Routing and charging locations for electric vehicles for intercity trips

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This study addresses two problems in the context of battery electric vehicles (EVs) for intercity trips: the EV routing and the EV optimal charging station location problems. The paper first shows that the EV routing on the shortest path subject to range feasibility for one origin-destination (O-D) pair, called the shortest walk problem, as well as a stronger version of the problem – the p-stop limited shortest walk problem, can be reduced to solving the shortest path problem on an auxiliary network. Second, the paper addresses optimal charging station location problems in which EVs are range feasible with and without p stops. We formulate the models as mixed integer multi-commodity flow problems on the same auxiliary network without path and relay pattern enumeration. The Benders decomposition is used to propose an exact solution approach. Numerical experiments are conducted using the Indiana state network.

**Keywords:** electric vehicles, routing, charging station locations, Benders decomposition

1. Introduction

Electric vehicles (EVs) have received much attention in the past few years with the promise of achieving reduced fossil fuel dependency, enhanced energy efficiency, and improved environmental sustainability. While EVs can achieve significantly lower operating costs and are more energy efficient (US Department of Energy, 2014a; Weaver, 2014), they have not yet been widely accepted by the traveling public. A primary reason is range anxiety which denotes the driver concerns that the vehicle will run out of battery power before reaching the destination. This is a serious issue, particularly for long or intercity trips (Mock et al., 2010; Yu et al., 2011).

Given the current battery technologies, an EV typically has a range of around 100 miles with a full charge, depending on the motor type, vehicle size and battery pack style (US Department of Energy, 2014b). For example, the 2015 Nissan Leaf and Chevrolet Spark EVs have a driving range of about 80 miles. The Tesla (model X and S) EV with its advanced battery technology has a higher range of around 250-350 miles which is expected to improve further (Tesla Motors, 2014). The charging time depends on the electric power connector, charging schemes, and battery capacity (Botsford and Szczepanek, 2009). It usually takes 6-10 hours for an EV to be fully charged in a slow charging mode (level I and II, see Table 1). For example, it takes the Nissan Leaf equipped with an 80kW motor 8 hours to recharge for a 240V 3.6 kW on-board charger, and 5.5 hours for a 240V 6.6 kW charger (level II). The Tesla Model S equipped with a 225kW motor with 60kWh battery pack takes about 10 hours for a 240V single charger (US Department of Energy, 2014b). Fast charging technology, which requires a level III power connector, can enable a range of over 100 miles with as little as a ten-minute charging time. It supplies direct current of up to 550A, 600V and specifies power level of the charger up to 500kW (Botsford and Szczepanek, 2009). Currently, the fast charging operation is much more costly than level II charging, and thus these facilities are sparsely deployed in the public domain. While fast charging can reduce the charging time, range anxiety concerns cannot be eliminated in the near future due to the fast charging's inability to provide a full charge in a limited amount of time. This is because batteries currently require longer time to charge fully the closer they are to full charge, and hence fast charging typically focuses on obtaining about 80% charge in a certain time duration.

<table>
<thead>
<tr>
<th>Types of charge</th>
<th>Level I</th>
<th>Level II</th>
<th>Level III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power (kW)</td>
<td>1.4</td>
<td>6.6</td>
<td>≥14.4</td>
</tr>
<tr>
<td>Charging voltage</td>
<td>120V, 15A</td>
<td>120V and 240V, 30A</td>
<td>Up to 600V, 550A</td>
</tr>
</tbody>
</table>

Another technology that has been investigated to address the EV range limitation is battery exchange/swapping (Senart et al., 2010). At battery exchange/swapping stations, a pallet of depleted batteries is removed from an EV and replaced with a fully recharged battery (Squatriglia, 2009). Battery exchange can be performed quickly, usually in minutes, but requires identical pallets and batteries. Battery exchange has been practiced in Israel, and is an available option in Denmark (Environmental Defense Fund, 2014). Recently, Tesla Motors announced plans to open battery swapping stations in California (Edelstein, 2014).

The range limitation specifies the maximum distance an EV can travel without stopping to recharge. The actual range of an EV is relevant to several factors including travel speed, terrain, battery state of charge, temperature, etc.
In this paper, the travel range is assumed to be a fixed quantity corresponding to the normally encountered traffic conditions in the rural context of typical intercity trips. The range limitation is an issue not only for EVs, but also for vehicles that use alternative fuels as they need to find alternative fuel refueling facilities to successfully complete the trip (Kang and Recker, 2012; Kuby and Lim, 2005, 2007; Ogden et al., 1999; Wang and Lin, 2009). Hence, alternative fuel vehicles are also subject to a range limitation, and similar to EVs, need to visit the appropriate refueling stations to complete a long or intercity trip.

This paper addresses two problems in the context of EVs for intercity trips. First, it analyzes the EV routing problem subject to the range limitation. EVs are assigned on the shortest path in an auxiliary network, where each arc represents a range feasible walk in the original network. The problem is similar to the EV range-constrained shortest walk problem (Adler et al., 2014; Mirchandani et al., 2014) analyzed for a single origin-destination (O-D) pair, but extended to the case of one origin to multiple destinations. It is shown that the EV shortest walk problem from one origin to multiple destinations, as well as the p-stop limited shortest walk problem (where an EV stops to recharge at most p times) from one origin to multiple destinations, have the same complexity as the corresponding single O-D pair cases. This enables computational efficiency for planners, who seek EV routing feasibility for multiple O-D pairs. Further, we show that the p-stop limited shortest walk problem can be solved efficiently using a dynamic programming, which is simpler than the existing method of creating an expanded network with multiple layers. Note that p, the number of stops, is a parameter herein rather than a decision variable. The associated EV shortest walk problem minimizing p is NP-complete (Ichimori et al., 1983). The p-stop limited problem is a restricted problem. Because en route charging can involve a significant amount of charging time (fast or slow charging), EVs aim to recharge as few times as possible to reduce the en route charging time. Thereby, the p-stop limited problem can be viewed as the EV drivers not accepting routes with more than p stops.

The unrestricted EV routing problem subject to range feasibility is a simplified version of bi-criteria weight constraint shortest path problem (Beasley and Christofides, 1989). In it, there are two arc attributes, arc cost and weight. The goal is to find the least cost path subject to a limit on the accumulation of weight. Similar to the EV range limitation context, replenishment can be set on either nodes or arcs to reset the accumulated weight to zero; the corresponding problems are known as the shortest path problem with relays (Laporte and Pascoal, 2011), or the shortest path problem with replenishment arcs (Smith et al., 2012). In Laporte and Pascoal (2011), nodes can be used as relays to reset the transported weight to zero. They present both a label-setting and a label-correcting algorithm for the problem where all costs and weights are assumed to be non-negative and integer. In Smith et al. (2012), replenishment occurs at arcs. They present a label-correcting method incorporating replenishment, and explore several different label structure and treatment strategies. Further, they use a high-level network, called meta-network, to exploit the inter-replenishment sub-path structure of feasible paths (Smith et al., 2012). Such solutions consist of sequences of weight-feasible paths between replenishment arcs. Their meta-network is similar to the auxiliary network in which we solve the EV routing problem. The methods discussed heretofore (Laporte and Pascoal, 2011, Smith et al. 2012) can solve the general weight constraint shortest path problem, including the unrestricted EV routing problem, efficiently. However, the routing problem in this study has specific characteristics associated with EVs. For example, an EV usually does not accept a route that has more than p stops, and thus we consider the p-stop limited problem in the EV context, which is a significant difference from the shortest path problem with relays (Laporte and Pascoal, 2011) or with replenishment arcs (Smith et al., 2012).

Second, the paper addresses the optimal charging station location problems (CSLPs) subject to range feasibility with and without p stops. The unrestricted problem without p stops is known as the network design problem with relays (NDPR) (Cabral et al., 2007, Konak, 2012) in the operations research literature. The NDPR locates relays at a subset of nodes in the graph such that there exists a path linking the origin and the destination for each commodity for which the length between the origin and the first relay, the last relay and the destination, or any two consecutive relays does not exceed a preset upper bound. The NDPR arises in telecommunications and logistic systems where the payload must be reprocessed at intermediate stations called relays (Cabral et al., 2007). Although conceptually similar to NDPR, a key difference exhibited by our CSLP is that the charging facilities (or relay nodes) in our study consider capacity. In the real-world, because an EV will consume time to charge its battery, the maximum number
of EVs that can be served at a charging facility within a certain period is subject to an upper bound due to service time. This feature makes the EV CSLP a capacitated network design problem while the NDPR is an uncapacitated one. Further, EVs may travel on multiple paths in the CSLP whereas in the NDPR each commodity is routed on a single path which simplifies the problem. The p-stop limited CSLP is even more complex than the NDPR due to the p-stop constraint.

Cabral et al. (2007) studied the NDPR in the context of telecommunication network design. They formulated the problem as a path-based integer programming model and proposed a column generation method for solving the problem. This column generation method is not practical because it requires the enumeration of all possible paths and relay locations a priori. Because each combination of possible path and relay pattern is represented as a binary variable, the model formulation entails a huge number of binary variables. Cabral et al. used column generation to solve a linear programming (LP) relaxation of the problem to generate the column set, and then solved the integer problem with the generated column set. This approach may not generate an optimal solution because the generated column set is restricted and does not guarantee the inclusion of the optimal column set. In the numerical experiments reported in Cabral et al. (2007), the optimality gaps are more than 100% in most instances. Konak (2012) proposed a set covering formulation with a meta-heuristic algorithm for the NDPR. It still enumerates the path set for each commodity, which precludes the efficient design of an exact solution algorithm.

In this study, we propose a different network flow based model formulation. Specifically, the EV CSLPs are formulated as multi-commodity mixed integer linear programs on the same auxiliary network developed for the EV routing problem. Solution algorithms based on Benders decomposition are proposed to determine the exact solution in a real-sized problem with small optimality gap. Due to their conceptual similarity in the problem context, “charging”, “refueling” and “relays” are viewed interchangeably in this study. Hence, the methodologies investigated here for EV charging also apply to the NDPR and alternative fuel vehicle refueling context.

The study contributions are as follows. First, it is shown that the p-stop EV routing can be solved using dynamic programming through one shortest-path computation on an auxiliary network, which is conceptually simpler than the method of Adler et al. (2014) that creates a multi-level expanded network containing \( p + 2 \) copies of the nodes. Second, most prior studies investigate EV routing/equilibria without considering the determination of the optimal charging station locations, and assume them to be pre-determined inputs. By contrast, this study investigates the CSLPs under range feasibility. The CSLP is different from NDPR due to limited capacity at charging facilities and p-stop constraint in EV routing. Third, unlike the existing NDPR studies, this study proposes a different network flow based mixed integer model formulation. As any feasible path on the established auxiliary network respects the range limitation constraint, our link-based formulation (rather than a path-based formulation) avoids enumerating the path set and the relay pattern set. A solution algorithm based on Benders decomposition is designed to determine the exact solution with small optimality gap.

The remainder of the paper is organized as follows. Section 2 reviews relevant literature on the EV routing and charging/refueling location problems. Section 3 addresses the EV routing problem. Section 4 addresses the charging station location problem. Numerical experiments are analyzed in Section 5. Section 6 provides some concluding comments.

2. Literature review of EV studies

Past studies relevant to EVs in the transportation domain can be broadly grouped into three categories: EV energy-efficient routing, EV equilibrium assignment, and EV facility location of charging stations. Sachenbacher et al. (2011) introduced the problem of determining the most energy-efficient path for EVs with recuperation in a graph-theoretical context. Here, a recuperation capability implies that EVs can be equipped with regenerative braking systems to recuperate a part of the kinetic energy lost during the deceleration phase so as to recharge the battery. Artimeier et al. (2010) and Storandt (2012) proposed revised shortest-path algorithms to address the energy-optimal routing. They formulated energy-efficient routing in the presence of rechargeable batteries as a special case of the constrained shortest path problem and presented an adaptation of a shortest path algorithm. Ichimori et al. (1983) and Adler et al. (2014) studied the EV shortest walk problem to determine the route from an origin to a destination.
with minimum detouring; this route may include cycles for detouring to recharge batteries. Ichimori et al. (1983) further showed that the EV shortest walk problem minimizing the number of stops is polynomial-time reducible to the vertex cover problem, and is hence NP-complete. In a dynamic context, Schneider et al. (2014) investigated the EV routing problem with custom time windows. Conrad and Figliozzi (2011) introduced the charging vehicle routing problem, wherein vehicles with limited range must service a predetermined set of customers and may recharge at specific customer locations in order to continue a tour. It can be used for EVs with fast charging capabilities carrying short distance. This model was extended by Jiang and Xie (2014) to account for economic factors. Jiang and Xie (2014) extended their model to the combined mode choice and assignment framework. In this model, the EV class still has no capability to recharge, and travelers select vehicle class and route jointly. This implies that travelers own both EV and ICE vehicles. Their analysis suggests that for the set of routes with length less than the range, all travelers select EVs; and, for the set of routes with length beyond the range, all travelers select ICE vehicles. Traffic equilibrium is then achieved for the two path sets. He et al. (2014) addressed single class network equilibrium by considering EVs with charging capabilities. Their objective is to minimize the traditional user equilibrium term plus the charging time. Most studies discussed heretofore consider network equilibrium with flow-independent energy consumption, that is, EV travel cost is only related to the distance traveled and independent of the traffic congestion. He et al. (2014) consider flow-dependent energy consumption to model EV equilibria as a variational inequality (VI). In general, the aforementioned EV equilibrium models do not consider the optimal deployment of the charging station locations.

Several studies have investigated the facility location problem of charging stations (Baouche et al., 2014; Chen et al., 2013; He et al., 2013; Sellmair and Hamacher, 2014; Xi et al., 2013) and battery swapping stations (Mak et al., 2013). Nie and Ghamami (2013) addressed the selection of the battery size and charging capacity to meet a given level of service such that the total social cost is minimized. Mirchandani et al. (2014) addressed several logistic issues of EVs with battery swap, which include EV routing, charging location, and scheduling problem for electric buses. They showed that planned scheduling of EV fleet of buses to the routes and charging stations can reduce the amount of energy used by the buses. The corresponding EV fleet scheduling problem, however, is NP-hard. Mirchandani et al. also investigated the problem of locating minimum number of swapping stations such that the total detouring cost for a single O-D pair flow is minimized. He et al. (2013) investigated the charging station location problem for plug-in hybrid electric vehicles (PHEVs), wherein there is no range constraint for PHEVs. Mirchandani argued that PHEVs are always charged at trip destinations. So, travelers are assumed to jointly select routes and destinations based upon charging prices at destinations.

Alternative fuel refueling models mainly focus on the refueling station location problem. Hodgson (1990) developed the first flow-capturing location model (FCLM), a flow-based maximal covering problem locates p facilities to maximize the flow volume captured. In FCLM, a flow is captured if at least one facility is located anywhere along the path of flow without considering the driving range limitation. To enhance FCLM, Kuby and Lim (Kuby and Lim, 2007; Lim and Kuby, 2010) studied the flow-refueling location model (FRLM) that is designed to locate p refueling stations so as to refuel the maximum volume of traffic flows traveling on their shortest paths subject to the range limitation. Upchurch et al. (2009) extended the model to account for the limited capacity of the refueling stations. A similar refueling location problem was investigated by Wang and Lin (2009) using the concept of set covering. Wang (2007, 2008) explored the battery swapping location model to optimally meet the needs of electric scooters used by tourists. In a nutshell, in the aforementioned alternative fuel refueling location studies, vehicles travel on the shortest path for each O-D pair without restriction on the maximum number of stops/refuels. Further, the model formulations are path-based. In our study, however, the charging location problem considers the maximum number of stops/refuels, and proposes different model formulations and solution algorithms.
3. Electric vehicle routing

This section discusses the EV routing problem. The EV routing is subject to the range constraint imposed by the battery technology. The battery range of an EV trip is denoted by \( C \), which represents the maximum length an EV can travel without charging. Here, “charging” is used to broadly represent battery recharge, battery exchange, or any other options to obtain a fully charged battery for the EV to continue its intercity travel.

Let \( G = (N, A) \) denote a directed graph, where \( N \) denotes a set of nodes and \( A \) a set of arcs. Let \( n = |N| \) and \( m = |A| \). Let \( s \) denote an origin and \( t \) a destination. EVs are assigned from \( s \) to \( t \) in \( G \). A walk \( r \) is represented by a set of nodes visited in sequence, i.e., \( r = \{n_1, n_2, ..., n_k\} \). If each node in \( r \) is visited only once, the walk is also labeled a path. Each arc in \( A \) is associated with a distance (or length) \( c_{ij} \), where \( (i, j) \in A \). The length of a path (or walk) is the summation of lengths of arcs contained in the path (or walk). Denote by \( V \subseteq N \) a set of charging stations whose geographic locations are known.

A walk \( r := \{n_1, n_2, ..., n_k\} \) is range feasible if the following condition holds: a sub-walk between any two successive nodes in \( \{s, t\} \cup V \) contained in \( r \) has a length at most \( C \). For example, consider a walk \( r := \{s, n_1, n_2, n_3, t\} \) and the charging location \( V := \{n_2\} \). Then, two pairs of successive nodes in \( \{s, n_2, t\} \) are \( \{s, n_2\} \) and \( \{n_2, t\} \). The EV routing problem assigns the EV on a walk from \( s \) to \( t \) such that the walk is range feasible and the length is the shortest. This EV routing problem was studied by Adler et al. (2014), where the problem is referred to as the EV shortest walk problem (EV-SWP). They show that EV-SWP is polynomially solvable. A stronger version of the problem, EV-SWP subject to \( p \) stops (charging stations are visited at most \( p \) times) is also polynomially solvable. In this paper, we extend the EV-SWP and the \( p \)-stop EV-SWP for a single O-D pair in Adler et al. (2014) to the case of multiple destinations. The proposed method leverages the fact that both EV-SWP and \( p \)-stop EV-SWP can be reduced to solving the shortest path problem (SPP) on an auxiliary network, as discussed hereafter.

**Problem 1: Electric vehicle shortest walk problem (EV-SWP) (Adler et al., 2014)**

Find a shortest walk in \( G \) starting at \( s \) and ending at \( t \) such that the walk is range feasible. □

The EV-SWP can be polynomially reduced to a general shortest path problem on an auxiliary graph. Build a complete graph \( G_e \) between the set of nodes \( \{s, t\} \cup V \) as follows. For any pair of nodes \( i, j \in \{s, t\} \cup V \), compute the shortest length from \( i \) to \( j \) in \( G \), denoted by \( d_{ij} \). If \( d_{ij} \leq C \), we create an arc \( (i, j) \in G_e \) such that the cost of the arc, denoted by \( c_{ij} \), equals the shortest length \( d_{ij} \). Therefore, each arc \( (i, j) \in G_e \) represents a shortest path from \( i \) to \( j \) in \( G \). Building \( G_e \) requires solving the all-to-all shortest path problem with \( |V| + 2 \) nodes, and thus the complexity is \( O((|V| + 2)^3) \) (for example, Floyd-Warshall algorithm). The EV-SWP then becomes the problem of finding the shortest path from \( s \) to \( t \) in \( G_e \). The rationale of the problem reduction is that if there is a walk from node \( i \in \{s, t\} \cup V \) to node \( j \in \{s, t\} \cup V \) and \( i \neq j \), then such a walk must follow one shortest path from \( i \) to \( j \) in \( G \).

**Proposition 1:** The EV-SWP can be solved in \( O((|V| + 2)^3 + (|V| + 2) \log_2 (|V| + 2) + (|V| + 2)^2) \).

**Proof.** Building the auxiliary graph \( G_e \) requires \( O((|V| + 2)^3) \) time to solve the all-to-all shortest path problem using the Floyd-Warshall algorithm. \( G_e \) contains at most \( n' = |V| + 2 \) nodes and \( m' = (|V| + 2)^2 \) arcs. Therefore, solving the shortest path problem in \( G_e \) requires \( O(n'^2 \log_2 n' + m') = O((|V| + 2) \log_2 (|V| + 2) + (|V| + 2)^2) \). The overall complexity of solving EW-SWP is \( O((|V| + 2)^3 + (|V| + 2) \log_2 (|V| + 2) + (|V| + 2)^2) \). □

This complexity is slightly worse than \( O(|V|(|n \log_2 n + m|)) \) in Adler et al. (2014). However, the approach illustrated heretofore can be trivially extended to the case with multiple destinations.

**Problem 2:** Electric vehicle shortest walk problem for multiple destinations (EV-SWP-M)

Given an origin \( s \) and a set of destinations \( T(s) \), find a walk \( r \) from \( s \) to each destination \( t \in T(s) \) such that any walk \( r \) from \( s \) to \( t \) is the shortest and range feasible. □
The case of multiple destinations is a little different from the single O-D pair case. We build an auxiliary graph $G_{em}$ as follows. For any pair of nodes $i \in \{s\} \cup V$ and $j \in T(s) \cup V$, $i \neq j$, compute the shortest length from $i$ to $j$ in $G$, denoted by $d_{ij}$. If $d_{ij} \leq C$, we create an arc $(i, j) \in G_{em}$ such that the cost of the arc equals $d_{ij}$. Different from the case of the single O-D pair, the set of destination nodes $T(s)$ cannot be visited as intermediate nodes in the path found in $G_{em}$; otherwise, it may violate the range feasibility. Therefore, in $G_{em}$ there are no outbound arcs originating from $T(s)$, which restricts that a node in $T(s)$ cannot be visited by a path found in $G_{em}$ unless it is the destination. This, however, does not restrict visiting the set of nodes in $T(s)$ in the shortest walk in graph $G$, because each arc $(i, j) \in G_{em}$ represents a shortest path from $i$ to $j$ in $G$ which may visit nodes in $T(s)$ in $G$.

Figure 1 illustrates an example of creating $G_{em}$ for multiple destinations, and its difference from the single O-D pair case. Suppose the driving range is 5. Applying the rule used to create $G_{e}$ for the single O-D pair, that is, for any pair of nodes $i, j \in \{s\} \cup T \cup V, i \neq j$, create an arc $(i, j) \in G_{e}$ if the shortest length from $i$ to $j$ in $G$ (shown in Figure 1(a)) is no more than $C$. The auxiliary graph shown in Figure 1(b) is created, where the shortest path from 1 to 7 is \{1,4,5,7\}. Here, the sub-walk \{4,5,7\} has a length 7, which is more than the range 5, violating the range limitation. Figure 1(c) illustrates the rule of creating the auxiliary graph $G_{em}$; there are no outbound arcs associated with destinations 5 and 7, hence the shortest path from 1 to 7 is \{1,4,6,7\}, which maintains range feasibility. To demonstrate that the destination node 5 could be visited as an intermediate node in the shortest walk in $G$, revise the length of arc (4,5) to 2. Then the auxiliary graph $G_{em}$ is as shown in Figure 1(d), where the shortest path from 1 to 7 is \{1,4,7\}, which represents the shortest walk \{1,2,4,5,7\} in $G$. It can be observed that the destination node 5 is visited by the shortest walk in $G$ although it is not visited in the auxiliary graph $G_{em}$ because the arc (4,7) in $G_{em}$ represents the shortest walk \{4,5,7\} in $G$.

The EV-SWP-M is equivalent to solving a shortest path tree in $G_{em}$. As solving the shortest path has the same effort as solving the shortest path tree, EV-SWP and EV-SWP-M have the same complexity.

**Proposition 2:** The EV-SWP-M can be solved in $O((|V| + |T| + 1)^3 + (|V| + |T| + 1) \log_2(|V| + |T| + 1) + (|V| + |T| + 1)2)$, where $|T|$ represents number of destinations.

**Proof.** Same as Proposition 1. □

![Network G](a.png)

1 is origin, 5 and 7 are destinations, 4 and 6 are charging locations, range is 5
Figure 1. An example of creating the auxiliary graph $G_{em}$ for multiple destinations.

Now, a restricted version of the problem is considered. If the number of charging stops (i.e., nodes visited in $V$) is restricted to a number $p \leq |V|$, the resulting problem is called the $p$-stop limited EV-SWP.

**Problem 3:** $p$-stop limited *electric vehicle shortest walk problem* ($p$-EV-SWP) (Adler et al., 2014)
Finding a shortest walk in $G$ starting at $s$ and ending at $t$ such that the walk is range feasible and charging occurs at most $p$ times.

We show that $p$-EV-SWP is polynomially solvable by solving the shortest path problem in $G_e$ using dynamic programming. The $p$-EV-SWP is equivalent to solving the shortest path in $G_e$ from $s$ to $t$ containing at most $p + 1$ arcs. Consider solving the shortest path problem using dynamic programming. Define the function $D_k(i)$ as the length of a shortest path from $s$ to $i$ containing at most $k$ arcs. Then, the following recurrence can be established:

$$
D_0(s) = 0 \\
D_0(j) = \infty, \forall j \in V \cup \{t\} \\
D_k(j) = \min\{D_{k-1}(j), \min_{(i,j) \in G_e}[D_{k-1}(i) + c_{ij}], \forall j \in V \cup \{t\}, 0 \leq k \leq p + 1\} \tag{1}
$$

Now, by increasing $k$ from 0 to $p + 1$, and each time calculating $D_k(i)$ for all $i \in G_e$ by the recursion, we obtain an $O((p + 1)n'm')$ algorithm and the length of the shortest walk is $\min_{0 \leq k \leq p + 1} D_k(t)$. When $p$ is small, the algorithm runs in $O(n'm')$.

**Proposition 3:** The $p$-EV-SWP can be solved in $O((p + 2)(|V| + 2)^3)$.

**Proof.** Building the auxiliary graph $G_e$ requires $O((|V| + 2)^3)$. Using dynamic programming to solve the shortest path in $G_e$ requires $O(pn'm') \approx O((p + 1)(|V| + 2)^3)$. Hence, the overall complexity is $O((p + 2)(|V| + 2)^3)$. □

Further, the above dynamic program can solve the one-to-all shortest path problem containing at most $k$ arcs. Hence, the result also applies to the $p$-stop limited shortest walk problem for multiple destinations.

**Problem 4:** $p$-stop limited electric vehicle shortest walk problem for multiple destinations ($p$-EV-SWP-M)

Given an origin $s$ and a set of destinations $T(s)$, find a walk $r$ from $s$ to each destination $t \in T(s)$ such that the walk is range feasible and charging occurs at most $p$ times. □

**Proposition 4:** The $p$-EV-SWP-M has complexity of $O((p + 2)(|V| + 2)^3)$.

**Proof.** The $p$-EV-SWP-M can be solved using the same dynamic programming method used to solve $p$-EV-SWP in the auxiliary graph $G_{em}$, and thus has the same complexity. □

This section illustrated that the EV-SWP, EV-SWP-M, $p$-EV-SWP, and $p$-EV-SWP-M are all related to the shortest path (tree) problem on the auxiliary graph $G_e$ (or $G_{em}$). Therefore, they are all polynomially solvable; the complexity is $O((|V| + 2)^3 + (|V| + 2)\log_2(|V| + 2) + (|V| + 2)^2)$ for unlimited stops and $O((p + 2)(|V| + 2)^3)$ for the $p$-stop restricted problem. This indicates that the EV shortest walk problem for multiple destinations is not computationally harder than the EV shortest walk problem for a single O-D pair.

### 4. Charging stations

#### 4.1. The base model

In this section, we consider the location problem of charging stations. Let $V$ denote a set of candidates of charging locations. Given $|V|$ charging station options with construction costs $g_i > 0$ for $i \in V$, the objective is to select a subset of $V$ such that EVs can have range feasible travel from origin to destination for each O-D pair. The base model does not consider the $p$-stop limitation.

**Problem 5:** Electric vehicle charging station location problem (EV-CSLP)

Find a subset set of $V$ with the minimum construction cost such that the EVs between each O-D pair are range feasible. □
Let $S$ be a set of origins and $T$ be a set of destinations to be connected. Let $y_i$ be a binary variable; $y_i = 1$ if the charging station is deployed at location $i$ and $y_i = 0$ otherwise. A set of O-D pairs is denoted by a vector $K := \{(s, t) \mid s \in S, t \in T\}$. Let $k \in K$ denote a commodity associated with the O-D pair $(s, t)$. Let $s(k)$ represent the origin of commodity $k$ and $t(k)$ represent its destination. Construct an auxiliary network $G_e$ as follows. For each node pair $i, j \in S \cup T \cup V, i \neq j$, compute the shortest length, denoted by $d_{ij}$, between $i$ and $j$ in $G$. If $d_{ij} \leq C$, create an arc $(i, j) \in G_e$ and let the arc cost equal $d_{ij}$. Let $x_{ij}^k > 0 | (i, j) \in G_e$ implies that arc $(i, j) \in G_e$ is used in the path from $s(k)$ to $t(k)$ for commodity $k$, and $x_{ij}^k = 0$ otherwise. Note that $x_{ij}^k$ is a real number between 0 and 1 rather than binary so as to simplify the model. Then, the base model of EV-CSLP can be formulated as a multi-commodity flow problem on the auxiliary graph $G_e$ as follows.

**EV-CSLP:**

$$\begin{align*}
\min \sum_{i \in G_e} g_i \cdot y_i \\
\sum_{(i,j) \in G_e} x_{ij}^k - \sum_{(j,i) \in G_e} x_{ji}^k &= \begin{cases} 
1 & \text{if } i = s(k) \\
0 & \text{if } i \neq s(k), t(k) \\
-1 & \text{if } i = t(k)
\end{cases} \quad \forall i \in G_e, k \in K \quad (2)
\\
\sum_{(j,i) \in G_e} x_{ji}^k & \leq y_i \quad \forall i \in V, k \in K \quad (3)
\\
\sum_{(j,i) \in G_e} x_{ji}^k &= 0 \quad \forall i \in S \cup T \setminus \{V \cup s(k) \cup t(k), k \in K \quad (4)
\\
x_{ij}^k & \geq 0, y_i \in \{0,1\} \quad \forall (i,j) \in G_e, i \in G_e, k \in K \quad (5)
\end{align*}$$

Equation (2) seeks to minimize the total construction cost of the charging stations. Equation (3) indicates that one unit of flow is sent for each commodity $k$ (O-D pair $k$) in $G_e$, and if the flow conservation condition (3) holds in $G_e$ it means that the EV routing is range feasible for every commodity $k$ in graph $G$. Note that if commodity $k$ sends flows on a single path, $x_{ij}^k$ equals either 1 or 0 on arc $(i, j)$. If $x_{ij}^k$ is fractional, it implies that commodity $k$ assigns flows on multiple paths; in such a case, each path maintains range feasibility. Equation (4) indicates that the inbound flow at node $i$ for commodity $k$ could be positive if charging station $i$ is constructed, i.e., $y_i = 1$, and is 0 if the station is not constructed, i.e., $y_i = 0$. Note that the formulated model requires $V \cap (S \cup T) = \emptyset$ (i.e., terminal nodes cannot be candidates for charging stations); otherwise, equation (4) will always require terminal nodes $T$ to have charging station constructed. However, this is not a limitation because if a terminal node $i$ is a candidate for charging station, one can split the terminal node into two nodes, $i'$ and $i''$, and create arcs $(i', i'')$ and $(i'', i')$ with zero length. Let $i'' \in V$ and $i'$ be the terminal node. Then, the condition that terminal nodes cannot be candidates for charging stations holds. Equation (5) indicates that any terminal node other than that in $V \cup s(k) \cup t(k)$ is not allowed as a pass-through node for commodity $k$. This is because the model is formulated in the auxiliary graph $G_e$, and if a terminal node other than $V \cup s(k) \cup t(k)$ is allowed as a pass-through node, then range feasibility may be violated as illustrated in the Figure 1 example. Finally, equation (6) specifies the binary variable $y_i$ and the non-negativity of $x_{ij}^k$.

**EV-CSLP** is a mixed-integer linear programming (MILP), which is difficult to solve if the problem size is large. Hence, we develop an exact solution algorithm based on Benders decomposition to solve the EV-CSLP. Observe that if $y$ is given, then the problem becomes a linear program and easy to solve. The idea of Benders decomposition is to solve an easier problem over $x$ by temporarily fixing $y$, and then use the parameterized solution $x(y)$ and the duality property to improve the estimate of the optimal value of $y$. The sub-problem is a linear program after $y$ are fixed, and the duality theorem in linear programming can be used to generate cutting planes to solve for the difficult problem over variable $y$.

The problem (2)-(6) has the following structure.

$$\begin{align*}
\text{min} \quad f^T y \\
\text{s.t.} \quad Dx &= d \quad (7) \\
Ax + By &\leq b \\
x \geq 0, y &\in Y = \{0,1\}
\end{align*}$$

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Benders decomposition partitions the problem (7)-(10) into two problems: (i) a master problem that contains the $y$ variables, and (ii) a sub-problem that contains the $x$ variables. The master problem is as follows.

$$\min f^T y + \theta(y)$$

s.t. $$y \in Y$$

where $\theta(y)$ is defined to be the optimal objective function value of:

$$\min 0$$

s.t. $$Dx = d$$

$$Ax \leq b - By$$

$$x \geq 0$$

Formulation (13)-(16) is a linear program for any given value of $y \in Y$. The objective function (13) indicates that the problem needs to solve for a feasible solution and does not have a specific objective. Instead of solving $\theta(y)$ directly, Benders decomposition solves $\theta(y)$ by solving its dual. This is based on the key observation that the feasible region of the dual formulation does not depend on the value of $y$, which only affects the objective function. Let us associate dual variable $\alpha$ (unrestricted) with Eq. (14) and $\beta$ with Eq. (15). Then the dual problem is:

$$\max \alpha^T d + \beta^T (b - By)$$

s.t. $D^T \alpha + A^T \beta \leq 0$

$\alpha$ unrestricted, $\beta \leq 0$

Let variable $y = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$, matrix $H = \begin{bmatrix} D \\ A \end{bmatrix}$, and vector $q = \begin{bmatrix} d \\ b - By \end{bmatrix}$; then the dual problem has the following structure.

$$\max \quad y^T q$$

s.t. $$H^T y \leq 0$$

Assuming that the feasible region of (20)-(21) is not empty, we can enumerate all extreme points $(y^1_p, ..., y^I_p)$ and extreme directions $(y^1_r, ..., y^J_r)$, where $I$ and $J$ are the number of extreme points and extreme directions, respectively. Using the extreme points and extreme directions, the dual problem can be reformulated as the following equivalent problem.

$$\min \theta$$

s.t. $$(y^j_r)^T q \leq 0 \quad \forall j = 1, ..., J$$

$$(y^i_p)^T q \leq \theta \quad \forall i = 1, ..., I$$

Now, the dual-sub problem has the form (22)-(24), and the original master problem (11)-(12) becomes the following formulation depending on the $\theta$ and $y$ variables.

$$\min f^T y + \theta$$

s.t. $$(y^j_r)^T q \leq 0 \quad \forall j = 1, ..., J$$

$$(y^i_p)^T q \leq \theta \quad \forall i = 1, ..., I$$

Since there is an exponential number of extreme points and extreme directions, enumerating all constraints of type (26) and (27) are not practical. Instead, Benders decomposition starts with a subset of these constraints, and solves a “relaxed master problem”, which obtains a candidate optimal solution $(y^*, \theta^*)$. It then solves the dual sub-problem (20)-(21) to calculate $\theta(y^*)$. If the dual-sub problem has an optimal solution with $\theta(y^*) = \theta^*$, then the algorithm stops with an optimal solution. If the dual-sub problem is unbounded, then a constraint of the type (26), labeled the Benders feasibility cut, is generated and added to the relaxed master problem. If the dual-sub problem has an optimal solution with $\theta(y^*) > \theta^*$, then a constraint of the type (27), labeled the Benders optimality cut, is generated and added to the relaxed master problem. The relaxed master problem is then resolved with the added cut. Since $I$ and $J$ are finite, new feasibility and optimality cuts are generated in each iteration, and the method converges to an optimal solution in a finite number of iterations.
We now use the Benders decomposition framework described heretofore to solve the EV-CSLP (2)-(6). A dual variable $\pi^k_i$ (unrestricted) is associated with equation (3), $\mu^k_i \leq 0$ with equation (4) and $\rho^k_i$ (unrestricted) with equation (5). The relaxed master problem (RMP) is:

$$\begin{align*}
\text{RMP-CSLP:} & \quad \min \sum_{i \in G_e} g_i \cdot y_i + \theta \\
& \quad \text{s.t.} \quad y_i \in \{0,1\} \quad \forall i \in G_e \\
& \quad \text{(3)} - (5) \quad x^k_{ij} \geq 0 \quad \forall (i,j) \in G_e, k \in K
\end{align*}$$

With the temporarily fixed $y$, the sub-problem becomes:

$$\begin{align*}
\text{Sub:} & \quad \min 0 \\
& \quad \text{(3)} - (5) \quad x^k_{ij} \geq 0 \quad \forall (i,j) \in G_e, k \in K
\end{align*}$$

The sub-problem is a linear program. Instead of solving the sub-problem directly, we solve its dual problem, which is:

$$\begin{align*}
\text{Dual-Sub:} & \quad \max \sum_{k \in K} \sum_{i \in s(k)} \pi^k_i - \sum_{k \in K} \sum_{i \in t(k)} \pi^k_i + \sum_{i \in G_e} \mu_i \cdot y_i \\
& \quad \pi^k_i - \pi^k_j \leq 0 \quad \forall (i,j) \in G_e, j \in G_e \setminus \{V \cup S \cup T\} \cup s(k) \cup t(k), k \in K \\
& \quad \pi^k_i - \pi^k_j + \mu_i \leq 0 \quad \forall (i,j) \in G_e, j \in V, k \in K \\
& \quad \pi^k_i - \pi^k_j + \rho^k_i \leq 0 \quad \forall (i,j) \in G_e, j \in \{S \cup T\} \setminus \{V \cup s(k) \cup t(k)\}, k \in K \\
& \quad \mu_i \leq 0 \quad \forall i \in G_e, k \in K
\end{align*}$$

With a temporarily fixed $y$, if the dual of the sub-problem is unbounded, then it generates a Benders feasibility cut:

$$\begin{align*}
\sum_{k \in K} \sum_{i \in s(k)} \pi^k_i - \sum_{k \in K} \sum_{i \in t(k)} \pi^k_i + \sum_{i \in G_e} \mu_i \cdot y_i \leq 0
\end{align*}$$

If the dual of the sub-problem is bounded, but its optimal value is more than the $\theta^*$ solved in the RMP, then it generates a Benders optimality cut:

$$\begin{align*}
\sum_{k \in K} \sum_{i \in s(k)} \pi^k_i - \sum_{k \in K} \sum_{i \in t(k)} \pi^k_i + \sum_{i \in G_e} \mu_i \cdot y_i \leq \theta
\end{align*}$$

The Benders cuts are added into the RMP and it is solved again until the optimal value of the dual-sub problem is equal to the optimal $\theta^*$ solved by the RMP. Then, the procedure stops. The Benders decomposition solution approach is summarized as follows:

**Step 1:** Solve the RMP-CSLP. Let $\theta^*$ be the optimal value of $\theta$, and $y^*$ be the optimal solution of $y$.

**Step 2:** Let $y = y^*$ solve the dual-sub problem. If the dual-sub problem is unbounded, add Benders feasibility cut (37) into the RMP-CSLP. If the dual-sub problem is bounded, and the optimal objective function value is more than $\theta^*$, add Benders optimality cut (38) into the RMP-CSLP.

**Step 3:** If the optimal value of the dual-sub problem equals $\theta^*$, stop; the problem is solved to optimality. Otherwise, go to Step 1.

### 4.2. The enhanced model

In the enhanced model we consider the $p$-stop limitation in the EV routing. In addition, the charging station $i$ has a capacity, denoted by $u_i$, representing the maximum number of EVs that can recharge or swap their batteries within a certain period. This feature differentiates the EV CSLP and NDPR.

**Problem 6:** Electric vehicle combined $p$-stops and charging station location problem (EV-$p$-Stop-CSLP)

Find a subset of $V$ with the minimum construction cost such that the EVs between each O-D pair are range feasible, and stop at most $p$ times. $\square$

Denote by $b^k$ the demand of EVs to be assigned for commodity $k$. Denote by $l^k_i$ the maximum number of arcs that have been visited from origin $s(k)$ to node $i$ for commodity $k$ in $G_e$. $l^k_i$ also represents the maximum number of
charging stations that have been visited by the EVs. Denote an auxiliary variable $e_{ij}^k$ which is 1 if arc $(i,j)$ is used by commodity $k$, and 0 otherwise. The EV-$p$-Stop-CSLP can be formulated in the following link-based formulation.

**EV-$p$-Stop-CSLP:**

\[
\min \sum_{i \in G_e} g_i \cdot y_i
\]  

\[
\pi_i^k = \begin{cases} 
  b^k & \text{if } i = s(k) \\
  0 & \text{if } i \neq s(k), t(k) \\
  -b^k & \text{if } i = t(k)
\end{cases} 
\quad \forall i \in G_e, k \in K
\]  

\[
\mu_i \leq 0 
\]  

\[
\rho_i^k = \sum_{(j,i) \in G_e} x_{ji}^k = 0 
\quad \forall i \in V, (i,j) \in G_e, k \in K
\]  

\[
\alpha_{ij}^k \leq 0 
\]  

\[
\lambda_{ij}^k \leq 0 
\]  

\[
\lambda_{ij}^k \leq 0 
\]  

\[
\lambda_{ij}^k \leq 0 
\]  

\[
\lambda_{ij}^k \leq 0 
\]  

\[
\lambda_{ij}^k \leq 0 
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\lambda_{ij}^k \leq 0 
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\[
\lambda_{ij}^k \leq 0 
\]  

\[
\lambda_{ij}^k \leq 0 
\]  

\[
\lambda_{ij}^k \leq 0 
\]  

\[
\lambda_{ij}^k \leq 0 
\]  

\[
x_{ij}^k \geq 0, y_i \in \{0,1\}, e_{ij}^k \in \{0,1\}, t_i^k \geq 0 
\]  

Equation (39) seeks to minimize the construction cost. Equation (40) assigns demand $b^k$ along paths in $G_e$. Note that due to the limited capacity at charging stations, $b^k$ may be assigned on multiple paths. The flow conservation condition being satisfied in $G_e$ implies that flow is feasible subject to the range limitation for each utilized path. Equation (41) indicates that flow $x_{ij}^k$ can be positive if the charging location $i$ is constructed, and the total commodity flow visiting charging location $i$ is no larger than its capacity $u_i$; $x_{ij}^k$ must be zero if the charging location $i$ is not constructed. Similar to the base model, equation (42) indicates that any node other than $V \cup s(k) \cup t(k)$ cannot represent a pass-through node for commodity $k$ in $G_e$. Equation (43) specifies that if $x_{ij}^k$ is positive, arc $(i,j)$ is utilized for commodity $k$, and hence $e_{ij}^k = 1$. Otherwise, $x_{ij}^k = 0$, which indicates $(i,j)$ is not utilized for commodity $k$. Equation (44) indicates that the inbound arcs for node $j \in V$ can be utilized only if the charging facility is constructed at $j$. Equation (44) links variables $e$ and $y$. Equation (45) specifies that if $e_{ij}^k = 1$, then $t_i^k$ increases by one. If $e_{ij}^k = 0$, then equation (45) becomes redundant. $t_i^k | i \in V$ implies the maximum number of charging stations visited from $s(k)$ to $i$ for commodity $k$. Equation (46) indicates that the maximum number of visited charging locations is bounded by $p$ at each node for each commodity. Equation (47) specifies the initial condition of $t_i^k$ which equals zero at the origin node for each commodity $k$. Equation (48) specifies the binary variables $y_i$ and $e_{ij}^k$, and the non-negativity of $x_{ij}^k$ and $t_i^k$.

The solution algorithm again applies the Benders decomposition. We associate a dual variable $\pi_i^k$ (unrestricted) with equation (40), and $\mu_i \leq 0, \rho_i^k, \alpha_{ij}^k \leq 0, \lambda_{ij}^k \leq 0, \varphi_i^k \leq 0, \omega_i^k$ with equations (41)-(43), (45)-(47), respectively. The relaxed master problem (RMP) is:

**RMP-$p$-CSLP:**

\[
\min \sum_{i \in G_e} g_i \cdot y_i + \theta
\]

\[
y_i \in \{0,1\}, e_{ij}^k \in \{0,1\} 
\quad \forall i \in V, (i,j) \in G_e, k \in K
\]  

With the temporarily fixed $y$ and $e$, the sub-problem becomes:

**p-Sub:**

\[
\min 0
\]

\[
x_{ij}^k \geq 0, t_i^k \geq 0 
\quad \forall (i,j) \in G_e, i \in G_e, k \in K
\]  

The $p$-sub problem is a linear program. For Benders decomposition, we solve the dual problem of $p$-sub as follows.
\[
\text{Dual-p-Sub:} \\
\max \sum_{k \in K} \sum_{i \in \mathcal{I}(k)} b^k \cdot \pi^k_i - \sum_{k \in K} \sum_{i \in \mathcal{I}(k)} b^k \cdot \pi^k_i + \sum_{i \in \mathcal{V}} \mu_i \cdot y_i \cdot u_i + \sum_{k \in K} \sum_{(i,j) \in \mathcal{E}_e} \alpha^k_{ij} \cdot e^k_{ij} \cdot M \\
+ \sum_{k \in K} \sum_{(i,j) \in \mathcal{E}_e} \lambda^k_{ij} \cdot M - \sum_{k \in K} \sum_{(i,j) \in \mathcal{E}_e} (M + 1) \cdot e^k_{ij} \cdot \lambda^k_{ij} + \sum_{k \in K} \sum_{(i,j) \in \mathcal{E}_e} (p + 1) \cdot \varphi^k_i 
\]
\[\text{s.t.} \]
\[\pi^k_i - \pi^k_j + \alpha^k_{ij} \leq 0 \quad \forall (i,j) \in \mathcal{G}_e, j \in \mathcal{G}_e \setminus \{V \cup S \cup T\} \cup \{s(k)\} \cup \{t(k)\}, k \in K \]
\[\pi^k_i - \pi^k_j + \mu_j \cdot u_j + \alpha^k_{ij} \leq 0 \quad \forall (i,j) \in \mathcal{G}_e, j \in \mathcal{V}, k \in K \]
\[\pi^k_i - \pi^k_j + \alpha^k_{ij} \leq 0 \quad \forall (i,j) \in \mathcal{G}_e, j \in \{S \cup T\} \setminus \{V \cup s(k) \cup \{t(k)\}, k \in K \}
\]
\[\alpha^k_{ij} + \varphi^k_i + \sum_{(i,j) \in \mathcal{A}} \lambda^k_{ij} \leq 0 \quad \forall i \in s(k), k \in K \]
\[\varphi^k_i - \sum_{(i,j) \in \mathcal{E}_e} \lambda^k_{ij} \leq 0 \quad \forall i \in s(k), k \in K \]
\[\lambda^k_{ij} \leq 0 \quad \forall i \in s(k), k \in K \]
\[\mu_i \leq 0, \lambda^k_{ij} \leq 0 \quad \forall i \in s(k), k \in K \]

With a temporarily fixed \(y\) and \(e\), if the dual of the sub-problem is unbounded, then it generates a Benders feasibility cut:
\[
\sum_{k \in K} \sum_{i \in \mathcal{I}(k)} b^k \cdot \pi^k_i - \sum_{k \in K} \sum_{i \in \mathcal{I}(k)} b^k \cdot \pi^k_i + \sum_{i \in \mathcal{V}} \mu_i \cdot y_i \cdot u_i + \sum_{k \in K} \sum_{(i,j) \in \mathcal{E}_e} \alpha^k_{ij} \cdot e^k_{ij} \cdot M + \sum_{k \in K} \sum_{(i,j) \in \mathcal{E}_e} \lambda^k_{ij} \cdot M \\
- \sum_{k \in K} \sum_{(i,j) \in \mathcal{E}_e} (M + 1) \cdot e^k_{ij} \cdot \lambda^k_{ij} + \sum_{k \in K} \sum_{(i,j) \in \mathcal{E}_e} (p + 1) \cdot \varphi^k_i \leq 0
\]

If the dual of the sub-problem is bounded, but its optimal value is more than the \(\theta^*\) solved in the RMP, then it generates a Benders optimality cut:
\[
\sum_{k \in K} \sum_{i \in \mathcal{I}(k)} b^k \cdot \pi^k_i - \sum_{k \in K} \sum_{i \in \mathcal{I}(k)} b^k \cdot \pi^k_i + \sum_{i \in \mathcal{V}} \mu_i \cdot y_i \cdot u_i + \sum_{k \in K} \sum_{(i,j) \in \mathcal{E}_e} \alpha^k_{ij} \cdot e^k_{ij} \cdot M + \sum_{k \in K} \sum_{(i,j) \in \mathcal{E}_e} \lambda^k_{ij} \cdot M \\
- \sum_{k \in K} \sum_{(i,j) \in \mathcal{E}_e} (M + 1) \cdot e^k_{ij} \cdot \lambda^k_{ij} + \sum_{k \in K} \sum_{(i,j) \in \mathcal{E}_e} (p + 1) \cdot \varphi^k_i \leq \theta
\]

The Benders cuts are added into the RMP and it is solved again until the optimal value of the dual-sub is equal to the optimal \(\theta^*\) solved by the RMP. Then, the procedure stops. The Benders decomposition solution approach can be stated as follows:

**Step 1:** Solve the RMP-p-CSLP. Let \(\theta^*\) be the optimal value of \(\theta\), and \(\hat{y}, \hat{e}\) be the optimal solutions of \(y\) and \(e\), respectively.

**Step 2:** Let \(y = \hat{y}, e = \hat{e}\) solve the dual-p-sub problem. If the dual-p-sub problem is unbounded, add Benders feasibility cut (62) into the RMP-p-CSLP. If the dual-p-sub problem is bounded, and the optimal objective function value is more than \(\theta^*\), add Benders optimality cut (63) into the RMP-p-CSLP.

**Step 3:** If the optimal value of dual-p-sub equals \(\theta^*\), stop; the problem is solved to optimality. Otherwise, go to Step 1.

**5. Numerical examples**

**5.1. An illustrative hypothetical network example**

We consider the small hypothetical network shown in Figure 2. \(s\) is the origin and \(t\) is the destination. Suppose the battery range is 8, and the charging stations \(V = \{5, 9, 11\}\). The auxiliary graph \(G_e\) is built in Figure 3. It can be
verified that the shortest walk from $s$ to $t$ is the shortest path in $G_e$, i.e., $\{s, 9, t\}$, which has a length 12. This path exhibits the shortest walk in $G$, that is, $\{s, 3, 6, 9, 13, t\}$. It can be verified that both sub-walks $\{s, 3, 6, 9\}$ and $\{9, 13, t\}$ satisfy the range limitation, and thus the walk is range feasible.

There are two 1-stop-limited walks in the network, i.e., $\{s, 5, t\}$ and $\{s, 9, t\}$ in $G_e$, or $\{s, 3, 5, 7, 11, t\}$ and $\{s, 3, 6, 9, 13, t\}$ in $G$. Among them, the latter is the walk with the shortest distance. It is range infeasible to travel from $s$ to $t$ without stops; so, the minimum number of stops is 1. Note that both paths in $G_e$ have two arcs in their paths. While the walk $\{s, 3, 5, 7, 11, t\}$ visits two charging stations, i.e., 5 and 11, the EV does not need to recharge at 11 because the distance from 5 to $t$ is 6 which is less than the feasible range. Hence, the walk $\{s, 3, 5, 7, 11, t\}$ is a 1-stop-limited walk as exhibited in $G_e$.

Next, we solve the charging location problem in this example. Suppose nodes 1 to 13 are candidates for charging stations, and their construction costs are as shown in Table 2. Five destinations are considered in this problem, $\{10, 11, 12, 13, t\}$. We solve the minimum charging location problem such that each O-D pair is range feasible and has 2-limited stops. The auxiliary graph $G_{em}$ is built in Figure 4.

Table 2. Cost of charging stations in the illustrative hypothetical network example.

<table>
<thead>
<tr>
<th>Charging station</th>
<th>Cost</th>
<th>Charging station</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>8</td>
<td>110</td>
</tr>
<tr>
<td>2</td>
<td>50</td>
<td>9</td>
<td>70</td>
</tr>
<tr>
<td>3</td>
<td>80</td>
<td>10</td>
<td>80</td>
</tr>
<tr>
<td>4</td>
<td>30</td>
<td>11</td>
<td>60</td>
</tr>
<tr>
<td>5</td>
<td>200</td>
<td>12</td>
<td>120</td>
</tr>
<tr>
<td>6</td>
<td>150</td>
<td>13</td>
<td>90</td>
</tr>
<tr>
<td>7</td>
<td>50</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 2. Example hypothetical network.
Figure 3. The auxiliary graph $G_e$ for EVs routing with charging locations $V = \{5, 9, 11\}$.

Figure 4. The auxiliary graph $G_e$ with candidates for charging locations 1-13.
5.2. A real-world network example

The Indiana state network, shown in Figure 6, is used as a real-world network to illustrate the proposed methods. The network consists of 17 cities with populations of more than 50,000, and 25 links. These cities are connected by I-65 and U.S. highways, as shown in the figure. There are 9 traffic analysis zones (TAZs) and 81 O-D pairs. The network is of medium size in the current practice of EV planning. As the EV demand in future years is not available, the O-D trips in this study are hypothetical as shown in Table 3, and the total number of trips is 16,400. All cities are candidates for charging locations. The capacities of the charging stations are assumed to be commensurate with the population and the cost. The capacity and cost data are shown in Table 4. The full battery range is assumed to be 150 miles, consistent with the optimal range forecasted in a recent study (Lin, 2014). The auxiliary graph has 34 nodes and 368 arcs; hence, the p-stop CSLP involves $368 \times 81 = 29,808$ binary variables of $e_{ij}^p$, and 17 binary variables of $y_{ij}$. Therefore the total number of binary variables is 29,825. The size of the formulated mixed-integer linear program is very large and the problem is difficult to solve. The Benders decomposition approach discussed in Section 4 is used to determine the optimal charging station locations with the minimum construction cost, under the assumption that each O-D pair is range feasible and EVs stop at most 2 times. The problem is solved using CPLEX 12.1 interfaced with Python 2.7 on a personal computer equipped with a 2.66-GHz Intel(R) Xeon(R) E5640 CPU with 24 GB of memory.

Table 3. O-D demand data.

<table>
<thead>
<tr>
<th>Origin/Destinations</th>
<th>Indianapolis</th>
<th>Fort Wayne</th>
<th>Evansville</th>
<th>South Bend</th>
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<th>Lafayette</th>
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<th>Louisville</th>
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Table 4. Cost and capacities of charging stations in the real-world example.

<table>
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<tr>
<th>City</th>
<th>Cost ($)</th>
<th>Capacity (number of vehicles)</th>
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<tr>
<td>Indianapolis</td>
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<td>5,000</td>
</tr>
<tr>
<td>Fort Wayne</td>
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<tr>
<td>Muncie</td>
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</tr>
<tr>
<td>Fishers</td>
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</tr>
<tr>
<td>Carmel</td>
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<td>Hammond</td>
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</table>

Figure 6. The Indiana state network.
When the capacity constraint is not considered, the results of the optimal charging locations in this case are Kokomo and Bloomington, as indicated in Figure 8(a). Only two charging stations are required to ensure the range feasibility. The total construction cost is $350,000. It can be verified that between each O-D pair EVs are range feasible, and stop to recharge at most twice.

When the capacity constraint is considered, Benders decomposition is applied to solve the $p$-stop problem where $p = 2$. The computational time to solve the problem to optimality is 74 minutes. The optimal charging locations are Kokomo, Fishers, Lafayette and Bloomington. Because of the limited capacity in Kokomo, two more charging stations are needed. Fishers and Lafayette are selected because of the low construction costs. Each O-D pair is range feasible and EVs recharge at most twice. The total construction cost is $1.1 million. In this case, Indianapolis is not selected because while it can provide enough capacity, its construction cost (which includes the land cost) is expensive. Also, Fishers is a suburb of Indianapolis. The results suggest that instead of constructing large charging stations with enough capacity in a single city, it is desirable to deploy the charging stations so that they geographically spread with limited capacities, due to the range feasibility limitation and potential facility costs.
6. Conclusions

In this paper, we investigate the EV routing and optimal deployment of charging locations subject to the range limitation. The EV routing subject to the range feasibility for the case of a single origin to multiple destinations is shown to have the same complexity as that of a single O-D pair. A stronger version of the problem, the $p$-stop problem for the case of single origin to multiple destinations is not harder than the case of the single O-D pair, either. Specifically, the four routing problems, EV shortest walk problem for a single O-D pair, EV shortest walk for a single origin, $p$-stop shortest walk problem for a single O-D pair, and $p$-stop shortest walk problem for a single origin, are all solvable by the shortest path algorithm on an auxiliary network. The difference is that the $p$-stop version of the problem uses a dynamic programming method to solve for the shortest path problem on the auxiliary network.

The problem of optimal charging locations is analyzed. Each O-D pair is subject to range feasibility and EVs are assumed to recharge at most $p$ times. The unrestricted problem (EVs can recharge with unlimited stops) is known as the NDPR. In this paper, the $p$-restricted problem is modeled as a mixed-integer multi-commodity flow problem on the auxiliary network. In contrast to the prior NDPR methodologies, we present a link-based formulation which circumvents feasible path and relay pattern enumeration. The problem involves a significant amount of binary variables and is hence difficult to solve. We develop a novel solution algorithm based on Benders decomposition,
which is designed to determine the exact solution. A numerical example on a real-world network (the Indiana state network) is analyzed, and shows that the Benders decomposition approach can determine the exact solution in a reasonable amount of time. Due to the conceptual similarity, the proposed methodologies apply to the network design problem with relays.

In the Benders decomposition method, the dual sub-problem is a feasibility problem. In our model formulation the method generates feasibility cuts which are significantly more in number than the number of optimality cuts, and thus the convergence could be slow, particularly at the initial stage where the feasibility cuts are not tight. The role of feasibility cuts is to guarantee that the lower bound obtained from the relaxed master problem is valid and the role of the optimality cuts is to restrict the lower bound. Thus, producing more optimality cuts would lead to faster convergence of the Benders decomposition method (Magnanti and Wong, 1981; Saharidis and Ierapetritou, 2010). Exploring algorithmic strategies to accelerate Benders decomposition by producing tight cuts is a future research direction.

The EV routing problem in this study assigns flows on the shortest path; there is no congestion effect and the operating cost is flow-independent. In the real world, traffic involves congestion in the network and the EVs will likely factor travel time in the selection of the shortest route. It leads to a traffic equilibrium model subject to the range feasibility. Investigating the equilibrium model of EVs subject to the range feasibility, combined with the optimal charging location deployment, is another future research direction.

References

Adler, J.D., Mirchandani, P.B., Xue, G., Xia, M., 2014. The electric vehicle shortest-walk problem with battery exchanges. Networks and Spatial Economics, in press.


