Nonlinear Consensus-Based Connected Vehicle Platoon Control Incorporating Car-Following Interactions and Heterogeneous Time Delays

Yongfu Li, Member, IEEE, Chuancong Tang, Srinivas Peeta, and Yibing Wang, Member, IEEE

Abstract—This paper proposes a distributed nonlinear consensus delay-dependent control algorithm for a connected vehicle (CV) platoon. In particular, considering that the behavior of the following vehicle is associated with the longitudinal inter-vehicle gap with respect to the preceding vehicle, a nonlinear function is designed to characterize the car-following interactions between CVs. Then, a nonlinear consensus algorithm is proposed by incorporating the car-following interactions and heterogeneous time delays. The delay-dependent convergence condition of the proposed control algorithm is analyzed using the Lyapunov–Krasovskii method, and an estimate of the delay bound is provided. Under the proposed algorithm, not only can the consensus of CVs be guaranteed but also the behavior of vehicles is consistent with traffic flow theory. Finally, an example using a 10-vehicle platoon is provided under three scenarios: no time delays, heterogeneous time delays, and homogeneous time delays. Results from extensive simulations verify the effectiveness of the proposed control algorithm in terms of the position, velocity, and acceleration/deceleration profiles.

Index Terms—Connected vehicle platoon, distributed control, consensus algorithm, time delay, car-following interaction, convergence analysis.

I. INTRODUCTION

A. Motivation

COOPERATIVE control of vehicular platoons has attracted considerable attention from the control theory and transportation engineering disciplines [1]. Platoon-based driving strategies have been identified as promising options to enhance traffic mobility, safety and energy conservation by leveraging the capabilities of the emerging connected vehicle (CV) technologies [2]–[4]. The purpose of platoon control is to regulate the vehicles in a string to travel together with a harmonized velocity and a small inter-vehicle gap, in which the small inter-vehicle gap will lead to a decrease in aerodynamic drag and an increase in road capacity [5]–[8]. Due to these potential benefits, the platoon control problem has been extensively studied [9]–[12]. However, control theory and transportation engineering have different focuses related to platoon control. From the control theory perspective, the main idea is to treat each vehicle in a traffic stream as an individual agent and characterize the motion of vehicle using the simple first/second order integrator model. The focus is on the design of appropriate control algorithms to ensure the string stability and consensus with respect to the position and velocity of vehicles. However, there is less attention to the interactions between vehicles, and velocity and traffic direction constraints are ignored, which can lead to negative spacing error, negative velocity, and unreasonable acceleration/deceleration rates. On the transportation engineering side, the focus is mainly on the interactions between vehicles and on the development of robust traffic flow models to capture the characteristics of vehicle dynamics in the traffic stream. However, for vehicle platoons in practice, not only should the consensus of vehicles be ensured, but also the behavior of vehicles should be consistent with traffic flow theory. Thereby, negative spacing error implies that the critical inter-vehicle gap cannot be guaranteed and rear-end collision may occur, while negative velocity implies that vehicles backup. Further, high acceleration/deceleration rates affect driving comfort. Hence, there is an emergent research need to bridge the gap between control theory and transportation engineering related to vehicle platoon control.

This study proposes a novel platoon control algorithm that incorporates car-following interactions between vehicles and considers heterogeneous time delays in the CV context. In particular, a nonlinear function is used to characterize the car-following interactions between a pair of CVs in a string. Then, a distributed nonlinear consensus algorithm is proposed for platoon control, which incorporates: (i) the car-following interactions between the following and preceding vehicles, (ii) the velocity consensus with respect to the follower and the predecessor as well as the follower and the leader, (iii) the position consensus with respect to the follower and...
the predecessor as well as the follower and the leader, and (iv) the heterogeneous time delays related to CV information transmission.

Based on the proposed delay-dependent control algorithm, the convergence condition is analyzed using the Lyapunov–Krasovskii method, and an estimate of the delay bound is provided. Through this algorithm, vehicles in a string can smoothly reach consensus with respect to position and velocity, and the behavior of vehicles is consistent with traffic flow theory, thereby avoiding negative spacing error, negative velocity, and unreasonable acceleration/deceleration rate.

B. Literature Review

Several approaches to the platoon control problem have been considered in the literature. These approaches fall into two broad categories: string stability-based approaches and consensus-based approaches. The main idea of the string stability-based approach is to ensure attenuation of the spacing error from the first to the last vehicle in the platoon under different spacing policies. The consensus-based approach focuses on the consensus of vehicles with respect to position and velocity using different control protocols.

1) String Stability-Based Approaches: This approach mainly includes two strategies to maintain a desired safe spacing, i.e., constant spacing (CS) policy and constant time headway (CTH) policy. The main difference between these two policies lies in whether the required spacing between vehicles is dependent on vehicle speed or not. A comparison of these two policies can be found in [13].

Based on the CS policy, Swaroop et al. [14] developed a decentralized adaptive control algorithm in the presence of parametric uncertainty. Seiler et al. [15] suggested that string instability would arise under the predecessor-follower topology. Dunbar and Caveney [16] proposed a distributed receding horizon control for vehicle platoons with nonlinear dynamics to investigate asymptotic stability, leader-follower string stability, and predecessor-follower string stability. However, these studies do not consider the effect of time delay on string stability. To address this issue, Guo and Yue [17] proposed a guaranteed-cost platoon control method considering constant actuator delays and the effects of sensing range limitation. Ge and Orosz [18] proposed an acceleration-based connected cruise control algorithm with heterogeneous time delays to study the string stability of vehicle platoons. However, these studies focus on the string stability of vehicle platoons by treating each vehicle as an individual agent and ignoring the interactions between vehicles, leading to negative spacing error between vehicles during the movement of the platoon, which implies that rear-end collisions may occur.

Related to the CTH policy, Naus et al. [19] proposed a decentralized controller for vehicles with a limited communication distance and derived a necessary and sufficient condition in the frequency-domain for string stability. Guo et al. [20] proposed a distributed finite-time adaptive integral-sliding-mode control approach to guarantee the finite time stability of each vehicle and the string stability. Rogge and Aeyels [21] proposed a novel strategy to guarantee platoon string stability where the vehicles are coupled in a unidirectional ring.

To investigate the effects of time delay on string stability, Besselink and Johansson [22] proposed a novel delay-based spacing policy that guarantees that all vehicles in the platoon track the same velocity profile in the spatial domain, and that guarantees disturbance string stability with respect to the varying reference velocity. However, negative spacing error and negative velocity issues exist with these studies.

2) Consensus-Based Approaches: This approach focuses on facilitating the vehicles in a string to reach a consensus state by designing a control protocol. The consensus problem of vehicle platoons has been extensively investigated. Lin et al. [23] proposed optimal localized feedback gains for one-dimensional formation in which vehicles only use information from their immediate neighbors to achieve position consensus with respect to the follower and the predecessor. Barooah et al. [24] proposed a decentralized bidirectional double-integrator controller for a platoon to achieve position and velocity consensus with respect to the follower and the leader using information from itself and its two nearest neighbors only. Zheng et al. [25] proposed a consensus algorithm based on the third-order state-space model by using feedback linearization to achieve position and velocity consensus as well as acceleration consensus with respect to the follower and the predecessor. Zhang et al. [26] proposed a hierarchical framework for the design of connected cruise control in the presence of heterogeneous time delays and uncertainties to achieve velocity consensus with respect to the follower and the predecessor. Di Bernardo et al. [27] proposed a distributed double-integrator controller incorporating a local action depending on the state variables of the vehicle itself (measured onboard) and an action depending on the information received from neighboring vehicles through the communication network to achieve position and velocity consensus with respect to the follower and the leader with time-varying heterogeneous delays. Li et al. [28] proposed a double-integrator feedback-based platoon control protocol for connected autonomous vehicles under different network topologies of initial states to achieve position and velocity consensus with respect to the follower and the predecessor as well as the follower and the leader. Jia and Dong [29] proposed a consensus-based control algorithm for multi-platoon cooperative driving using second-order vehicle dynamics with heterogeneous time delays. However, the consensus algorithms designed in these studies are mostly linear consensus algorithms which cannot capture the interactions between vehicles effectively. Also, while these studies focus on consensus with respect to position and velocity, they do not guarantee non-negative spacing error, non-negative velocity, or avoidance of unreasonable acceleration/deceleration rates.

C. Contributions

To capture the car-following interactions between vehicles and avoid negative spacing error, negative velocity, and unreasonable acceleration/deceleration rate, this paper proposes a more realistic and effective distributed nonlinear consensus algorithm for vehicle platoons in the presence of heterogeneous time delays. The study contributions are as follows,
and are illustrated in the context of controllers proposed in the current literature:

(i) Unlike the controllers proposed in [15], [20], [21], [24], [25], and [27]–[29] that focus on the states of individual vehicles, we propose a distributed nonlinear delay-dependent controller incorporating the car-following interactions between the following and preceding vehicles in a traffic string. By doing so, vehicles can smoothly reach the consensus state with respect to position and velocity, and the behavior of vehicles is consistent with traffic flow theory, thereby avoiding negative spacing error, negative velocity, and unreasonable acceleration rate drop.

(ii) Differing from the past studies [15], [20], [21], [24], [25], [28] that only perform convergence analysis without time delays, we rigorously analyze convergence for the proposed delay-dependent consensus algorithm. First, we transform the proposed distributed nonlinear controller to a time-varying delay state-space system. Then, the theoretical analysis is performed based on the Lyapunov-Krasovskii stability theorem. Finally, the upper bound of time delay for platoon stability is derived, which is a key factor for it.

(iii) Extensive analyses related to time delay, including for no time delay, heterogeneous time delays, and homogeneous time delays, with disturbances acted on the lead vehicle, are performed using numerical simulation to verify the effectiveness of the proposed method in terms of position, velocity, and acceleration/deceleration profiles.

D. Organization

The paper is organized as follows. Section II presents preliminaries and the problem statement. Section III describes the distributed nonlinear consensus algorithm. Section IV performs the convergence analysis. Section V conducts extensive simulation-based analyses and discusses the results. Concluding remarks are provided in the final section.

II. PRELIMINARIES AND PROBLEM STATEMENT

A. Graph Theory

This study considers a vehicle platoon consisting \( n + 1 \) vehicles traveling on a straight road that includes a leader (labeled as vehicle \( L \)) and \( n \) followers (labeled as vehicles \( 1 \) to \( n \)). To use the communication capacity efficiently, we propose the predecessor-leader following (PLF) topology to characterize the connectivity between vehicles under the connected environment. That is, each following vehicle has access to real-time information (i.e., position and velocity) from its immediate predecessor and the leader through a V2V communication link.

To specify the topology, a directed graph \( G = \{V, E, A\} \) is used in this study, where \( V = \{1, 2, \ldots, n\} \) is the set of nodes, and \( E \subseteq V \times V \) is the set of edges. \( A = \{a_{ij}\}_{i,j=1}^{n} \in \mathbb{R}^{n \times n} \) is a weight adjacency matrix with nonnegative elements, where \( a_{ij} = 1 \) in the presence of a communication link from node \( j \) to node \( i \), and \( a_{ij} = 0 \) otherwise. In addition, we assume no self-loops in the directed graph, i.e., \( a_{ii} = 0 \) for all \( i = 1, \ldots, n \). Define \( U = D - A \), where \( D = \text{diag}(d_1, \ldots, d_n) \) is an adjacency matrix with nonnegative elements, whose diagonal elements are given by \( d_i = \sum_{j=1}^{n} a_{ij} \). In addition, define matrix \( K = \text{diag}(k_{1L}, \ldots, k_{nL}) \in \mathbb{R}^{n \times n} \) to be a leader adjacency matrix associated with the follower vehicles and the leader vehicle, where \( k_{iL} = 1 \) in the presence of a communication link from node \( i \) to leader \( L \), and \( k_{iL} = 0 \) otherwise.

B. Mathematical Preliminaries

We now introduce some lemmas that will be instrumental for the proof of convergence discussed later.

**Lemma 1** [30]: Given a complex-coefficient polynomial

\[
r(z) = z^2 + (a + ib)z + c + id,
\]

where \( a, b, c, d \in \mathbb{R} \), \( r(z) \) is Hurwitz stable if and only if \( a > 0 \) and \( ab + a^2c - d^2 > 0 \).

**Lemma 2** [31]: For any \( a, b \in \mathbb{R}^n \) and any positive-definite matrix \( F \in \mathbb{R}^{n \times n} \), then:

\[
2a^Tc \leq a^TFa + c^TF^{-1}c.
\]

**Lemma 3** (Hermite-Hadamard Inequality) [32]: Let \( f : I \subseteq \mathbb{R} \rightarrow \mathbb{R} \) be a convex mapping defined on the interval \( I \) of real numbers; then, the following inequality holds:

\[
f\left(\frac{a + b}{2}\right) \leq \frac{1}{b - a} \int_{a}^{b} f(x) dx \leq \frac{f(a) + f(b)}{2},
\]

where \( a, b \in I \) with \( a < b \).

**Proposition 1** (Uniform Norm): Denote \( \phi(s) \in C([a, b], \mathbb{R}^n) \) be the set of continuous functions mapping the interval \([a, b]\) to \( \mathbb{R}^n \), then the uniform norm of \( \phi \) is defined as

\[
||\phi||_{\infty} = \max_{t \in [a, b]} ||\phi(t)||.
\]

The vector norm \( || \cdot || \) represents the 2-norm \( || \cdot ||_2 \).

Let \( C([a, b], \mathbb{R}^n) \) be a Banach space of continuous functions defined in an interval \([-r, 0]\), and which takes values in \( \mathbb{R}^n \) with a norm \( ||\phi||_{\infty} = \max_{t \in [-r,0]} ||\phi(t)|| \). Consider the following time-delay system:

\[
\begin{aligned}
\dot{x}(t) &= f(t, x_t), \quad t \geq t_0 \\
x(\theta) &= \phi(\theta), \quad \theta \in [-r, 0],
\end{aligned}
\]

where \( x_t(\theta) = x(t + \theta), \forall \theta \in [-r, 0], f : \mathbb{R} \times C([-r, 0], \mathbb{R}^n) \rightarrow \mathbb{R} \) is a continuous function and \( f(t, 0) = 0, \forall t \in \mathbb{R} \). The following (Lemma 4) holds:

**Lemma 4** (Lyapunov-Krasovskii Stability Theorem) [33]: Suppose that \( f : \mathbb{R} \times C([-r, 0], \mathbb{R}^n) \rightarrow \mathbb{R} \) given in (5) maps \( \mathbb{R} \) (bounded sets of \( C([-r, 0], \mathbb{R}^n) \)) into a bounded sets of \( \mathbb{R}^n \), and suppose that \( u(s), v(s) \) are continuous non-negative and non-decreasing functions with \( u(s) > 0 \) for \( s \neq 0 \) and \( u(0) = 0 \), if there exists a continuously differentiable function \( E : \mathbb{R} \times C_{n,h} \rightarrow \mathbb{R}^n \) such that:

\[
u(||\phi(0)||) \leq E(t, \phi) \leq \nu(||\phi||_{\infty}),
\]

and

\[
\dot{E}(t, \phi) = \lim_{\epsilon \rightarrow 0^+} \sup_{t \in \Omega} \frac{E(t + \epsilon, x_{t + \epsilon}(t, \phi)) - E(t, \phi)}{\epsilon}
\]

\[
\leq -\omega(||\phi||_{\infty}),
\]

then the solution \( x = 0 \) is uniformly stable.
Moreover, if $s > 0$ for $s > 0$ then it is uniformly asymptotically stable. In addition, if
\[
\lim_{s \to \infty} u(s) = +\infty
\]
then it is globally uniformly asymptotically stable.

C. Consensus Problem Statement

For the scenario in Fig. 1, the information flow topology shown in Fig. 1(b) is labeled the PLF topology. The longitudinal dynamics of each following vehicle in the string can be formulated as follows [27], [28]:
\[
\begin{align*}
\dot{x}_i(t) &= v_i(t), \\
\dot{v}_i(t) &= u_i(t), \\
&i = L, 1, 2, \ldots, n,
\end{align*}
\]
where $x_i(t)$ and $v_i(t) \in \mathbb{R}$ are the position and velocity of vehicle $i$ in the platoon at time $t$, respectively. $u_i(t) \in \mathbb{R}$ is the control input of the vehicle $i$ at time $t$. If $i = L$, it implies that $x_L(t)$ and $v_L(t)$ are the position and velocity of the lead vehicle at time $t$, respectively. $u_L(t) \in \mathbb{R}$ is the control input of the lead vehicle at time $t$. In this paper, we assume the lead vehicle moves with a constant velocity, i.e., $v_0$. It implies $u_L(t) = 0$.

The goal of this study is to design a distributed nonlinear delay-dependent control algorithm to address the consensus problem for a vehicle platoon. That is, for each vehicle $i \in \{1, 2, \ldots, n\}$, the platoon control goal can be expressed as solving the following consensus problem:
\[
x_i(t) \to x_L(t) - i \cdot (l_c + l_e), \quad v_i(t) \to v_L(t),
\]
where $l_e$ is the length of vehicle, and $l_c$ is the desired safe inter-vehicle gap. Accordingly, the position and velocity errors with respect to the desired platooning equilibrium (10) can be defined as follows:
\[
\begin{align*}
\dot{\bar{x}}_i(t) &= x_L(t) - x_i(t) - i \cdot (l_c + l_e), \\
\dot{\bar{v}}_i(t) &= v_L(t) - v_i(t).
\end{align*}
\]

For simplicity, we denote $\bar{x} \Delta [\bar{x}_1, \ldots, \bar{x}_n]^T$, $\bar{v} \Delta [\bar{v}_1, \ldots, \bar{v}_n]^T$, and $\bar{e} \Delta [\bar{x}^T \bar{v}^T]^T$. Hence, we can rewrite (10) as follows:
\[
\lim_{t \to \infty} ||\bar{e}|| = 0.
\]

III. NONLINEAR CONSENSUS ALGORITHM

Based on traffic flow theory, one assumption of car-following model is that each vehicle will seek a safe velocity determined by the distance from its preceding vehicle. Therefore, to address the above consensus problem and guarantee the consistency of the behavior of vehicles with traffic flow, the distributed nonlinear delay-dependent control algorithm is designed as:
\[
u_i(t) = \sum_{j=1}^{n} a_{i,j} \left( \alpha (V_i(h_{i,j}(t)) - v_i(t)) \right) + \beta (v_j(t) - v_{ij}(t)) - v_i(t)) \\
+ \gamma (x_j(t) - x_{ij}(t)) - x_i(t) \\
+ v_L(t - \tau_{ij}(t)) + v_L(t - \tau_{ij}(t)) \tau_{ij}(t) - v_i(t)) \\
\]
where $a_{i,j}$ is the communication link from vehicle $j$ to vehicle $i$. $k_{i,j}$ is the communication link from the leader vehicle $i$. $\alpha, \beta, \gamma > 0$ are constant control gains. $\tau_{ij}(t)$ and $\tau_{ij}(t)$ are the time-varying delays affecting the communication with vehicle $i$ when information is transmitted from vehicle $j$ and the leader, respectively (in general $\tau_{ij}(t) \neq \tau_{ij}(t)$).

Remark 1: The proposed algorithm (13) is as follows: (13a) represents the car-following interactions between vehicles $i$ and $j$; (13b) denotes the velocity difference between vehicles $i$ and $j$; (13c) denotes the position difference between vehicles $i$ and $j$ with respect to the desired gap $\tau_{ij}(t)$. Due to the time delay $\tau_{ij}(t)$ of $x_j$ with respect to $x_i$, $v_L(t - \tau_{ij}(t))$ is the gap supplement; (13d) denotes the velocity difference between vehicle $i$ and the leader and vehicle $j$ and the leader, respectively; and (13e) denotes the position difference between vehicle $i$ and the leader with respect to the desired gap $r_{ij}(t)$. Similarly, due to the time delay $\tau_{ij}(t)$ of $x_L$ with respect to $x_i$, $v_{ij}(t - \tau_{ij}(t))$ is the gap supplement.

Remark 2: In this study, we assume that all neighboring vehicles transmit their real-time information (i.e., position and velocity) to vehicle $i$, together with their ID and time stamp. The time stamp represents the time instant at which information was sent [34]. It implies that clock synchronization can be guaranteed across all the strings via differential global positioning system [27], [35].

In addition, the following nonlinear function is defined to capture the interactions between vehicles $i$ and $j$, which is associated with the average bumper-to-bumper distance [11]:
\[
V_i(h_{i,j}(t)) = V_1 + V_2 \tanh(C_1(h_{i,j}(t)) - C_2),
\]
where $V_1, V_2, C_1, C_2$ are positive constants.

The average bumper-to-bumper distance between vehicles $i$ and $j$ is:
\[
h_{i,j}(t) = (x_j(t) - x_i(t) - (i - j)l_e)/(i - j).
\]
Based on (11) and (13), (9) can be rewritten as follows:

\[
\begin{align*}
\dot{x}_i(t) &= \tilde{v}_i(t) \\
\dot{\tilde{v}}_i(t) &= \sum_{j=1}^{n} a_{i,j}[\alpha(V_i(h_{i,j}(t)) - V_i(h_{i,j}^*(t))) - \tilde{v}_i(t)] \\
&+ \beta(\tilde{v}_j(t) - \tau_j(t)) - \tilde{v}_i(t) + \gamma(\tilde{x}_j(t) - \tau_j(t)) - \tilde{x}_i(t)) \\
&- k_{i,L}(\beta \tilde{v}_i(t) + \gamma \tilde{x}_i(t)) + \sum_{j=1}^{n} \beta(v_{i,L}(t - \tau_j(t)) - v_{i,L}(t)) \\
+ \gamma(v_{i,L}(t - \tau_{i,L}(t))\tau_j - (x_{i,L}(t) - x_{i,L}(t - \tau_j(t))) \\
+ k_{i,L}(\beta(v_{i,L}(t - \tau_{i,L}(t)) - v_{i,L}(t)) \\
+ \gamma(v_{i,L}(t - \tau_{i,L}(t))\tau_L - (x_{i,L}(t) - x_{i,L}(t - \tau_{i,L}(t))))).
\end{align*}
\]

(16)

Since the leader moves with a constant velocity, it implies that \(v_{L}(t - \tau_{L}(t)) = v_{L}(t)\), \(x_{L}(t - \tau_{L}(t)) = v_{L}(t)\), and \(x_{L}(t) - x_{L}(t - \tau_{L}(t)) = v_{L}(t)\). Then, it follows from (16) that

\[
\begin{align*}
\dot{x}_i(t) &= \tilde{v}_i(t) \\
\dot{\tilde{v}}_i(t) &= \sum_{j=1}^{n} a_{i,j}[\alpha(V_i(h_{i,j}(t)) - V_i(h_{i,j}^*(t))) - \tilde{v}_i(t)] \\
&+ \beta(\tilde{v}_j(t) - \tau_j(t)) - \tilde{v}_i(t) + \gamma(\tilde{x}_j(t) - \tau_j(t)) - \tilde{x}_i(t)) \\
&- k_{i,L}(\beta \tilde{v}_i(t) + \gamma \tilde{x}_i(t)),
\end{align*}
\]

(17)

where \(h_{i,j}^*(t) = h_{e}, V_i(h_{i,j}^*(t)) = v_{L}(t)\).

Based on (14), the following equation can be obtained using Taylors expansion:

\[
V_i(h_{i,j}(t)) = V_i(h_{i,j}^*(t)) + V_i'(h_{i,j}^*(t))(h_{i,j}(t) - h_{i,j}^*(t))
\]

(18)

Note that \(h_{i,j}(t) - h_{i,j}^*(t) = (\tilde{x}_j(t) - \tau_j(t)) - (\tilde{x}_i(t))\); it follows from (18) that,

\[
V_i(h_{i,j}(t)) - V_i(h_{i,j}^*(t)) = (\tilde{x}_j(t) - \tau_j(t)) - \tilde{x}_i(t)) \\
\times V_i'(h_{i,j}^*(t)).
\]

(19)

Define

\[
\phi_{i,j}(\tilde{z}_{i,j}(t)) = V_i'(h_{i,j}^*(t)) / (i - j)
\]

(20)

Then, (17) can be further rewritten as:

\[
\begin{align*}
\dot{x}_i(t) &= \tilde{v}_i(t) \\
\dot{\tilde{v}}_i(t) &= \sum_{j=1}^{n} a_{i,j}[\alpha \phi_{i,j}(\tilde{z}_{i,j}(t)) + \gamma(\tilde{x}_j(t) - \tau_j(t)) \\
&- \tilde{x}_i(t)) + \beta \tilde{v}_i(t) - \tau_j(t)) - (\alpha + \beta)\tilde{v}_i(t)] \\
&- k_{i,L}(\beta \tilde{v}_i(t) + \gamma \tilde{x}_i(t))
\end{align*}
\]

(21)

To describe platoon dynamics in the presence of heterogeneous time delays associated with different communication links in a more compact form, we define \(\tau_p(t), p = 1, 2, \ldots, m(m \leq n(n - 1))\) as an element of the sequence \(\{\tau_{i,j}(t) : i, j = 1, \ldots, n, i \neq j\}\).

Following the approach used in [27] and [32], (21) yields:

\[
\begin{align*}
\dot{\hat{z}}(t) &= A_0 \hat{z}(t) + \sum_{p=1}^{m} A_p \hat{z}(t - \tau_p(t))
\end{align*}
\]

(22)

where for \(p = 1, 2, \ldots, m,\)

\[
A_0 = \begin{bmatrix} 0_{n \times n} & 0 \ 0 & I_{n \times n} \end{bmatrix}
\]

(23)

\[
A_p = \begin{bmatrix} 0_{n \times n} & 0 \ 0 & \alpha \phi_{i,j}(\tilde{z}_{i,j}(t)) + \gamma B_p \beta B_p \end{bmatrix}
\]

with

\[
B_p = \begin{bmatrix} 0 & \cdots & \cdots & a_{1,p} & \cdots & \cdots & a_{n,p} \\
\vdots & \ddots & \cdots & \vdots & \ddots & \cdots & \vdots \\
a_{n,p} & \cdots & \cdots & a_{(n-1),p} & \cdots & \cdots & 0 \\
\end{bmatrix}
\]

(24)

\[
a_{i,j,p} = \begin{cases} a_{i,j}, & p = i \\
0, & p \neq i \end{cases}, \quad i, j = 1, \ldots, n, \quad B_p \in \mathbb{R}^{n \times n}, \quad p = 1, 2, \ldots, m \quad \mathbb{B} = \sum_{p=1}^{m} B_p, \quad B = A.
\]

IV. CONVERGENCE ANALYSIS

A. Convergence Analysis

This section analyzes the convergence of the algorithm (22) proposed in Section III. Starting from solving the delay-dependent consensus problem, a transformation called the Leibniz-Newton formula [36] is given as follows:

\[
\hat{z}(t - \tau_p(t)) = \hat{z}(t) - \int_{-\tau_p(t)}^{0} \hat{z}(t + s)ds.
\]

(25)

Then, substituting (22) into (25) yields:

\[
\hat{z}(t - \tau_p) = \hat{z}(t) - \sum_{q=0}^{m} A_1 \int_{-\tau_p(t)}^{0} \hat{z}(t + s - \tau_q(t + s))ds,
\]

(26)

where \(A_0, A_1, \ldots, A_m\) are defined in (23) and \(\tau_0(t + s) \equiv 0\).

Using the above transformation, (22) can be transformed into

\[
\dot{\hat{z}}(t) = A_0 \hat{z}(t) + \sum_{p=1}^{m} A_p \hat{z}(t) - \sum_{p=1}^{m} \sum_{q=0}^{m} A_p A_q \int_{-\tau_p(t)}^{0} \hat{z}(t + s - \tau_q(t + s))ds.
\]

(27)

From (23) it follows that \(A_p A_q = 0\) when \(p = 1, \ldots, m\) and \(q = 1, \ldots, m(q \neq 0)\). Hence, (27) can be rewritten as follows:

\[
\dot{\hat{z}}(t) = M \hat{z}(t) - \sum_{p=1}^{m} C_p \int_{-\tau_p(t)}^{0} \hat{z}(t + s)ds,
\]

(28)

where

\[
M = A_0 + \sum_{i=1}^{m} A_i
\]

(29)
Thus, the \textit{Hurwitz} stability of matrix $M$ is equivalent to that of polynomial $R(\lambda) = \lambda^2 + \theta_1 \lambda + \zeta_i$, for all $\theta_1 \in \sigma(H_1)$, $\zeta_i \in \sigma(H_2)$. Based on Lemma 1, we have:

(1) $\text{Re}(\theta_1) > 0$, which holds by the positive stable matrix $M$.
(2) $\text{Re}(\theta_1) \text{Im}(\theta_1) \text{Im}(\zeta_i) + \text{Re}^2(\theta_1) \text{Re}(\zeta_i) - \text{Im}^2(\zeta_i) > 0$, which can be satisfied by the condition (32). Thus, Theorem 1 holds.

Theorem 2 can address the aforementioned consensus of a vehicle platoon in the presence of heterogeneous time delays.

\textbf{Theorem 2:} Consider the delay-dependent system in (22). Set the control parameters $a_{i,j}, b_{i,j}, \alpha, \beta, \gamma$ in (13) to satisfy Theorem 1. Assume delays $\tau_p(t)$ to be bounded, i.e., $\tau_p(t) \in [0, \tau_{\text{max}}]$, $\tau_p(t) \in (-\infty, d_p(\nu_t, \nu_p) \text{ and } d_p \leq 1$. Then, there exists a constant $\tau^* > 0$ such that, for $\tau_p(t) < \tau^*, \forall p, \forall t$ consensus can be achieved as in (12). That is,

$$\lim_{t \to \infty} \tilde{z}(t) = 0,$$

if and only if the leader $L$ is globally reachable in $G_{n+1}$.

\textbf{Proof (Sufficiency):} According to Lemma 4, choose the Lyapunov-Krasovskii function for (22) as follows:

$$V(\tilde{z}(t)) = \tilde{\epsilon}^T(t) P \tilde{z}(t) + \sum_{p=1}^{m} \int_{t-\tau_p(t)}^{t} \tilde{\epsilon}^T(\eta) S_p \tilde{\epsilon}(\eta) d\eta,$$ (35)

where $P = P^T > 0$ and $S_p > 0 (p = 1, \ldots, m)$ are appropriately chosen matrices. Define the following continuous non-decreasing and positive functions, satisfying the hypotheses of Lemma 4:

$$g(\tilde{z}(t)) = \tilde{\epsilon}^T(t) P \tilde{z}(t),$$

$$z(\tilde{z}(t - \tau^*)) = \tilde{\epsilon}^T(t) \tilde{\epsilon}(t),$$

$$+ \sum_{p=1}^{m} \int_{t-\tau_p(t)}^{t} \tilde{\epsilon}^T(\eta) S_p \tilde{\epsilon}(\eta) d\eta,$$ (36)

where $\tau^*$ is the maximum time delay.

Based on definitions in (35) and (36), we know that condition (6) in Lemma 4 is fulfilled, i.e.,

$$g(\tilde{z}(t)) \leq V(\tilde{z}(t)) \leq z(\tilde{z}(t - \tau^*)).$$ (37)

Then, differentiating Lyapunov-Krasovskii in (35) yields:

$$\dot{V}(\epsilon) = \tilde{\epsilon}^T(t) P \tilde{z}(t) + \tilde{\epsilon}^T(t) \tilde{p}(t)$$

$$+ \sum_{p=1}^{m} \tilde{\epsilon}(\eta) S_p \tilde{\epsilon}(\eta) \eta = 0 - \tau_p(t).$$ (38)

Substituting (28) into (38), it follows that:

$$\dot{V}(\epsilon) = \tilde{\epsilon}^T(t) (PM + M^T P) + \sum_{p=1}^{m} \tilde{\epsilon}(\eta) \tilde{p}(t) (t - \tau_p(t))$$

$$- (1 - \tau_p(t)) \sum_{p=1}^{m} \tilde{\epsilon}(t - \tau_p(t)) \tilde{p}(t - \tau_p(t))$$

$$- 2 \epsilon^T(t) P \sum_{p=1}^{m} C_p \int_{t-\tau_p(t)}^{t} \tilde{z}(t + s) ds.$$ (39)

As the leader is globally reachable in $G$, the matrix $M$ in (29) is Hurwitz stable according to Theorem 1. Further, we have $PM + M^T P = -Q$ with $Q > 0$ and $P > 0$, $P = P^T$ according to the Lyapunov theory.

In addition, define $\alpha^T = \tilde{\epsilon}^T P C_p, c = \tilde{\epsilon}(t + s), F = P^{-1}$ and integrate both sides of the inequality in Lemma 2. One can derive from (39) that:

$$2 \epsilon^T(t) P C_p \tilde{z}(t + s)$$

$$\leq \tilde{\epsilon}^T(t) P C_p \tilde{p}(t + s)$$

$$= \tilde{\epsilon}^T(t) P C_p P^{-1} P^T \tilde{z}(t + s) + \tilde{\epsilon}^T(t + s) P \tilde{z}(t + s).$$ (40)

Integrating both sides of (40), and it follows from (39) that,

$$\dot{V}(\epsilon) \leq - \tilde{\epsilon}^T(t) \tilde{Q} \tilde{z}(t) + \tilde{\epsilon}^T(t) \sum_{p=1}^{m} \tilde{p}(t) - (1 - \tau_p(t))$$

$$\times \sum_{p=1}^{m} \tilde{\epsilon}(t - \tau_p(t)) \tilde{p}(t - \tau_p(t))$$

$$- \sum_{p=1}^{m} \tilde{\epsilon}(t - \tau_p(t)) P C_p P^{-1} P^T \tilde{z}(t)$$

$$- \sum_{p=1}^{m} \tilde{\epsilon}(t - \tau_p(t)) P C_p P^{-1} P^T \tilde{z}(t)$$

$$- \sum_{p=1}^{m} \tilde{\epsilon}(t - \tau_p(t)) P C_p P^{-1} P^T \tilde{z}(t)$$

$$\leq \sum_{p=1}^{m} \tilde{\epsilon}(t - \tau_p(t)) P C_p P^{-1} P^T \tilde{z}(t)$$

$$\leq \sum_{p=1}^{m} \tilde{\epsilon}(t - \tau_p(t)) P C_p P^{-1} P^T \tilde{z}(t)$$

$$\leq \sum_{p=1}^{m} \tilde{\epsilon}(t - \tau_p(t)) P C_p P^{-1} P^T \tilde{z}(t)$$

According to Lemma 3, the last part of (41) satisfies,

$$\int_{t-\tau_p(t)}^{t} \tilde{\epsilon}^T(t + s) P \tilde{z}(t + s) ds$$

$$\leq \frac{\tau_p(t)}{2} \tilde{\epsilon}(t) P \tilde{z}(t) + \tilde{\epsilon}^T(t + \tau_p(t)) P \tilde{z}(t + \tau_p(t))$$

$$< \frac{\tau^*}{2} \tilde{\epsilon}(t) P \tilde{z}(t) + \tilde{\epsilon}^T(t + \tau_p(t)) P \tilde{z}(t + \tau_p(t)),$$ (42)

where $\tau^*$ is the maximum delay, and $\tau_p(t) < \tau^*, \forall p, \forall t.$
Based on (42), it follows from (41) that:
\[
\dot{V}(\bar{e}) < -\bar{e}^T(t)Q\bar{e}(t) + \bar{e}^T(t)\sum_{p=1}^{m}S_p\bar{e}(t) - (1 - d_p)
\times \sum_{p=1}^{m}\left(\bar{e}^T(t - \tau_p(t))S_p\bar{e}(t - \tau_p(t))\right) - \sum_{p=1}^{m}\left[\tau^*_p\bar{e}^T(t)P
\times CP_p^{-1}(\bar{e}(t)) - \frac{\tau^*_p}{2}\bar{e}^T(t)P\bar{e}(t)
+ \bar{e}^T(t + \tau_p(t))P\bar{e}(t + \tau_p(t))\right]
\]
(43)

Now, defining an augmented state error vector \(v(t) = [\bar{e}(t), \bar{e}(t - \tau_1(t)), \ldots, \bar{e}(t - \tau_m(t))]^T\), (43) can be rewritten in a more compact form as follows:
\[
\dot{V}(t) < v^T(t)\Lambda v(t),
\]
(44)
where \(\Lambda = \text{diag}(\Lambda_1, \Lambda_2, \ldots, \Lambda_{m+1})\) with
\[
\Lambda_1 = -Q + \sum_{p=1}^{m}S_p - \sum_{p=1}^{m}\frac{\tau^*_p}{2} \left[2PC_p^{-1}C^TP + P\right],
\Lambda_2 = -\frac{\tau^*_p}{2}P - S_1(1 - d_1),
\Lambda_{m+1} = -\frac{\tau^*_p}{2}P - S_m(1 - d_m).
\]
(45)

To guarantee uniform stability of (22), matrix \(\Lambda\) should be negative definite according to Lemma 4. Since \(d_p \leq 1, \forall p = 1, \ldots, m\), all \(\Lambda_i(i = 2, \ldots, m+1)\) defined in (45) are negative definite. Hence, \(\Lambda\) is negative definite when \(\Lambda_1\) in (45) is negative definite. The following will guarantee \(\Lambda_1\) is negative definite, that is,
\[
\Lambda_1 = -Q + \sum_{p=1}^{m}S_p - \sum_{p=1}^{m}\frac{\tau^*_p}{2} \left[2PC_p^{-1}C^TP + P\right] < 0.
\]
(46)

Consequently, we can derive that
\[
\tau^* < ||\left\{\sum_{p=1}^{m}S_p - Q|| + \sum_{p=1}^{m}PC_p^{-1}C^TP + \frac{P}{2}||\right\}^{-1}
\]
(47)

Note that matrices \(S\) and \(P\) can be chosen independently to avoid the numerator becoming zero.

Given the choice made for \(g(\bar{e}(v))\) in (36), (22) is also globally asymptotically stable according to Lemma 4.

(Necessity): (22) is asymptotically stable for any time delay \(\tau_p(t) < \tau^*, \forall p, \forall t; \) Let \(\tau_p(t) = 0, p = 1, \ldots, m\), it follows from (28) that the system \(\hat{\xi}(t) = M\hat{\xi}(t)\) with \(M\) defined in (29), is asymptotically stable. Hence, according to Theorem 1, Theorem 2 holds accordingly. Consequently, we have
\[
\lim_{t \to \infty} ||\hat{\xi}(t)|| = 0. \text{ It implies that } \lim_{t \to \infty} ||\hat{x}_l(t) - x_i(t) - i \cdot (l_c + h_2)|| = 0, \lim_{t \to \infty} ||\hat{b}_l(t)|| = \lim_{t \to \infty} ||\hat{b}_l(t) - L_1(t)|| = 0. \text{ This completes the proof.}
\]

Remark 3: According to Theorem 2, we know that vehicles in a string described in (9) can form a platoon under control algorithm (13). Particularly, the inter-vehicle gap between vehicles approaches the desired gap, i.e., \(r_{ij}\), while the velocity of vehicles can also converge to the desired velocity, i.e., \(v_L\).

It implies that the consensus of vehicles with respect to position and velocity in the string under the proposed control algorithm (13) is guaranteed.

Remark 4: As the consensus with respect to position can be achieved, the safe desired inter-vehicle gap (i.e., \(r_{ij}\)) between vehicles will be ensured. It implies that the rear-end collisions between vehicles in the string can be avoided.

Remark 5: The convergence of the delay-dependent system is rigorously analyzed using the Lyapunov-Krasovskii method. Hence, the stability of the system is also guaranteed.

B. Convergence Speed Analysis

To facilitate the convergence speed analysis, (22) can be rewritten as follows using Taylor’s expansion:
\[
\dot{\hat{\xi}}(t) = (I + \tau_p(t)\sum_{p=1}^{m}A_p)^{-1}M\hat{\xi}(t)
\]
(48)
where \(M = A_0 + \sum_{p=1}^{m}A_p\) is defined in (29). Let \(\hat{\Lambda} = (I + \tau_p(t)\sum_{p=1}^{m}A_p)^{-1}M\). It yields:
\[
\dot{\hat{\xi}}(t) = \hat{\Lambda}\hat{\xi}(t)
\]
(49)
The solution to (49) is:
\[
\hat{\xi}(t) = e^{\hat{\Lambda}t}\hat{\xi}(0)
\]
(50)
Therefore, we have
\[
||\hat{\xi}(t)|| = ||e^{\hat{\Lambda}t}\hat{\xi}(0)||
\]
(51)
Since \(\hat{\Lambda}\) is stable (Hurwitz matrix), so \(\exists \Omega, \zeta > 0\) and \(\zeta \in \text{s.t.} ||e^{\hat{\Lambda}t}|| \leq e^{-\zeta t}\), which implies that \(e^{\hat{\Lambda}t}\hat{\xi}(0) \to 0\).

Remark 6: Note that the convergence time is determined by \(\hat{\Lambda}\). Thus, the convergent speed of the system can be changed by choosing the appropriate values of parameters \(\alpha, \beta, \gamma\).

V. NUMERICAL EXPERIMENTS

This section presents the numerical experiments to verify the effectiveness of the proposed platoon control algorithm. For this purpose, according to the scenario shown in Fig. 1, we use the example of a ten-vehicle platoon including one lead vehicle and nine follower vehicles in a straight lane. Further, to investigate the impacts of time delay on the performance, three scenarios are considered: no time delays, heterogeneous time delays, and homogeneous time delays.

A. Simulation Setting

For the simulations, the sampling interval is set as \(\Delta t = 0.01s\). The initial positions are \(x(0) = [0,10,20,5,3,1,5,43,55,67,5,80,5,94,108]^T\) m on a lane. The initial velocities are set as \(= [7,7,7,7,7,7,7,7,7]^T\) m/s. The heterogeneous time delays are selected as \(= [0,0.15,0.18,0.19,0.2,0.21,0.22,0.23,0.27,0.30]^T\) s, and the homogeneous time delays are set as \(= 0.20 \) s. The desired gaps between the followers and the leader are set as \([45,40,35,30,25,20,15,10,5]^T\) m. In addition,
ties, the desired velocity for the leader is given by

\[
v_L(t) = \begin{cases} 
7 \text{m/s}, & 0 \leq t < 30 \text{s} \\
(7 + \frac{8}{1 + e^{-0.5r}}} \text{m/s}, & 30 \leq t < 70 \text{s} \\
15 \text{m/s}, & 70 \leq t < 80 \text{s} \\
(15 - \frac{15}{1 + e^{-0.5r}}} \text{m/s}, & \text{otherwise}
\end{cases}
\]

The disturbance on the leader under three scenarios is specified as follows:

\[
\zeta_L(t) = 0.3 \sin(2\pi(t - 30))e^{-0(t-30)/10}, \quad t \geq 30 \text{s}
\]

where the disturbance is only acted on the lead vehicle under three scenarios: no time delays, heterogeneous time delays, homogeneous time delays.

In the simulation scenario, the time-varying heterogeneous delays increase with the increase in communication time \(\tau_i(t) \leq \tau^*; \tau_i(t) \in [\tau_{\text{min}}, \tau_{\text{max}}]\), with \(\tau_{\text{min}} = 0\) s, and \(\tau_{\text{max}} \leq \tau^* = 30 \times 10^{-2}\) s, where the theoretical upper bound \(\tau^*\) is computed, as in Theorem 2. Note that \(\tau^*\) is within the average end-to-end communication delay that is typical of an IEEE 802.11p vehicular network, which is of the order of hundredths of a second (i.e., \(10^{-2}\) s) [37].

**B. Discussion of Results**

The drive cycles of vehicles in the platoon are presented in Figs. 2, 3, 4, 5 and 6. Fig. 2 shows the position profile of vehicles in the platoon under the proposed control algorithm (13). Based on Fig. 2, the follower vehicles can follow the lead vehicle smoothly and then gradually stop behind the leader, while the vehicles can maintain a certain safe constant inter-vehicle gap with each other under the three scenarios considered. It is further illustrated by the spacing errors (i.e., \(x_{i-1} - x_i - h_c\)) shown in Fig. 3. From Fig. 3, the spacing errors from different initial values converge to positive constant value during the time 20s \(\leq t \leq 30\) s. After an acceleration process, the spacing errors converge to another positive constant value during the time 70s \(\leq t \leq 80\) s. Finally, the spacing errors converge to zero smoothly. It also shows that the inter-vehicle gaps (i.e., \(x_{i-1} - x_i\)) between vehicles in the platoon are always safe and converge to the desired safety gap accordingly, i.e., \(h_c = 5\) m. This implies that the vehicle platoon with the proposed control algorithm can avoid rear-end collisions and achieve consensus in terms of the position profile.

Based on Fig. 4, if the lead vehicle starts to accelerate from 7 m/s to 15 m/s during the time 30s \(\leq t \leq 80\) s, and then eventually decelerate to zero, the vehicles in the platoon start to accelerate from the initial velocity. When the vehicles reach the peak velocity, they gradually slow down to 7 m/s. during this process, the leader (i.e., \(v_L\)) maintains a constant velocity and the follower vehicles will adjust their velocities according to the car-following interactions shown in (13a). In addition, the maximum velocities are different (i.e., 11 m/s, 11.8 m/s, 12.1 m/s) under the scenarios of no time delays, heterogeneous time delays, and homogeneous time delays. Then, they will accelerate to reach 15 m/s followed by the lead vehicle. Finally, they will stop with 0 m/s. In addition, Fig. 4(c) shows that fluctuations appear on the velocity profile under larger homogenous time delays (0 \(\leq t < 30\) s) and the disturbance (shown in (53)) acting on the leader (\(t \geq 30\) s). It implies that the time delay will impact the stability of vehicle platoon if it exceeds a critical value.
Fig. 4. Velocity profile for a platoon with: (a) no time delays, (b) heterogeneous time delays, (c) homogeneous time delays.

Fig. 5. Velocity errors profile for a platoon with: (a) no time delays, (b) heterogeneous time delays, (c) homogeneous time delays.

Fig. 6. Acceleration profile for a platoon with: (a) no time delays, (b) heterogeneous time delays, (c) homogeneous time delays.

Fig. 5 shows the velocity error profile. From Fig. 5, the maximum differences are 4 m/s, 4.3m/s, 5.2m/s under no time delays, heterogeneous time delays and homogeneous time delays, respectively. Besides, the fluctuation in velocity error in Fig. 5(c) also illustrates insights similar to those discussed in Fig. 4 (c).

Fig. 6 shows acceleration/deceleration profile. From Fig. 6, the maximum decelerations are less than 4m/s². The fluctuation appears on the acceleration profile under the heterogeneous and homogeneous time delays.

In summary, Figs. 2-6 show that the proposed platoon control algorithm can ensure consensus of vehicles in the platoon with respect to position, velocity, and acceleration/deceleration profiles. Also, it circumvents negative spacing error, negative velocity, and unreasonable acceleration/deceleration rate. An analysis of the effect of time delays suggest that time delay beyond the upper bound will deteriorate the performance (stability) of platoon control.

C. Comparison to Existing Approaches

Note that the previous studies (such as [14], [15], [17], [27], and [29]) show that negative spacing errors exist, which implies that collisions may occur during platoon movement. In addition, some spacing errors are larger than even 20m in [25], which is unreasonable in practice. Compared to these studies, the spacing errors (i.e., $x_{i-1} - x_i - h_e$) in this study shown in Fig. 3 are positive. This is because car-following interactions between vehicles (shown in (13a)) are incorporated in this study.

Related to the velocity profile, negative velocity exists in [20] and [26], which implies that vehicles in the platoon move backwards on a road, which is not realistic. Compared to [20] and [26], Fig. 4 shows that this study can avoid negative velocity. This is also because car-following interactions between vehicles (as shown in (13a)) are considered. Also, the disturbance will impact the stability of vehicle platoon.

In [25], however, the fluctuation in velocity profile is very significant, and a similar process costs about 30s from -20m/s...
to 40 m/s. This is because the control algorithm designed in [25] ignores the car-following interactions between vehicles. In the context of the acceleration/deceleration profile, the acceleration in [18] is not smooth enough as a large acceleration rate drop occurs, which leads to uncomfortable drive level. Compared to [18], the acceleration profile shown in Fig. 6 is relatively smoother. Also, the maximum acceleration rate does not exceed 6 m/s² and the minimum deceleration rate is smaller than 4 m/s² although the disturbance is considered. Comparing the three scenarios, we note that Fig. 6(a) is the smoothest with no time delays, followed by Fig. 6(b) with heterogeneous time delays, and then Fig. 6(c) with larger homogeneous time delays.

Table I summarizes the performance of the proposed control algorithm with different time delays.

VI. CONCLUDING REMARKS

This study focuses on cooperative control of vehicle platoons. Unlike the existing linear consensus approaches, this study proposes a nonlinear consensus method by considering the interactions between vehicles. In particular, a distributed nonlinear delay-dependent control algorithm is developed to achieve platooning of vehicles in the presence of heterogeneous time-varying delays. Further, the interactions between vehicles in the platoon are incorporated to avoid negative spacing error, negative velocity, and unreasonable acceleration/deceleration rate. The convergence of the proposed control algorithm is analyzed rigorously based on the Lyapunov-Krasovskii method. Also, the upper bound of time delay for platoon stability is provided. Finally, three scenarios, corresponding to no time delays, heterogeneous time delays, and homogeneous time delays, are analyzed extensively. Results from the simulations demonstrate the effectiveness of the proposed algorithm with respect to the position, velocity, and acceleration/deceleration profiles.

This paper bridges the gaps between control theory and transportation engineering related to platoon control, by designing a control algorithm that considers consensus and vehicular interactions simultaneously. In addition, this study serves as a starting point for our future research that includes field experiments, to generate more practical insights.

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**Yongfu Li** (M’16) received the Ph.D. degree in control science and engineering from Chongqing University, Chongqing, China, in 2012. He is currently an Associate Professor of control science and engineering with Chongqing University of Posts and Telecommunications. His research interests include intelligent transportation systems, connected and autonomous vehicles, and control theory.

**Chuancong Tang** received the B.S. degree in automation from Chongqing University of Posts and Telecommunications, Chongqing, China, in 2016. He is currently pursuing the M.S. degree in control science and engineering with Chongqing University of Posts and Telecommunications. His research interests include intelligent transportation systems and cooperative system and control.

**Srinivas Peeta** received the Ph.D. degree in civil engineering from the University of Texas at Austin, Austin, TX, USA, in 1994. He is currently the Frederick R. Dickerson Chair and a Professor with the Schools of Civil and Environmental Engineering and Industrial and Systems Engineering, Georgia Institute of Technology. He is also a Principal Research Faculty with Georgia Tech Research Institute. His research interests include intelligent transportation systems, operations research, control theory, and computational intelligence techniques.

**Yibing Wang** (M’03) received the Ph.D. degree in control theory and applications from Tsinghua University, Beijing, China. He is currently a Full Professor with the College of Civil Engineering and Architecture, Institute of Transportation Engineering, Zhejiang University, Hangzhou, China. His research interests include traffic flow modeling, freeway traffic surveillance, ramp metering, urban traffic signal control, and vehicular ad hoc networks.