Overview Paper

Nonlinear finite-time consensus-based connected vehicle platoon control under fixed and switching communication topologies

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ABSTRACT

This paper proposes nonlinear consensus-based control strategies for a connected vehicle (CV) platoon under different communication topologies. In particular, pinning control based consensus protocols are proposed by incorporating the car-following interactions between CVs under fixed and switching communication topologies. The finite-time stability and consensus of the proposed protocols are rigorously analyzed using the LaSalle’s invariance principle and Lyapunov technique. The theoretical analyses investigate the impacts of communication topology on convergence and stability of CV platoon. This study conducts numerical experiments for a CV platoon under four scenarios: (i) Fixed communication topology with time-invariant leader, (ii) fixed communication topology with time-variant leader, (iii) switching communication topology with time-invariant leader, and (iv) switching communication topology with time-variant leader. Simulations results illustrate the effectiveness of the proposed protocols in terms of convergence time and stability with respect to position and velocity profiles.

1. Introduction

The recent advances in information communication technology, especially the vehicle-to-vehicle (V2V) communication, have attracted considerable attention in transportation domain (Alsabaan et al., 2013; Yan and Olariu, 2011). Vehicles with common interests (e.g., destination, partly overlapping path) can cooperate on the road through V2V communication, for instance, to form a platoon (Jia et al., 2014). Benefits of the platoon-based driving pattern include the road throughput increase, traffic congestion mitigation, and energy consumption and exhaust emissions reduction (Jia et al., 2015; Shao et al., 2015; Farah and Koutsopoulos, 2014). Hence, many studies seek to develop effective control protocols for platoon formation (Jia and Ngoduy, 2016; Santini et al., 2017; Zheng et al. 2015; Zhang et al. 2011; Shen et al. 2018).

The main objective of vehicle platoon formation control is to facilitate a string of vehicles traveling together to achieve a harmonized speed and a pre-specified inter-vehicle gap between adjacent vehicles. The formation control problem can be typically formulated as either an asymptotic stabilization problem or an infinite-time control problem, with collision-free constraints to ensure...
vehicle safety (Li et al., 2015). Most existing studies on platoon formation control mainly focus on the design of appropriate control protocols to assist vehicles to achieve the infinite-time consensus state asymptotically. In practice, however, a vehicle platoon is expected to form in a finite time interval, leading to the crucial concern about the convergence time of platoon control. Moreover, compared to asymptotic stability, finite-time control has a faster convergence rate and better disturbance rejection to system uncertainty and external disturbance (Cao and Ren, 2014; Zhao and Hua, 2014; Lu et al., 2017). Therefore, it entails the investigation of finite-time control for vehicle platoon formation.

Most of the previous studies on finite-time control assume each vehicle as an individual agent and characterize the vehicle movement using the simple first/s order integrator model. However, these studies ignore the car-following interactions between vehicles, and velocity and traffic direction constraints, resulting in negative spacing errors and negative velocity. Also, most studies mainly consider a fixed communication topology (Milanes et al., 2014; Zheng et al. 2015). However, the communication topology among vehicles under the connected environment may not be maintained fixed, partially due to the communication constraints and interferences. Hence, there is also a need to investigate the performance of vehicle platoon control under switching communication topologies.

Motivated by the abovementioned research needs, this paper proposes finite-time control protocols for CV platoon to guarantee the finite-time stability and consensus under switching communication topologies. The vehicle platoon control has been addressed in the literature from three aspects: (1) control protocol design, (2) impacts of communication topology, and (3) consensus analysis.

(1) Control protocol design

Several approaches to the platoon formation control have been considered in the literature, including linear control, sliding mode control (SMC), and model predictive control (MPC). In the category of linear control approach, Ghasemi et al. (2013) proposed a feedback-based decentralized control framework for a platoon of vehicles. Guo and Yue (2012) proposed a control protocol considering the constant actuator delays and the effects of sensing range limitation. In the category of SMC approach, Kwon and Chwa (2015) proposed a coupled SMC for vehicle platooning. In the category of MPC approach, Wang et al. (2015) dealt with vehicle platooning under a congested traffic condition. Dunbar and Caveney (2012) proposed a distributed receding horizon control for vehicle platoons with nonlinear dynamics. Zheng et al. (2017) proposed a distributed model predictive control (DMPC) algorithm for heterogeneous vehicle platoons with nonlinear dynamics and unidirectional topologies and derived a sufficient condition to guarantee asymptotic stability. Tuchner and Haddad (2017) proposed an interpolating control strategy for vehicle platoon formation. Chen et al. (2017) investigated the impact of grade on platooning on uphill grades and found that truck platoons were asymptotically unstable beyond some critical grade. Recently, Bernardo et al. (2015) proposed a distributed protocol for platooning control considering heterogeneous delays. Gao et al. (2016) proposed an H-infinity control method for a platoon of heterogeneous vehicles with uncertain vehicle dynamics and uniform communication delay. Li et al. (2016a) proposed a new robust acceleration tracking control of vehicle longitudinal dynamics for the purpose of platoon-level automation. Beselink and Johansson (2017) proposed a delay-based spacing policy that guarantees disturbance string stability with respect to the reference velocity. In addition, Li et al. (2017a) introduced a decomposition framework to model, analyze, and design the platoon system, which incorporates four interrelated components. Luo et al. (2018) proposed coordinated platooning model with multiple speed options that integrates scheduling, routing, speed selection, and platoon formation/dissolution that minimizes the total fuel consumed by a set of vehicles. Mai et al. (2017) proposed an ontological model of platooning objects and properties and abstract basic building blocks of platoon operations that can then be aggregated to complex platooning behavior. Larsson et al. (2015) developed a framework for modeling platooning vehicles traveling in road networks and proposed heuristics that can solve large instances of the platooning problem. Lioris et al. (2017) assessed the potential mobility benefits of platooning and found that saturation flow rates and intersection capacity could be doubled or tripled by platooning.

The abovementioned studies mainly focus on the asymptotic stability of platoon control. Compared to asymptotic stability control, finite-time control has a faster convergence rate and better disturbance rejection to system uncertainty and external disturbance (Cao and Ren, 2014; Zhao and Hua, 2014; Zhao et al., 2015). In the category of finite-time control, Guo et al. (2016) proposed a distributed adaptive SMC to address the string stability of vehicle platoon with nonlinear acceleration uncertainty. Cao and Ren (2014) analyzed the continuous consensus of a multi-agent system (MAS) with unknown inherent nonlinear dynamics. Guan et al. (2012) studied the finite-time consensus problem using the pinning control strategy. Zhao and Hua (2014) proposed a finite-time consensus tracking control protocol for MAS via SMC. However, these studies do not consider the car-following interactions among CVs, which may result in negative spacing errors and negative velocity.

(2) Impacts of communication topology

Early platoon control studies mainly applied radar technology, implying that a vehicle can only obtain the information of its nearest neighbors. The communication topologies upon this sensing system represent the predecessor following (PF) and bidirectional (BD) structures. Naus et al. (2010) proposed a cooperative adaptive cruise control (CACC) for platoon under the PF topology. Ploeg et al. (2014) studied the string stability of a CACC-equipped vehicle platoon under the PF topology. Kwon and Chwa (2014) proposed a coupled SMC for vehicle platoon under the BD topology. Recently, more communication topologies have emerged with the advancements of V2V communications, including the predecessor-leader following (PLF) topology, bidirectional-leader (BDL) topology, and two-predecessor following (TPF) topology (Zheng et al., 2015). Several studies have examined the influence of communication topology on platoon performance, including stability and scalability. Li et al. (2017b) presents a robust distributed control method for vehicular platoons with bounded parameter uncertainty and a broad spectrum of interaction topologies. Peters
et al. (2014) proposed a control protocol to achieve string stability while maintaining a tight space with time delays under the PLF topology. Li et al. (2017b) investigated the impacts of communication topology with different initial states on the performance of platoon control. These studies mainly focus on platoon control under a fixed communication topology. However, due to the communication constraints and environmental disturbances, the communication topology between vehicles may change or switch. Furthermore, it can be difficult to timely transfer the leader information to the other vehicles that are far from the leader, which leads to only a small fraction of follower vehicles receiving the information of the lead vehicle.

(3) Consensus analysis

Vehicle platoon control is a specific consensus control problem, which manages all vehicles in a string to converge to a consensus state. From the consensus control problem, Olfati-Saber and Murray (2004) proposed a linear controller to achieve position consensus considering communication time-delay. Ren (2007) proposed a multi-vehicle control algorithm for position consensus. Lin et al. (2012) designed an optimal localized-feedback controller to achieve the position consensus. Saeednia and Menendez (2017) proposed a cooperative distributed approach for forming/modifying platoons of trucks based on consensus algorithms. However, these works only focus on the first-order dynamic system. The second-order consensus problem is more complicated and challenging. To address this issue, Barooah et al. (2009) proposed a decentralized control protocol for vehicle platoon to achieve both position and velocity consensus. Bernardo et al. (2015) proposed a distributed controller to achieve position and velocity consensus with time-varying heterogeneous delays. Jia and Ngoduy (2016) proposed a consensus-based control algorithm for multi-platoon cooperative driving with second-order vehicle dynamics. Recently, Zegers et al. (2016) proposed distributed consensus-based control approach for vehicle platoons factoring vehicle dynamics, velocity-dependent spacing-policies and generic communication topologies. Zhang et al. (2018) proposed a hierarchical framework for the design of connected cruise control considering communication delay and uncertain vehicle dynamics. Zheng et al. (2016) proposed a consensus algorithm based on the third-order state-space model using feedback linearization to achieve position and velocity consensus as well as acceleration consensus. However, the consensus protocols designed in these studies are mostly linear, which cannot effectively capture the car-following interactions between vehicles. While these studies focus on position and velocity consensus, they do not guarantee the non-negativity of spacing error and velocity, which is required for practical road system.

Some recent studies synthesize the above three aspects to reveal the comprehensive mechanism of vehicle platoon control. Bernardo et al. (2016) designed a novel control framework for vehicle platooning which is validated in experiments with time-varying heterogeneous delays. Salvi et al. (2017) proposed a third-order-based consensus control protocol to investigate the performance of vehicle platoon while considering the impact of the heterogeneous time delays. They also derived the condition of the convergence and string stability for vehicle platoon under the scenarios of fixed and switching communication topology. However, the convergence time of platoon consensus in these two studies are infinite while the leader is assumed with constant states (such as constant speed). Compared to these control protocols, the proposed controller in this study can achieve consensus in finite time under the fixed and switching communication topologies with different leader states.

The primary objective of this paper is to propose control protocols to achieve nonlinear finite-time platoon consensus for CVs under fixed and switching communication topologies. The contributions are summarized as follows.

Novel pinning control based nonlinear consensus protocols are proposed which factors the car-following interactions between CVs. In particular, the proposed control protocols incorporate: (i) the car-following interactions between the following and preceding vehicles, (ii) the inter-vehicle gaps and velocity differences between follower vehicles, and (iii) the inter-vehicle gap and velocity differences between follower vehicles and the lead vehicle. The proposed protocols ensure the consensus of inter-vehicle gap and velocity of vehicles, and the consistency of vehicular flow with traffic flow theory, thereby prevents negative spacing errors and negative velocity.

The finite-time stability and consensus of the proposed protocols are rigorously analyzed using the LaSalle’s invariance principle and Lyapunov technique. The theoretical analysis guarantees that the consensus associated with spacing error and velocity can be reached in a finite time.

The numerical experiments conducted in this study provide extensive analyses on the CV platoon performance under the following four scenarios: (i) Fixed communication topology with time-invariant leader, (ii) fixed communication topology with time-variant leader, (iii) switching communication topology with time-invariant leader, and (iv) switching communication topology with time-variant leader. The simulation results verify the effectiveness of the proposed control protocols in terms of convergence time and stability with respect to position and velocity profiles.

The rest of the paper is organized as follows. Section 2 presents the preliminaries for problem formulation. Section 3 proposes the control protocols for fixed and switching topologies and analyzes the consensus of the proposed protocols. Section 4 performs numerical experiments. Section 5 concludes this study.

2. Preliminaries

2.1. Notation

\( i, j \): The number of followers in a vehicle string on road and \( i, j = 1, 2, \ldots, n \);

\( x_l(t) \): Position of the leader in a vehicle string at time \( t \);

\( x_i(t) \): Position of vehicle \( i \) in a vehicle string at time \( t \);
u_0(t): Control input of the leader in a vehicle string at time t;
u_i(t): Control input of vehicle i in a vehicle string at time t;
r_i = i(h + m): The desired inter-vehicle gap between vehicle i and the leader; h is the desired safety inter-vehicle gap and m is the length of vehicle;
r_ij = r_i - r_j: The desired inter-vehicle gap between vehicle i and vehicle j in a vehicle string.

2.2. Problem statement

This paper considers CVs in a string running on a straight road (see Fig. 1). The platoon includes n followers and one leader. To describe the connections between CVs, the communication topology is defined based on the algebraic graph theory. Let the linkage matrix be \( D = \text{diag}(d_1, d_2, \ldots, d_n) \), where \( d_i > 0 \), \( i = 1, 2, \ldots, n \), if follower i can receive information from the leader; otherwise \( d_i = 0 \) (Zheng et al., 2015; Li et al., 2016b). The topology of follower vehicles in a string can be described by a graph \( G = [V, E, A] \), where \( V \) is the set of n nodes and \( E \subseteq V \times V \) is the set of edges. Each vehicle is regarded as a node in the weighted graph and edges denote the communication links between CVs. The topology of the graph is associated with a weighted adjacency matrix \( A = [a_{ij}] \) consisting of nonnegative elements \( a_{ij} \geq 0 \). Follower vehicle i can obtain information from follower vehicle j if \( a_{ij} > 0 \), otherwise \( a_{ij} = 0 \) (Bernardo et al., 2015; Olfati-Saber and Murray, 2004). If a graph \( G \) with \( \forall (i, j) \in E \Rightarrow (j, i) \in E \), the graph \( G \) is called an undirected graph. A graph is connected if there is a path between every pair of nodes. Let \( P = \text{diag}(p_1, p_2, \ldots, p_n) \), where \( p_i \) is the pinning control gain satisfying \( p_i > 0 \) if vehicle i is pinned and otherwise \( p_i = 0 \) (Guan et al., 2012).

The following lemmas will be used in further analyses of the proposed control protocols.

Lemma 1 (LaSalle’s Invariance Principle—Rouche et al., 1977). Let \( x(t) \) be a solution of \( \dot{x}(t) = f(x) \), \( x(0) = x_0 \in \mathbb{R}^n \), where \( f: U \rightarrow \mathbb{R}^n \) is continuous with an open subset \( U \) of \( \mathbb{R}^n \), and \( V: U \rightarrow \mathbb{R} \) be a locally Lipschitz function such that \( D^+V(x(t)) \leq 0 \), where \( D^+ \) denotes the upper Dini derivative. Then \( \Theta^+(x_0) \cap U \) is contained in the union of all solutions that remain in \( S = \{x \in U : D^+V(x) = 0\} \), where \( \Theta^+(x_0) \) denotes the positive limit set.

The following definition of homogeneity with dilation will be used in the finite-time convergence analysis (Rosier, 1992). Consider the n-dimensional system

\[
x = f(x), \ x = (x_1, x_2, \ldots, x_n)^T \in \mathbb{R}^n.
\]  

(1)

A continuous vector field \( f(x) = (f_1(x), f_2(x), \ldots, f_n(x))^T \) is homogeneous of degree \( \kappa \in \mathbb{R} \) with dilation \( (\tau_1, \tau_2, \ldots, \tau_n) \), if

\[
f(\varepsilon^{\tau_1}x_1, \ldots, \varepsilon^{\tau_n}x_n) = \varepsilon^{\kappa}\hat{f}(x), \ \tau = 1, 2, \ldots, n, \ \varepsilon > 0.
\]  

(2)

is called locally homogeneous if \( f(x) \) is homogeneous of degree \( \kappa \in \mathbb{R} \) with the dilation \( (\tau_1, \tau_2, \ldots, \tau_n) \) and \( \hat{f} \) is a continuous vector field satisfying

\[
\lim_{\varepsilon \to 0} \frac{\hat{f}(\varepsilon^{\tau_1}x_1, \ldots, \varepsilon^{\tau_n}x_n)}{\varepsilon^{\tau+n}} = 0, \ \forall x \neq 0, \ i = 1, 2, \ldots, n.
\]  

(3)

Lemma 2 (Wang and Hong, 2010). Suppose that Eq. (1) is homogeneous of degree \( \kappa \in \mathbb{R} \) with dilation \( (\tau_1, \tau_2, \ldots, \tau_n) \), the function \( f(x) \) is continuous and \( x = 0 \) is its asymptotically stable equilibrium. If homogeneity degree \( \kappa < 0 \), then the equilibrium of Eq. (1) is finite-time stable. Moreover, if Eq. (3) holds, then the equilibrium of Eq. (2) is locally finite-time stable.

Lemma 3 (Wang and Hong, 2010). Consider system \( \dot{x} = f_k(x), f_k(0) = 0, x \in \mathbb{R}^n \). Let \( \Gamma \) denote the finite switching index set, \( \sigma(t): [0, \infty) \rightarrow \Gamma \) be a piecewise constant function of time, \( f_k \) be continuous with respect to x for fixed \( k \in \Gamma \), and \( \tau \) be the dwell time. If the switched system is asymptotically stable, and for a fixed \( k \in \Gamma \), \( \dot{x} = f_k(x) \) is finite-time stable, then the system is finite-time stable.

2.3. Vehicle model

This study applies the kinematic model to design the control protocol based on the leader-follower approach (Hu and Feng, 2011;
Guan et al., 2012; Bernardo et al., 2015; Li et al., 2016b). The dynamics of vehicle \( i \) is formulated as:

\[
\begin{aligned}
\dot{x}_i(t) &= v_i(t) \\
\dot{v}_i(t) &= u_i(t)
\end{aligned}
\]

(4)

Similarly, the dynamics of the leader in a string is:

\[
\begin{aligned}
\dot{x}_l(t) &= v_l(t) \\
\dot{v}_l(t) &= u_l(t)
\end{aligned}
\]

(5)

3. Finite-time platoon control protocol design

3.1. Finite-time platoon control under the fixed communication topology

Pinning control is an effective control scheme for a large complex network by pinning parts of nodes (Guan et al., 2012). In this study, a pinning control based nonlinear consensus protocol is proposed as follows:

\[
\begin{aligned}
\dot{x}_i(t) &= v_i(t) \\
\dot{v}_i(t) &= u_i(t) - \sum_{j=1}^{n} a_{ij} [\beta_1 (V_j (h_j(t)) - v_i(t))] - \sum_{j=1}^{n} a_{ij} [\beta_2 \varphi_1 (\text{sign}(x_i(t) - x_j(t)) - \eta_j)]
\end{aligned}
\]

(6)

where \( a_{ij} \) is communication link from vehicle \( i \) to vehicle \( j \); \( \beta_1, \beta_2 \) are positive constants; \( \varphi_1 \) is a continuous odd function satisfying \( \varphi_1(x) > 0 \) for some \( c_k > 0 \); \( \eta_j \) is the pinning control gain of vehicle \( j \); and \( d_i \) is the communication link from the lead vehicle to vehicle \( i \). Notation \( \text{sign}(x) = \frac{|x|}{x} \) denotes the sign function and \(|x|\) denotes the absolute value of \( x \).

The function \( V(h_j(t)) \) is defined to capture the interactions between vehicles \( i \) and \( j \), which is associated with the average bumper-to-bumper distance (Jiang et al., 2001; Li et al., 2018):

\[
V_i(h_j(t)) = V_l + V_2(\tanh(C_i(h_j(t)) - C_2)),
\]

(7)

where \( V_1, V_2, C_1, C_2 \) are the positive constants and \( h_j(t) = (x_i(t) - x_j(t)) - (i - j)m_0/(i - j) \). Define the position error and velocity error as:

\[
\begin{aligned}
\vec{x}_i(t) &= x_i(t) - x_j(t) - \eta_i \\
\vec{v}_i(t) &= v_i(t) - v_j(t)
\end{aligned}
\]

(8)

Hence, it follows from Eq. (6) that:

\[
\begin{aligned}
\dot{\vec{x}}_i(t) &= \vec{v}_i(t) \\
\dot{\vec{v}}_i(t) &= \sum_{j=1}^{n} a_{ij} [\beta_1 (V_j (h_j(t)) - V_i (\vec{v}_i(t))) - \sum_{j=1}^{n} a_{ij} [\beta_2 \varphi_1 (\text{sign}(\vec{x}_i(t) - \vec{x}_j(t)))]]
\end{aligned}
\]

(9)

where \( h_j^*(t) \) is the desired safety gap between adjacent vehicles, and \( h_j^*(t) = h \).

Applying the mean value theorem (Ben-Israel and Gilbert, 2002), we have:

\[
V_j(h_j(t)) - V_i(h_j(t)) = \frac{V_i(\xi_j(t))) - V_j(\xi_j(t)))}{l-j}(\vec{x}_j(t) - \vec{x}_i(t)),
\]

(10)

where the value of \( \xi_j(t) \) depends on \( h_j(t) \) and \( h_j^*(t) \). We remark that the expression of \( \xi_j(t) \) is unique if \( V_i(h) \) is invertible, while the value of \( \xi_j(t) \) may not be unique if \( V_i(h) \) is not invertible. The subsequent analysis only relies on the existence of \( \xi_j(t) \). Substituting Eq. (10) into Eq. (9) yields:

\[
\begin{aligned}
\dot{\vec{x}}_i(t) &= \vec{v}_i(t) \\
\dot{\vec{v}}_i(t) &= \sum_{j=1}^{n} a_{ij} [\beta_1 (B_j (\vec{x}_j(t) - \vec{x}_i(t))) - \vec{v}_i(t)))] - \sum_{j=1}^{n} a_{ij} [\beta_2 \varphi_1 (\text{sign}(\vec{x}_i(t) - \vec{x}_j(t)))] + \beta_2 \varphi_2 (\text{sign}(\vec{v}_i(t) - \vec{v}_j(t)))
\end{aligned}
\]

(11)

where \( B_j = V_j'(\xi_j(t))/(i-j) \).

**Theorem 1.** Suppose that the communication topology between follower vehicles is undirected. For the pinning control system (6) with \( 0 < c_k < 1 \), \( c_k = 2c_k/(1 + c_k) \), if there exist continuous odd functions \( \varphi_2 \) satisfying \( \varphi_2(x) > 0 \) for all \( x \neq 0 \) and \( \varphi_2(x) = c_k x + o(x) \) around \( x = 0 \) for some \( c_k > 0 \), \( k = 1, 2, 3, 4 \), then the vehicle platoon can achieve the finite-time consensus under fixed communication topology, i.e.:
\[ \lim_{t \to \infty} \| \mathbf{x}(t) \| = 0; \quad \lim_{t \to \infty} \| \mathbf{y}(t) \| = 0. \]

**Proof.** For Eq. (11), the Lyapunov function candidate is chosen as (Guan et al., 2012):

\[ H(t) = H_1(t) + H_2(t) + H_3(t) + H_4(t), \]

where

\[ H_1(t) = \frac{1}{2} \sum_{i=1}^{n} \int_{0}^{t} \alpha_i \eta_i \phi_i(s) \phi_i(s) \, ds, \quad H_2(t) = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \int_{0}^{t} \beta_{ij} \phi_i(s) \phi_j(s) \, ds, \quad H_3(t) = \sum_{i=1}^{n} \int_{0}^{t} \beta_{i0} \phi_i(s) \phi_i(s) \, ds, \quad \text{and} \]

\[ H_4(t) = \frac{1}{2} \sum_{i=1}^{n} \int_{0}^{t} \dot{a}_i \phi_i(s) \phi_i(s) \, ds. \]

Based on Eq. (11), we have

\[
\begin{align*}
\dot{H}_1(t) &= \sum_{i=1}^{n} \int_{0}^{t} \alpha_i \phi_i(s) \phi_i(s) \, ds - \sum_{i=1}^{n} \int_{0}^{t} \alpha_i \phi_i(s) \phi_i(s) \, ds \\
&= -\sum_{i=1}^{n} \int_{0}^{t} \alpha_i \phi_i(s) \phi_i(s) \, ds \\
&\leq 0.
\end{align*}
\]

If \( H(t) = 0 \), it yields

\[
\sum_{i=1}^{n} \phi_i(t) = 0.
\]

Hence, we have \( \mathbf{v}(t) = \dot{\mathbf{y}}(t) = 0 \). It implies that \( \mathbf{v}(t) = \dot{\mathbf{y}}(t) = 0 \). Further, we have

\[
\mathbf{v}(t) = \sum_{j=1}^{n} \alpha_j \phi_j(s) \phi_j(s) \phi_j(s) \phi_j(s) \, ds - \sum_{j=1}^{n} \int_{0}^{t} \phi_j(s) \phi_j(s) \phi_j(s) \phi_j(s) \, ds.
\]

It follows from Eq. (17) that

\[
\sum_{i=1}^{n} \phi_i(t) = \sum_{j=1}^{n} \alpha_j \phi_j(s) \phi_j(s) \phi_j(s) \phi_j(s) \, ds - \sum_{j=1}^{n} \int_{0}^{t} \phi_j(s) \phi_j(s) \phi_j(s) \phi_j(s) \, ds.
\]

Eq. (18) implies that \( \mathbf{x}(t) = \mathbf{y}(t) = 0 \). According to Lemma 1, we have \( x(t) - x(t) \to 0, \quad v(t) - v(t) \to 0 \) as \( t \to \infty \). Let \( \psi(t) = (\mathbf{x}(t), \mathbf{y}(t), \mathbf{v}(t), \mathbf{w}(t), \ldots, \mathbf{v}(t)) = (\psi_1(t), \psi_2(t), \ldots, \psi_{n-1}(t), \psi_n(t), \psi_{n+1}(t), \ldots, \psi_m(t)) \). Note that \( \varphi_k(x) = c_kx + o(x), \quad k = 1, 2, 3, 4 \), we have
Table 1

Adjacency matrices for four topologies.

<table>
<thead>
<tr>
<th>Topology</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
</tr>
</thead>
</table>
| Adjacency matrix | \[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\] | \[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\] | \[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\] | \[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\] |
| Linkage matrix | \[D = \text{diag}(1, 1, 1, 1, 1)\] | \[D = \text{diag}(1, 1, 0, 0, 0)\] | \[D = \text{diag}(1, 1, 0, 0, 0)\] | \[D = \text{diag}(1, 1, 0, 0, 0)\] |

Table 2

The initial state and desired distance of each vehicle.

<table>
<thead>
<tr>
<th>Vehicle</th>
<th>Leader</th>
<th>Follower 1</th>
<th>Follower 2</th>
<th>Follower 3</th>
<th>Follower 4</th>
<th>Follower 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>x((t_0))</td>
<td>100 m</td>
<td>80 m</td>
<td>60 m</td>
<td>40 m</td>
<td>20 m</td>
<td>0 m</td>
</tr>
<tr>
<td>v((t_0))</td>
<td>10 m/s</td>
<td>6 m/s</td>
<td>8 m/s</td>
<td>10 m/s</td>
<td>12 m/s</td>
<td>14 m/s</td>
</tr>
<tr>
<td>r((t_0))</td>
<td>0</td>
<td>-10 m</td>
<td>-20 m</td>
<td>-30 m</td>
<td>-40 m</td>
<td>-50 m</td>
</tr>
</tbody>
</table>
It implies that Eq. (11) is locally homogeneous with degree $\kappa = \alpha_1 - 1 < 0$. In addition, Eq. (11) can also be rewritten as $\ddot{u}_i(t) = \ddot{u}_i(t) + \ddot{u}_i(t)$ with

$$
\lim_{\varepsilon \to 0} \frac{\sum_{j=1}^{n} a_j [\beta_j \nu_j^2 (\phi_i(t) - \psi_j(t))] - \lambda_2 \phi_i(t)}{\varepsilon^{\lambda_2 + \sigma}} = \lim_{\varepsilon \to 0} \frac{\sum_{j=1}^{n} a_j [\beta_j \nu_j^2 (\phi_i(t) - \psi_j(t))] - \lambda_2 \phi_i(t)}{\varepsilon^{\lambda_2 + \sigma}} = 0.
$$

(23)
Then, Eq. (11) with variables \(\tilde{\delta}_t^i\) is homogeneous of degree \(\kappa = \alpha_i - 1 < 0\) with the dilation \((2, 2, ..., 2, \alpha_i + 1, \alpha_i + 1, ..., \alpha_i + 1)\) and \(\tilde{u}_t^i\) satisfies Eq. (3). Hence, we know that the invariant manifold \(\tilde{\delta}_t^i = 0, \tilde{\nu}_t^i = 0\) is Lyapunov stable. Thereby, Eq. (11) is globally and asymptotically stable and locally homogeneous with degree \(\kappa = \alpha_i - 1 < 0\). According to Lemma 2, the origin is a globally finite-time stable equilibrium of Eq. (11), which implies that \(\lim_{t \to T} ||\tilde{\delta}_t^i|| \to 0\) and \(\lim_{t \to T} ||\tilde{\nu}_t^i|| \to 0\). This completes the proof. \(\square\)

3.2. Finite-time platoon control under switching communication topology

Let \(\Gamma = [V, E, A_\sigma(t)]\) be a set of undirected graph. Note that \(\Gamma\) is a finite-time set if all weights are chosen from a finite-time set. Denote \(\Gamma = [0, 1, 2, ..., N]\) as the index set of graph \(\Gamma\). We introduce a switching signal \(\sigma(t)\): \(R^+ \to \Gamma\) and a switching time sequence \(t_0 = 0, t_1, ..., t_s\) at which the communication topology changes. For any \(i \in [t_n, t_{n+1})\), the topology \(\Gamma_{\alpha(i)} = G_{\sigma(i)} \in \Gamma\) is active. Accordingly, the adjacency weight between vehicles \(i\) and \(j\) is \(a_{ij}^\alpha\), the link connection between vehicle \(i\) and the leader is \(d_i^\alpha\), the pinning control gain for vehicle \(i\) is \(p_i^\alpha\), and \(\tilde{p}_t^i = d_i^\alpha + p_i^\alpha\). Thereby, the finite-time control protocol for the switching communication topology is designed as follows:
For any $t \in [t_i, t_{i+1})$, we have $\bar{x}_i(t) = x_i(t) - x_i(t-\tau)$, $\bar{v}_i(t) = v_i(t) - v_i(t)$. Then, Eq. (25) can be rewritten as

$$
\begin{align*}
\ddot{\bar{x}}_i(t) &= \bar{v}_i(t) \\
\ddot{\bar{v}}_i(t) &= u_i(t) - \sum_{j=1}^{n} a_{ij}^\tau [\beta_1 (v_j(t) - \bar{v}_i(t))] - \sum_{j=1}^{n} a_{ij}^\tau [\beta_2 \varphi_1 (\text{sign}(x_j(t) - x_i(t)) \eta_j) + \beta_1 \varphi_2 (\text{sign}(v_j(t) - \bar{v}_i(t)))]
- \beta_1^\tau [\beta_2 \varphi_1 (\text{sign}(x_j(t) - x_i(t)) \eta_j) + \beta_3 \varphi_2 (\text{sign}(v_j(t) - \bar{v}_i(t)))]. 
\end{align*}
$$

(25)

Theorem 2. Suppose that the communication topology among vehicles is undirected. For the pinning controlled switching system (26) with $0 < \alpha_1 < 1$, $\alpha_2 = 2\alpha_1(1 + \alpha_1)$ and the switching signal $\sigma(t): \mathbb{R}^+ \to \Gamma$, if there exists a continuous odd function $\varphi_k$ satisfying $x\varphi_k(x) > 0$ for $x \neq 0$ and $\varphi_k(x) = c_k x + o(x)$ around $x = 0$ for $c_k > 0$, $k = 1, 2, 3, 4$, then the vehicle platoon can achieve finite-time consensus under the switching communication topology.

Proof. For any $t \in [t_i, t_{i+1})$, let $\sigma(t) = p \in \Gamma$. The Lyapunov function candidate is formulated as:

$$
H(t) = H_1(t) + H_2(t) + H_3(t) + H_4(t),
$$

(27)

where $H_1(t) = \frac{1}{2} \sum_{i=1}^{n} \ddot{x}_i(t)^2$, $H_2(t) = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}^\tau \beta_1 \varphi_1 (\text{sign}(s))^2 ds$, $H_3(t) = \sum_{i=1}^{n} \int_{0}^{\infty} \dot{\bar{x}}_i(t)^2 \beta_2 \varphi_2 (\text{sign}(s))^2 ds$, and
Similar to the proof of Theorem 1, for any $p \in \Gamma$, the origin is a globally finite-time stable equilibrium of Eq. (26). According to Lemma 3, Eq. (26) converges in finite time. This completes the proof. □

4. Numerical experiments

This section provides the numerical experiments to verify the effectiveness of the proposed control protocols. To investigate the performance, the numerical experiments were conducted upon four communication topologies shown in Table 1. In addition, the effects of communication topologies on the platoon control (e.g. the convergence time and stability) are also investigated. In these experiments, six vehicles, consisting of a lead vehicle and five follower vehicles, are considered.

For comparison, the initial conditions are set as follows:

(i) The odd function: $\varphi_k(x) = x, k = 1, 2, 3, 4$;
(ii) Parameters: $\alpha_1 = 0.60, \alpha_2 = 2\alpha_i/(1 + \alpha_i) = 0.75$;
(iii) The initial state and desired distance of each vehicle are presented as in Table 2.
(iv) The values of relevant parameters related to the proposed protocols are listed in Table 3.
(v) Pinning control gains: $P_i = \text{diag}(0, 0, \ldots, 1); P_2 = \text{diag}(0, 0, \ldots, 1); P_3 = \text{diag}(0, 0, \ldots, 1); P_4 = \text{diag}(0, 0, \ldots, 0)$;
(vi) The ratio between the amplitude and convergence time is defined as

$$R = \frac{\psi_{\text{max}}(t_f)}{t_f}$$

where $\psi_{\text{max}}(t_f) = |\psi(t_f) - \psi(t_f)|_{\text{max}}, \psi(t_f)$ is the error at time $t_f$ and $t_f$ is the convergence time.
There are three levels of ratio: large ($R \geq 1.0$), medium ($0.25 \leq R < 1.0$), and small ($R < 0.25$).

4.1. Case 1 fixed communication topology and time-invariant leader

The drive cycles of vehicles in the platoon are presented in Figs. 2–5. Fig. 2 shows the position profile of platoon vehicles under the proposed control protocol (6). Based on Fig. 2, the followers can follow the leader smoothly while vehicles can maintain a certain safety constant inter-vehicle gap with each other with time-invariant leader under the fixed communication topology. It is further illustrated by the spacing errors shown in Fig. 3. Note that the previous studies (Guo and Yue, 2012; Bernardo et al., 2015; Jia and Ngoduy, 2016) show that negative spacing errors exist, which implies that rear-end collisions may occur. In addition, the spacing errors are larger than 20 m in Zheng et al., (2016), which is unrealistic. Compared to these studies, the spacing errors shown in Fig. 3(a)–(c) are positive. The reason is that car-following interactions between CVs are factored into the protocol (6) and the pinning control is applied to achieve consensus with respect to position. It is further illustrated by Fig. 3(d), where negative spacing error exists. That is because the pinning control is not applied to vehicles 2, 4, and 5 under the communication topology (d) in Table 1. In addition, Fig. 3 shows that the spacing errors from different initial values can converge to zero. It implies that the platoon controlled by the proposed protocol can avoid rear-end collisions and achieve the consensus in terms of position.

Regarding the velocity profile, negative velocity exists in Guo et al., (2016) and Zhang et al. (2017), which implies that vehicles in the platoon move backwards on a road. It is unrealistic. Compared to these studies, Fig. 4 shows that the proposed control protocol can prevent negative velocity. This is also because car-following interactions between CVs are considered. Based on Fig. 4, each vehicle in the platoon gradually accelerates from the different initial velocity. When the vehicle reaches the peak velocity, it gradually slows down to the velocity of the leader (i.e. 10 m/s). The leader maintains a constant velocity and the follower vehicles will adjust their velocities according to the car-following behavior. In addition, the maximum velocities are different under different topologies. It implies that the velocities of vehicles in the platoon will be impacted by the communication topology. Fig. 4(d) shows...
the fluctuations of the velocity profile. This is because the pinning control is not applied to vehicles 2, 4, and 5 under the communication topology (d). It suggests that the proposed protocol can improve the stability of vehicle platoon. Fig. 5 shows the velocity error profile, where the velocity error converges to zero and reaches the consensus asymptotically. From Figs. 2–5, we can conclude that vehicles can form a platoon in finite time under different communication topologies except topology (d). Also, negative spacing errors and negative velocity can be avoided except topology (d).

4.2. Case 2 fixed communication topology and time-variant leader

This section provides comparisons with time-variant leader. Assume the time-variant leader follows a speed trajectory defined by $v_L(t) = 0.4\cos(0.05t) + 0.1$. Fig. 6 shows that followers can track the time-variant leader while maintaining a constant inter-vehicle safety gap. Note that negative spacing error and negative velocity exists in the studies of Zhao and Hua (2014) and Zhao et al., (2015). Compared to these studies, the spacing errors shown in Fig. 7(a) and (b) are positive while negative spacing errors may occur in Fig. 7(c) and 7(d). This is because no connections are constructed between vehicles 2 and 3, and vehicles 4 and 5 in topology (c). Also, no connections exist between vehicles 1 and 2, vehicles 2 and 3, and vehicles 3 and 4 in topology (d). Fig. 7 shows that the spacing errors from different initial values also converge to zero (except topology (d)). It implies that the inter-vehicle gaps converge to the desired gap, i.e., $h = 5m$. Related to the velocity profile, Figs. 8 and 9 show that followers gradually accelerate to the peak velocity, and then they gradually slow down to the leader’s velocity. No negative velocity exists due to the incorporation of the car-following interactions. Thereby, the platoon controlled by the proposed protocol can avoid rear-end collisions (except topologies (c) and (d)) and achieve the consensus with respect to position and velocity.

Applying Eq. (28) and the definition of ratio level between the amplitude and convergence time, Table 4 summarizes the performance of the proposed control protocol under different communication topologies. The main findings are: (i) The proposed control
protocol can facilitate CVs to form a desired platoon under different topologies; (ii) The convergence time is affected by the communication topology. That is, the stronger interconnected communications among vehicles, the less convergence time; and (iii) different communication topologies will lead to different levels of platoon control effectiveness that is defined by the level of ratio, i.e., Eq. (28).

4.3. Case 3 switching communication topology with time-invariant leader

Fig. 10 shows the scenario of the switching communication topology. The initial communication topology is topology (a). The communication topology rotates in the order of (a), (b), (c), and (d) with time intervals of 10 s, 5 s, 5 s, 10 s. Fig. 11(a) shows the position profile with time-invariant leader. Fig. 11(a) shows that the followers can gradually trace the leader smoothly while vehicles maintain a certain safety constant inter-vehicle gap with each other. In this scenario, negative velocity exists in the past studies (Guan et al., (2012), Li et al., (2013), Zhao and Hua (2014), Zhao et al., (2015)). Compared to these studies, Fig. 11(b) shows that all vehicles in the platoon travel with non-negative velocity with the proposed protocol (23). Also, Fig. 12 shows that the spacing errors and velocity errors can converge to zero in finite time. It implies that the protocol (23) can not only achieve the finite-time consensus with respect to position and velocity but also ensure the vehicle behavioral consistency with traffic flow theory.

4.4. Case 4 switching communication topology with time-variant leader

This section provides comparisons with the time-variant leader where the acceleration rate \(a(t) = 0.4 \cos(0.05t) + 0.1\). Fig. 13(a) shows that the followers can follow the time-variant leader while maintaining a constant safety inter-vehicle gap. However, negative
velocity exists in Wang and Wu (2012). Compared to this study, as shown in Fig. 13(b), each follower accelerates from the initial velocity to the peak, and then it will reduce to 10 m/s under switching topology. In addition, compared to Wang and Wu (2012) and Dong and Hu (2016), Fig. 14 shows that the spacing errors and velocity errors can converge to zero and the spacing errors (except error 3 in Fig. 14(a)) are positive, implying that the proposed protocol can prevent rear-end collisions. Fig. 14(b) shows that the
Fig. 10. Switching communication topology.

Fig. 11. The state trajectories with time-invariant leader of switching topology: (a) the position trajectories, (b) velocity trajectories.

Fig. 12. The error trajectories with time-invariant leader of switching topology: (a) the spacing error, (b) the velocity error.
velocity error also converges to zero and reaches the consensus. Hence, from Figs. 13 and 14, vehicles in a string can form a platoon pattern in finite time by the proposed protocol under switching communication topology. Also, negative spacing error and negative velocity can be avoided.

Table 5 summarizes the performance of the proposed control protocol under switching topology. The main findings are as follows. (i) The proposed protocol can facilitate CVs to form a platoon under the switching communication topology; (ii) The convergence time in the case of time-variant leader is smaller than that in the case of time-invariant leader; and (iii) The spacing error and velocity error show different levels of ratio.
5. Conclusions

This paper focuses on the finite-time control of vehicle platoon formation under fixed and switching communication topologies. Unlike the existing control protocols, this study proposes nonlinear consensus-based control strategies for a CVs platoon based on the pinning control theory. In particular, the finite-time pinning control protocols are developed to achieve the consensus. Further, the car-following interactions between CVs are incorporated to avoid negative spacing error and negative velocity to ensure the vehicle behavioral consistency with traffic flow theory. The finite-time stability and consensus of the proposed protocols are analyzed using the LaSalle's invariance principle and Lyapunov technique. Finally, we investigate the proposed control protocols through extensive numerical experiments under different scenarios, including fixed topology with time-invariant leader, fixed topology with time-variant leader, switching topology with time-invariant leader, and switching topology with time-variant leader. The simulation results demonstrate the effectiveness of the proposed control protocols in terms of the convergence time and stability with respect to position and velocity.

This study highlights the design of cooperative control protocol to facilitate vehicles to form a desired platoon in finite time. Also, this paper bridges the gap between control theory and transportation engineering related to vehicle platoon control by that integrating the car-following interactions into the design of control protocol. In addition, this study provides new research directions in the field of platoon control under a travel environment equipped with emerging technologies.

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