied under Condition 2. In the second subcase, the perceived future CP of class $g$ is equal to or greater than the multi-period equilibrium CP in period $t$. Because $\gamma_{t,r_{1}} \geq 0$, Eq. (59) can be re-written as

$$\nu^{r}_{t,r_{1}} = \frac{\sum_{a \in A} c_{a}(\nu^{r}_{a}) (\delta_{a,r_{1},w} - \delta_{a,r_{2},w})}{\eta (\gamma_{t,r_{1}} - \zeta_{t,r_{2}})}$$

(65)

Hence, the multi-period equilibrium CP may not be unique in this subcase. This concludes the proof.

Data Availability Statement

All data, models, and code generated or used during the study appear in the published article.

References


