Robustness of the Off-line A Priori Stochastic Dynamic Traffic Assignment Solution for On-line Operations

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Abstract – This paper focuses on the off-line stochastic Dynamic Traffic Assignment (DTA) problem as part of a hybrid framework that combines off-line and on-line strategies to solve the on-line DTA problem. The primary concept involves the explicit recognition of stochasticity in O-D demand and/or network supply conditions to determine a robust off-line a priori solution that serves as the initial solution on-line. This strategy ensures that the computationally intensive components, which exploit historical data, are executed off-line while circumventing the need for very accurate on-line O-D demand forecast models. Thereby, efficient on-line reactive strategies could be used to address unfolding traffic conditions. The paper investigates the robustness of the off-line a priori DTA solution under plausible on-line situations. The results illustrate the superiority of the a priori solution over the currently used mean O-D demand based solution for on-line route guidance applications.

Keywords: on-line DTA, stochasticity, a priori optimization, robustness
INTRODUCTION

Advanced Traffic Management and Information Systems (ATMIS) aim at managing traffic in a network through real-time measurement, detection, communication, information provision, and control. A key component of ATMIS is dynamic traffic assignment (DTA), which aims to determine for origin-destination (O-D) trip desires, the optimal time-dependent paths that satisfy some system-wide and/or individual user class objectives. Peeta (1994) provides a comprehensive review of DTA models and their limitations in terms of their suitability for online ATMIS applications. Most existing DTA models are deterministic. They assume the time-dependent O-D trip demands and/or network configuration to be known a priori for the entire time horizon of interest. Such models are not suitable for online ATMIS applications as these assumptions ignore real-time variations in O-D desires and/or network characteristics (for example, due to incidents). A second concern is that most formulations consider a single class of users that are identical in terms of information availability, information supply strategy, and/or user response to supplied information. Mahmassani et al. (1993) formulate a deterministic DTA model called the Multiple User Classes Time-Dependent Traffic Assignment (MUCTDTA) model and propose a simulation-based solution algorithm to address the latter concern. While unrealistic for online applications, the MUCTDTA solution provides the best network performance if the assumed O-D demands and network characteristics are realized, thereby representing a benchmark for online control strategies.

Peeta and Mahmassani (1995a) use a rolling horizon framework to propose an online partial information availability DTA approach. The rolling horizon approach requires highly reliable forecasts of O-D desires and network conditions for the near-term future only, to determine nearly optimal paths in each stage of the horizon. From an operational perspective, these information requirements are more realistic compared to the perfect knowledge assumptions of deterministic models. Since it is stage-based, the rolling horizon approach ensures that unpredicted variations in online traffic conditions can be adequately accounted for in subsequent stages. However, if the actual O-D desires in a stage are significantly different from the forecasts, the solution is sub-optimal. Also, despite being a stage-based approach, it can be computationally intensive online in a centralized architecture, a primary concern for existing DTA models in general networks.

To address the online computational burden, Hawas and Mahmassani (1995) propose a non-cooperative reactive decentralized architecture where spatially distributed controllers make routing decisions independent of each other. Local control rules using artificial intelligence techniques and currently available partial information in the local area (for example, link travel times, concentrations) are used to determine user routes within the control territory. An attractive feature of the approach is the flexibility in defining the territorial size of each controller based on its processing capabilities, thereby circumventing the issue of computational burden. Another significant aspect is that the approach does not require any O-D demand predictions unlike the centralized frameworks. However, since controllers act independently based only on local rules, there is no coordination between them. If the proportion of inter-territory vehicles is large, the solution under this scheme deviates substantially from the optimal solution (Hawas and Mahmassani, 1995). Hawas (1995) extends the above approach to develop a cooperative scheme that enables exchanging non-local information between neighboring controllers.
decentralized architecture is more robust under incident situations because the local rule heuristics are more responsive to current network conditions, there can be substantial degradation in performance (Hawas and Mahmassani, 1997) under non-incident conditions compared to the benchmark centralized deterministic DTA solution. Since the approach is reactive, it does not exploit available historical data, especially on time-dependent O-D demand and incidents.

The above highlights the complexity of the on-line DTA problem. While perfect a priori knowledge (on O-D demand and incidents) and system-wide coordination are highly desirable, they remain the obstacles to on-line operability in general networks because they introduce unrealistic expectations and computational intractability, respectively, based on the current technological progress. This motivates the development of a methodology which is both robust and computationally efficient on-line. We propose a hybrid approach (Peeta and Zhou, 1997; Peeta and Zhou, 1999) that combines off-line and on-line components to solve the on-line DTA problem. The primary concept of this approach involves the development of an a priori solution using a centralized deterministic DTA based mechanism off-line, which serves as a robust initial solution on-line updated if necessary using efficient reactive strategies (Hawas and Mahmassani, 1997; Peeta and Zhou, 1999; Pavlis and Papageorgiou, 1998).

This paper focuses on the off-line component, called the a priori stochastic DTA problem. Our primary objective is to investigate the effectiveness of the a priori optimization strategy (Jaillet, 1988; Bertsimas et al., 1990; Laporte et al., 1994) in achieving this robustness and providing a robust initial on-line solution. To do so, we compare the performance of the a priori and mean O-D demand matrix based solutions under incidents, different O-D demand distributions, and various congestion levels. The time-dependent mean O-D demand matrix solution is an appropriate benchmark because existing DTA models use only the mean O-D demand matrix to determine the optimal path assignment proportions. Also, state-of-the-art dynamic O-D demand estimation and prediction models provide only mean O-D trip desires.

The paper contains eight sections. The next section briefly summarizes the hybrid framework. This is followed by sections on the a priori stochastic DTA problem and the associated solution methodology. Next, the experimental set-up to analyze the robustness of the a priori solution is discussed. This is followed by an analysis of results, a detailed discussion on broader insights on the a priori solution, and concluding comments.

THE HYBRID FRAMEWORK

By explicitly recognizing the inherent stochasticity in O-D demand and/or network supply conditions, O-D desires and/or non-recurrent congestion characteristics can be treated as random variables with known distributions (based on a historical database updated on-line or on a day-to-day basis). The associated stochastic on-line DTA problem aims at assigning time-dependent O-D trip desires paths to their destinations on-line while satisfying system-wide and/or individual user class objectives and explicitly incorporating the randomness in time-dependent O-D desires and/or network supply conditions. It is solved using the hybrid solution framework (Peeta and Zhou, 1999), shown in figure 1, which addresses the computationally intensive procedures off-line to reduce the processing time on-line. A comprehensive discussion of the conceptual and theoretical framework of the stochastic on-line DTA problem and the associated solution algorithm is presented in Peeta and Zhou (1999). It involves the determination of an off-line solution that serves as a “good” initial solution for on-line
application, to which adjustments are made on-line as warranted by unfolding traffic conditions. To achieve computational efficiency online, the solution to the off-line stochastic DTA problem should be robust enough that only minimal on-line adjustments are needed. A robust solution is defined here as one that has minimal deviation, in terms of the expected system-wide travel time, from the corresponding optimal benchmark (deterministic DTA) solution, on average, across the range of likely O-D demand matrices. The hybrid approach circumvents the need for robust on-line O-D demand and incident likelihood prediction models while fully exploiting current and historical data on the associated variables.

**Figure 1 about here**

We model only the time-dependent O-D demand explicitly as random variables while determining a robust off-line *a priori* solution. While incidents can also be treated as random variables in our framework, and associated time-dependent likelihood models exist (Madanat and Liu, 1995), they can be robustly managed on-line using reactive strategies (Hawas and Mahmassani, 1997; Peeta and Zhou, 1999). Also, the marginal effect of the occurrence or non-occurrence of a presumed incident on the degradation of the robustness of the off-line solution is, in general, relatively much higher compared to that of an O-D desire, as individual incidents can impact network performance significantly. This is because an incident generally affects several paths directly whereas the lack or presence of an O-D desire mainly affects one path. Consequently, we perceive incidents as on-line events with on-line management strategies.

We first solve the off-line *a priori* stochastic DTA problem using Monte Carlo simulation based O-D demand realizations and *a priori* optimization techniques, then use its solution, called the *a priori* solution, as the initial solution for the on-line problem. The reactive on-line strategy is responsive to demand and/or supply (incident) conditions being realized on a given day. Implemented within a rolling horizon framework, the on-line strategy consists of two heuristics. The first is a variant of the off-line *a priori* optimization strategy whereby O-D demand realizations that are unlikely to occur on a given day based on unfolding on-line conditions are excluded to update in real-time the off-line *a priori* solution for that stage of the planning horizon. An additional heuristic is incorporated to dynamically re-compute path assignment proportions under incidents using the mean O-D demand based single incident scenarios solved off-line.

**THE A PRIORI STOCHASTIC DTA PROBLEM**

**Definition of variables**

The following notation is used to represent variables in the formulation:

- $i$ = subscript for origin node, $i \in I$
- $j$ = subscript for destination node, $j \in J$
- $n$ = node in the network, $n \in N$
- $a$ = superscript for a link in the network $a \in A$
- $\tau$ = superscript denoting departure time interval, $\tau = 1, \ldots, T$; also called the assignment time interval
- $t$ = superscript denoting current time interval, $t = 1, \ldots, T$; (can be defined at a finer resolution than $\tau$)
- $u$ = subscript for user class, $u \in U$
Problem statement

Consider a traffic network represented by a directed graph G(N, A), where N is the set of nodes and A is the set of directed arcs. A network with multiple origins \(i \in I\) and destinations \(j \in J\) is considered for generality. The analysis period of interest, taken here as the peak period or the planning horizon \(T'\), is discretized into small equal intervals \(\tau = 1, \ldots, T\). Given the time-dependent O-D trip distributions \(R_{ij}^{\tau}(\mu, \sigma)\) for the planning horizon, \(\forall i, j, \text{ and } \tau\), determine the time-dependent proportions of vehicles to be assigned to network paths and corresponding arcs so as to minimize the expected total system travel time while satisfying individual user class objectives. The a priori solution is given by the proportions of the expected number of vehicles \(f_{ijk(u)}^{\tau u}\) that depart along path \(k(u)\) in realization \(\ell\); hence \(K_{ij}^{\tau u} = f_{ijk(u)}^{\tau u}\).

SOLUTION METHODOLOGY

A Monte Carlo simulation (Rubinstein, 1981) based approach is used to solve the a priori stochastic DTA problem. It incorporates the a priori optimization strategy (Jaillet, 1988) to determine the off-line a priori solution. The off-line a priori solution, represented by the proportions of vehicles assigned to paths to the destinations for the horizon of interest, is applied on-line to the actual time-dependent O-D demand realized for a given horizon of interest.

The a priori optimization strategy

A priori optimization is a strategy that can be used to address a specific family of combinatorial optimization problems whose common characteristic is the explicit inclusion of probabilistic elements in the problem definitions (Jaillet, 1988; Bertsimas et al., 1990; Laporte et al., 1994). Such problems have two motivations directly applicable to the stochastic DTA
problem. The first is the desire to define and analyze models for real-world problems in which randomness is not only represented, but a key concern (for example, O-D desires and incidents). The second is the need for investigating the robustness with respect to optimality, of optimal solutions to deterministic problems, when the instances for which these problems have been solved are modified. The analogy to our problem is the robustness of the off-line solution to unfolding O-D demand on-line.

The most direct approach to address such problems is the brute force reoptimization strategy, where each potential instance is optimally solved for. However, this may involve solving exponentially many instances and prove computationally intractable. The a priori optimization strategy addresses the problem in a different way. Rather than exploring every potential instance of the problem, the a priori optimization approach first finds an a priori solution to the original problem, then updates this a priori solution based on the particular instance or variation. In our case, the particular instance is reflected by the actual O-D demand matrix that is realized on-line. The a priori solution should satisfy two characteristics. It should be robust vis-à-vis the optimal solution to the particular instance. In this context, robustness implies a particular solution that performs better than others, on average, over several instances. It should also be robust enough so that the update solution on-line for each instance can be obtained efficiently.

Figure 2 about here

The a priori solution algorithm

Figure 2 illustrates the overall logic and framework to solve the a priori stochastic DTA problem. The known time-dependent O-D demand distributions are used to generate several realizations (or instances) of the time-dependent O-D demand. The DTA problem associated with each realization represents a deterministic DTA problem, as O-D desires are known a priori. A Multiple User Classes Time-Dependent Traffic Assignment (MUCTDTA) algorithm (Peeta, 1994; Mahmassani et al., 1993) is used to solve the deterministic DTA problem. Illustrated within the dotted outline in Figure 3, it addresses the problem faced by a central controller seeking to optimize overall network performance through the provision of real-time routing information to equipped motorists, taking into account different user classes in terms of information availability, information supply strategy, and driver response behavior. In particular, four user classes are considered: (1) equipped drivers who follow prescribed system optimal paths (SO); (2) equipped drivers who follow user optimal routes (UE); (3) equipped drivers who follow a boundedly-rational switching rule in response to descriptive information on prevailing conditions (BR); and (4) non-equipped drivers who follow externally specified paths, which may be projected from historical databases or solved for exogenously (PS). The boundedly rational path switching rule states that users switch from the current path at a decision point (typically a node) if travel time savings based on current traffic conditions on an alternative route exceed a certain threshold. A comprehensive description of the MUCTDTA algorithm is provided in Peeta (1994). The a priori optimization strategy is used to obtain the off-line a priori solution, which consists of the path assignment proportions for planning horizon. Figure 3 depicts the algorithmic logic:

Figure 3 about here
Step 1: Set the realization counter $\ell = 1$. Generate a valid realization (or instance) of the time-dependent O-D demand $r_{ij}^{\tau,\ell}$ from $R_{ij}^{\tau}(\mu, \sigma)$, $\forall i \in I, j \in J$, and $\tau = 1, \ldots, T$. Using the mean user class fractions, obtain $r_{ij}^{\nu,\ell}$, $u = 1, \ldots, 4$.

Step 2: Set the iteration counter $\alpha = 0$ for the MUCTDTA algorithm. Assign the generated O-D desires of the equipped user classes $r_{ij}^{\nu,\alpha,\ell}$, $\forall i \in I, j \in J, u=1,\ldots,3$, and $\tau = 1, \ldots, T$, to a time-dependent initial set of feasible paths $K(In)$. Hence, the initial solution is given by the assignment $r_{ijk}^{\alpha,\ell,0}$, $\forall i \in I, j \in J, \tau = 1, \ldots, T, u = 1, 2, 3$, and $k(u) \in K(In)$. The paths of all unequipped vehicles are known a priori and are part of the initial conditions. Hence, $r_{ijk}^{4,\ell,0}$, $\forall i \in I, j \in J, \tau = 1, \ldots, T$, are known and in the absence of information accessibility are assumed to remain unchanged throughout the iterative search process.

Step 3: The set of path assignments $r_{ijk}^{\alpha,\ell,\nu}$ for the entire duration are simulated using a traffic simulator (in our case DYNASMART, Jayakrishnan et al., 1995; Hawas et al., 1996). The simulation results provide several link level and aggregate performance measures.

Step 4: Compute the link marginal travel times for SO users using the time-dependent experienced link travel times and the number of vehicles on links obtained as post-simulation data from Step 3.

Step 5: Compute the time-dependent least marginal travel time paths and the shortest average travel time paths.

Step 6: Perform an all-or-nothing assignment of all O-D desires $r_{ij}^{\nu,\ell,\alpha}$ for given $i$, $j$, $\tau$, for the SO and UE classes. Assign the SO users to the least marginal travel time paths and the UE users to the shortest average travel time paths. The result is a set of auxiliary path assignments.

Step 7: Update the paths and the number of vehicles assigned to the paths for the SO and UE users. The update is obtained through a convex combination of the current paths assignments $r_{ijk}^{\alpha,\ell,\nu}$ and the auxiliary path assignments, $\forall i, j, \tau$, using the Method of Successive Averages (MSA).

Step 8: Check for convergence. The convergence criteria are based on the difference in the number of vehicles assigned to various paths over successive iterations for the SO and UE classes. If the convergence criteria are satisfied, go to Step 9. Otherwise, set $\alpha = \alpha + 1$ and go to Step 3.

Step 9: If $\ell \geq L$ (L is determined using the Central Limit Theorem), go to Step 10. Otherwise, set $\ell = \ell + 1$, generate a valid realization $r_{ij}^{\tau,\ell}$ and go to Step 2.

Step 10: Compute the a priori stochastic DTA solution as the proportions of the expected number of vehicles of user class $u$ who wish to depart from node $i$ to node $j$ at time $\tau$ assigned to path $k(u)$, $\forall i, j, \tau, u$, and $k(u) \in K_{ij}^{nu}$. $K_{ij}^{nu}$ is obtained by aggregating the corresponding time-dependent paths, $K_{ij}^{\nu,\ell}$, over all $L$ realizations.

Computation of the a priori solution
The computation of the *a priori* solution represents the final step of the *a priori* stochastic DTA algorithm. The *a priori* solution is defined as the vector of proportions of the expected number of vehicles that are assigned to the paths $k(u) \in K_{ij}^\tau$ in the network, $\forall i, j, u,$ and $\tau$. The probability of occurrence $p(\ell)$ of each realization $\ell$ is required to calculate these proportions. Two methods are used to compute $p(\ell)$, resulting in two *a priori* solutions.

**Method I:** If all generated O-D demand realizations are assumed equally likely to occur, then $p(\ell) = 1/L$. As discussed later, this may not be a restrictive assumption given the mechanism to determine feasible O-D demand realizations. Then, the proportion of the expected number of vehicles on path $k(u) \in K_{ij}^\tau$, $\forall i, j, u,$ and $\tau$, over all the generated realizations can be obtained as follows:

$$f_{ijk}^u = \frac{\sum_{l=1}^{L} r_{ijk}^u \pi_{i,j,k(u),\tau}}{\sum_{l=1}^{L} \sum_{k(u)} r_{ijk}^u \pi_{i,j,k(u),\tau}}, \forall i, j, u, k(u), \text{ and } \tau$$

(1)

where $k(u) \in K_{ij}^\tau$ on the right hand side of the equation.

**Method II:** This method to compute $p(\ell)$ is based on deviation of the time-dependent O-D desires generated in a realization from their corresponding means. A description of the method is provided in Peeta and Zhou (1997). It allocates a higher penalty when the deviation from the corresponding mean, $\mu_{ij}^\tau$, is larger. The aggregate weighted deviation is computed for each realization as the sum of the individual weighted deviations. Higher probabilities are assigned to realizations with lower aggregate weighted deviations. The *a priori* solution here is obtained as:

$$f_{ijk}^u = \frac{\sum_{\ell=1}^{L} p(\ell)r_{ijk}^u \pi_{i,j,k(u),\tau}}{\sum_{\ell=1}^{L} \sum_{k(u)} p(\ell)r_{ijk}^u \pi_{i,j,k(u),\tau}}, \forall i, j, u, k(u), \text{ and } \tau$$

(2)

where $k(u) \in K_{ij}^\tau$ on the right hand side of the equation and $k(u) \in K_{ij}^\tau$ on the left hand side.

Once the *a priori* solution is determined, $\{ f_{ijk}^u \}, \forall i, j, k(u), \tau, \text{ and } u \}$, the proportions are used to determine the actual number of vehicles assigned to paths for the O-D demand pattern, $v$, realized on-line as follows:

$$\tilde{r}_{ijk}^u,v = r_{ij}^v f_{ijk}^u, \forall i, j, u, k(u) \in K_{ij}^\tau, \text{ and } \tau$$

(3)

**O-D demand generation**

The time-dependent O-D demand distributions can be obtained from historical O-D demand data, updated over time if necessary. The planning horizon is divided into (equal) assignment intervals. The O-D demand distributions are assumed constant within each assignment interval. We use a discretized version of the probability density function shown in
Figure 4 to obtain the O-D demand distribution for a particular i, j, and τ in our experiments. The actual shape of the distribution depends on the historical data for the associated network. While the state-of-the-art for dynamic O-D demand estimation and prediction focuses only on the mean number of O-D desires \( \mu \), upper and lower bounds can be determined from historical data, as represented in the figure. Also, to the extent that the number of O-D desires is a discrete quantity, Figure 4 can be replaced by a probability mass function without loss of generality. Alternatively, a continuous distribution could be used with a continuous quantity (such as the O-D flow).

The time-dependent O-D demand distributions are used to generate L likely O-D demand realizations. Potentially, L is equal to the total number of possible outcomes for O-D demand realizations based on these O-D demand distributions. However, this precludes computationally feasibility vis-à-vis the \textit{a priori} solution procedure, even off-line, in practice because it a large number for general networks. Also, because of the temporal dependence of the O-D demand for an O-D pair over the planning horizon, the actual number of likely O-D demand patterns is significantly less than the total number of possible outcomes suggested by the O-D demand distributions. The value of L for an actual problem can be determined using the statistical principles of the Central Limit Theorem (CLT). One approach is to compute the sample sizes necessary to bound the error in the average demand for each O-D pair for each assignment interval, and setting L equal to the maximum sample size value obtained across all O-D pairs and assignment intervals. Here, the means and variances of demand for each O-D pair for each assignment interval from the historical database can be used as the best estimates of their true values. The approach may involve computing a large number of sample sizes depending on the number of O-D pairs and the planning horizon length. Another approach is to use CLT and the maximum tolerable error in average travel time to approximately obtain L. To the extent that an O-D demand realization in the \textit{a priori} solution procedure implies an average system travel time, the variation in average system travel time from the historical data can be used in a proxy manner. If the mean and variance of average system travel times from the historical database can be used as the best estimates of their true values, CLT can be employed to obtain the minimum value of L based on the tolerable error in average system travel time. L is set to 43 in our experiments.

Since the network-wide demand for the planning horizon does not vary substantially from day-to-day (unless there are special events), two criteria are used to check whether a generated realization of the time-dependent O-D demand for the planning horizon is realistic. The first criterion compares the generated demand for each O-D pair for the planning horizon with pre-set threshold values calibrated from historical O-D data. If:

\begin{equation}
\eta_{1ij} \leq \sum_{\tau} \hat{r}_{ij}^{\tau} \leq \eta_{2ij}
\end{equation}

where \( \hat{r}_{ij}^{\tau} \) is the number of vehicles in the generated potential realization who wish to depart from i to j in period \( \tau \) for a given O-D demand realization, \( \eta_{1ij} \) and \( \eta_{2ij} \) are calibrated threshold values, then the generated realization satisfies the first criterion and is checked using the second
criterion. Otherwise, it is rejected and re-sampled using $R^T_{ij}(\mu, \sigma)$. The second criterion checks the total number of O-D desires generated by the potential realization. If:

$$\beta_1 \leq \sum_{\tau} \sum_{i} \sum_{j} \tilde{r}_{ij}^\tau \leq \beta_2$$

(5)

where $\beta_1$ and $\beta_2$ are threshold values, then the generated realization is accepted. Otherwise, it is rejected and re-sampling is done.

The bounds in Equations (4) and (5) are used to ensure realism in the sampling process so that a generated O-D demand realization is consistent with actual data. Equations (4) ensure that the variation in the total O-D demand for a given O-D pair for a given day corresponds to the range observed from actual historical data. Similarly, equations (5) ensure that the range of the overall O-D demand over all O-D pairs under the generated realizations is consistent with field observations. In our experiments, due to the lack of actual data, the $\eta$ and $\beta$ values are artificially created. We set $\eta_1$ and $\eta_2$ for an O-D pair as the: (sum of mean time-dependent O-D demand values of that O-D pair over all assignment intervals) ± 2*[square root of (sum of the O-D demand variances for that O-D pair over all assignment intervals)]. Similarly, $\beta_1$ and $\beta_2$ are given by $(1 \pm 0.15)\times$(mean total demand). These settings imply that the O-D demand of a given O-D pair can vary to some degree on different days while the total O-D demand within the network is relatively more stable. They ensure that 96 percent of the randomly generated realizations are accepted for developing the 500 historical O-D demand patterns database for our simulation experiments. It should be emphasized here that in a real-world implementation, the determination of $\eta$ and $\beta$ values is relatively easier compared to the simulated experiments, and is based on the historical O-D demand data. The actual observed ranges of individual O-D pair demands and the total network demand over the planning horizon in the historical database can be directly used to represent the $\eta$ and $\beta$ values.

The two criteria ensure that unrealistic O-D demand realizations are excluded from consideration; thereby only likely O-D scenarios are used in determining the a priori solution. Hence, the assumption of equal likelihood in Method I may not be unduly restrictive. Also, though sampling for a given O-D pair is independent across assignment intervals, (4) implicitly ensures the dependence of O-D desires across assignment intervals, ensuring consistency with reality.

SIMULATION EXPERIMENTS

Experimental set-up

Figure 5 illustrates the network structure. It consists of 50 nodes, 168 links, and 320 O-D pairs. All links are 0.4 kilometers long and have two lanes. Freeway ramps are single-laned. The freeway links have a mean free speed of 88 km/h while all other links have a mean free speed of 48 km/h. The 35 minute horizon of interest is divided into seven 5-minute assignment intervals. An average of 21856 vehicles are generated across realizations over the 35 minutes, representing a medium congestion level for this network. Each user class constitutes approximately 25% of the vehicles generated.

Figure 5 about here
Experimental scenarios

Four scenarios are used to investigate the robustness and effectiveness of the \textit{a priori} solution. For each experimental scenario, a number of time-dependent O-D demand realizations (representing L) are used to obtain the \textit{a priori} solutions. These realizations are selected randomly from the historical database of 500 O-D demand patterns generated from the corresponding time-dependent O-D demand distributions, which are arbitrarily modeled as truncated discretized versions of the normal distribution.

**Scenario 1:** In this scenario, the \textit{a priori} solutions are computed using 43 O-D demand realizations. Its objective is to analyze the effectiveness of the \textit{a priori} solution when assumptions on O-D demand distributions and network conditions are correct. Hence, no incidents are assumed to occur over the horizon of interest. A second set of 43 O-D demand realizations from the historical database, different from the set used to compute the \textit{a priori} solution, is used to evaluate the robustness of the \textit{a priori} solution when the actual O-D demand patterns are different from those used to compute the solution. This is another objective addressed under Scenario 1.

**Scenario 2:** In this scenario, a different set of 43 time-dependent O-D demand realizations are generated and incidents are assumed to occur during the horizon of interest in the network. Two severe incidents, starting at time 5 minutes, are present for 30 and 35 minute durations, respectively, and are assumed to block eighty-five percent of the associated link capacities. The incident locations are highlighted in Figure 5. This scenario aims to analyze the robustness of the \textit{a priori} solution determined in Scenario 1 under incidents. However, the \textit{a priori} solution does not consider the presence of incidents as discussed earlier, since it is an off-line solution.

**Scenario 3:** This scenario addresses situations where the actual O-D demand distributions are different from the assumed ones. If the normal distributions in Scenario 1 represent the assumed distributions, positively and negatively skewed distributions shown in Figures 6 are assumed to represent the actual distributions under two different situations. Both the skewed distributions have identical means and bounds as the corresponding normal distributions. One set of 43 time-dependent O-D demand realizations each are generated from the positively and negatively skewed distributions. The \textit{a priori} solution obtained from the first scenario was employed to study the robustness of the solution. The objective of this scenario is to assess the robustness of the \textit{a priori} solution when the assumed shape of the O-D demand distribution is incorrect.

**Scenario 4:** This scenario evaluates the performance of the \textit{a priori} solution when the actual network congestion levels are different from the level used to obtain the \textit{a priori} solution. Two cases are considered: (i) the mean number of vehicles in the network is 15\% more than the number used to obtain the \textit{a priori} solution, and (ii) the mean number of vehicles in the network is 15\% less than that used to obtain the \textit{a priori} solution. In this scenario, the shape of the time-dependent O-D demand profile is assumed to be conserved. In each case, 43 realizations were generated and the \textit{a priori} solution obtained in Scenario 1 was applied.

Performance Measures
This section discusses the performance measures used to analyze the robustness of the \textit{a priori} solution. The performance measures are:

(i) MUCTDTA Solution (MUCTDTA): The MUCTDTA solution represents the best network performance (in terms of the average system travel time) when the time-dependent O-D demand matrix and network supply conditions are known deterministically. Hence, the MUCTDTA solution for each realization represents the benchmark by which other solutions are compared. A more robust solution approach will deviate lesser from the corresponding MUCTDTA solution, on average, across realizations.

(ii) Method I \textit{A priori} Solution (M1): The Method I solution is represented by the number of vehicles assigned to various paths as per equations (3) where the proportions $f_{ijk}^{\text{M1}}(u)$ are obtained from equations (1). These time-dependent path assignments are simulated using DYNASMART to obtain the average system travel time, which represents the M1 solution.

(iii) Method II \textit{A priori} Solution (M2): This solution is similar to that of Method I, except that the proportions $f_{ijk}^{\text{M2}}(u)$ are obtained using equations (2). Hence, M1 and M2 represent two different \textit{a priori} solutions.

(iv) Smallest MUCTDTA Solution Proportions (M3): Here, the path assignment proportions of the smallest average travel time MUCTDTA solution among the $L$ realizations are used to determine the vehicular path assignments. These are then simulated using DYNASMART. This solution can be used to clearly evaluate the robustness of the \textit{a priori} solution because presumably the smallest average travel time MUCTDTA solution has a “favorable” path assignment proportioning.

(v) Mean O-D Demand Based Optimal Solution (M4): The mean O-D demand matrix is solved using the MUCTDTA solution algorithm to obtain the corresponding path assignment proportions. Based on the state-of-the-art in DTA, these proportions would be used to determine the solution for any O-D demand pattern realized on-line. Hence, the most direct indicator of the utility of the \textit{a priori} solution is the corresponding mean O-D demand matrix based solution.

(vi) Method I + User Optimal Solution (M5): The UE class users seek paths that minimize their individual travel times. To analyze this aspect, the Method I solution for a realization $\ell$ is used as the initial solution and again optimized using the MUCTDTA algorithm where the paths of all user classes except the UE and BR classes are assumed fixed. The Method I BR and UE class proportions serve as the initial solution for those classes in the MUCTDTA algorithm. While the UE paths are updated based on the direction finding mechanism, the BR class vehicle paths are determined by the current network conditions based on boundedly-rational behavior as before. M5 does not represent a realistic generic initial solution as it involves further optimization of the \textit{a priori} solution for each specific realization $\ell$. It is also not a realistic on-line solution as it presumes that the specific O-D demand matrix for realization $\ell$ is deterministically known \textit{a priori}. Its primary value here is in providing useful insights. It represents a benchmark for the best UE class performance under the \textit{a priori} solution and can be compared to the \textit{a priori} solution M1 itself. By contrast, M1 could represent a scenario in which a for-profit private traffic information provider provides paths to individual UE class users.

(vii) Rolling Horizon Solution (RH): The rolling horizon framework is the primary solution approach currently proposed for on-line DTA implementation. In our experiments, the most favorable scenario is assumed for determining the system performance under the RH solution, whereby the predicted O-D demands to obtain the path assignment proportions for the next stage are assumed to be actually realized on-line. This is equivalent to assuming either that the time-
dependent O-D demands are deterministically known \textit{a priori} for the entire planning horizon or that there exists a dynamic O-D demand prediction model that is 100% accurate. Both are unrealistic assumptions in practice. A stage length of 20 minutes and a roll-period of 5 minutes are assumed.

While several solution methods are discussed here, the focus of the experimental analysis is on the comparison of the \textit{a priori} (M1) and the mean O-D demand (M4) solutions. It aims to investigate the following question: between the off-line \textit{a priori} solution and the currently used mean O-D demand solution, which represents a robust initial solution for on-line DTA deployment? In all the above methods, the paths of PS class vehicles are assumed to be fixed and identical to those in the corresponding MUCTDTA solution in each realization. This is consistent with the definition of the PS class defined earlier. Similarly, the initial paths of the BR class users in all methods are identical to the corresponding MUCTDTA solution BR paths, providing a consistent basis for the comparison of the various methods.

**SIMULATION RESULTS**

Table 1 illustrates the average vehicular travel times under Scenario 1 for the optimal solution (MUCTDTA) and the methods being evaluated. They are the average vehicular travel times by user class, further averaged over the 43 realizations. The numbers in the parentheses below represent the associated percentages where the corresponding benchmark MUCTDTA solutions are assumed to be 100%. The average on-line CPU times per stage for implementing various methods are illustrated. In addition, the table shows the standard deviations of vehicular travel times over the 43 realizations. As expected, the MUCTDTA solution over all user classes represents the best possible performance. The average travel times for the SO and UE classes are substantially superior to the corresponding times for the BR and PS classes even under the \textit{a priori} solution, indicating their inherent worth. The results highlight the limitations of the use of the mean O-D demand based solution on-line while emphasizing the robustness of the \textit{a priori} solution with respect to the most likely O-D demand scenarios.

**Table 1 about here**

The average travel times under the \textit{a priori} solutions M1 and M2 deviate the least from the MUCTDTA solution. While the M3 solution performs favorably, it is not as robust as the \textit{a priori} solutions. However, the mean O-D demand based solution performs worse, indicating that the more robust \textit{a priori} solution is better suited as an initial solution for the on-line DTA problem, contrary to current approaches which use the mean O-D demand based solution. The analysis of variance at the 0.05 significance level shows that the M4 solution is significantly different from the MUCTDTA, M1 and M2 solutions, though they are not significantly different from each other. There is no significant difference between the M1 and M2 solutions. This is because the procedure to generate the O-D demand realizations discards unlikely O-D demand patterns. Hence, the equal likelihood assumption of Method I is reasonable as reflected by the results. The solution M5 illustrates that the UE class users can improve upon the initial \textit{a priori} paths provided to them. It represents a benchmark for the UE class performance under the \textit{a priori} solution. However, it is not significantly different from the \textit{a priori} solution. Also, as discussed earlier, M5 is not a realistic on-line solution. The RH solution performs worse than all other solutions despite assumptions of full \textit{a priori} knowledge of O-D demands. This is because
of the truncation of the future horizon in each stage. Its value compared to the mean O-D demand based solution M4 is emphasized under incidents and other special events. Of greater significance vis-a-vis on-line implementation, the computational time for the RH solution is an order of magnitude greater than that for a priori solutions M1 and M2. This represents a key operational advantage for the a priori solution strategy.

Table 2 illustrates the effectiveness of the a priori solution strategy when the O-D demand realizations used to evaluate M1 are different from those used to compute it. The MUCTDTA solution corresponding to the new set of 43 realizations is used as the benchmark. The results mirror the trend in Table 1, and emphasize the robustness of the a priori solution while reiterating its superiority over M4. They validate the notion that the a priori solution ensures robustness, on average, for any set of O-D demand patterns realized on-line that are consistent with the historical distributions.

Table 2 about here

Scenario 2 analyzes the robustness of the a priori solution under incidents. Table 3 shows the performance of the MUCTDTA, M1, and M4 solutions under incidents. The MUCTDTA solution with incidents is benchmarked at 100%. In addition, for the purpose of generating insights, the a priori solution that considers incidents is also obtained. This is done by combining the MUCTDTA solutions under incidents for all O-D demand realizations using equations (1). It should be noted that this is not a realistic on-line solution as it would require a priori knowledge on the occurrence of these incidents. The results highlight the superior performance of the a priori solution over the mean O-D solution, despite incidents not being considered in the a priori solution. This is because the MUCTDTA path assignment proportions under some O-D demand realizations are favorably suited for the arbitrary incident scenario considered. The a priori solution is 50% worse compared to the MUCTDTA solution whereas the mean O-D solution is almost 100% worse. While not a realistic on-line strategy, the a priori solution with incidents indicates that M1 can be further improved if incidents are appropriately incorporated into the a priori solution procedure. While this is possible through the use of incident likelihood prediction models, it is not a desirable approach for two reasons. The multiple combinations of plausible incidents make it unrealistic to solve off-line the prohibitively large number of incident scenarios. Of greater importance, as discussed earlier, incidents are on-line events with potentially large impacts on network performance. Hence, incorporating them in a predictive mode off-line can significantly deteriorate the robustness of the associated solution, as opposed to addressing them on-line as and when they occur. Peeta and Zhou (1999) illustrate the advantages of efficient on-line reactive strategies to manage incidents using this logic. Other recent studies also indicate that on-line reactive strategies perform more robustly under incidents (Hawas and Mahmassani, 1997). Thereby, a robust initial solution can substantially enhance the effectiveness of on-line incident management.

Table 3 about here

Scenario 3 investigates the robustness of the a priori solution where the assumed distribution (normal) is different from the actual one (either positively or negatively skewed). Tables 4 and 5 illustrate the average system performance under the various methods. As expected, the MUCTDTA solutions perform the best under the skewed distributions. The
corresponding skewed \textit{a priori} solutions perform almost as well, emphasizing again the robustness of the \textit{a priori} approach. The performance of the normal \textit{a priori} solution is only slightly worse, indicating its robustness. However, the mean O-D solution is substantially worse, for both the positive and negative skewed distributions. Hence, even when the actual O-D demand distributions are not accurately known, a robust performance can be achieved using the \textit{a priori} strategy by considering reasonably approximate distributions.

\textit{Tables 4 and 5 about here}

Scenario 4 explores the robustness of the \textit{a priori} solution (M1) under network congestion levels that are different from those used to obtain the \textit{a priori} solution. Tables 6 and 7 illustrate the associated results. For higher congestion levels, the robustness of the \textit{a priori} solution is further emphasized. The performance of the mean OD matrix solution is worse than that of the \textit{a priori} solution. At lower congestion levels, the difference between the MUCTDTA, \textit{a priori}, and mean OD solutions is less pronounced. This is because the network is not sufficiently congested to exhibit sensitivity to unpredicted variations in O-D demand.

\textit{Table 6 and 7 about here}

\textbf{A PRIORI DTA SOLUTION: ISSUES AND BROADER INSIGHTS}

The primary focus of this paper is to illustrate that the off-line \textit{a priori} solution is more robust than the mean O-D demand based solution as an initial solution for on-line applications. The results strongly support this conjecture. Deeper insights can be obtained by analyzing the associated solution procedures. The MUCTDTA algorithm, which determines the optimal solution for each individual realization, is based on the enumeration of “good” paths for that O-D demand pattern. Since the \textit{a priori} solution is obtained by appropriately combining (using equations (1)) the path assignments from several realizations, a larger relevant path set is obtained compared to that of a single realization. Hence, the \textit{a priori} solution is more favorably representative of a broader domain of O-D demand patterns, on average, compared to single O-D demand pattern based solutions (such as M3 and M4). It also explains the robustness of the \textit{a priori} solution under randomly generated incidents, compared to the mean O-D solution. It is important to note here that an arbitrary large path set does not imply a better solution whether in the single O-D demand realization or the \textit{a priori} solution context. The \textit{relevance} of the path set and the \textit{proportioning} of vehicular assignments across the path set represent the critical factors. This emphasizes the value of the MUCTDTA and \textit{a priori} solution procedures. While the (local) optimal nature of the MUCTDTA solution has been previously addressed (Peeta, 1994; Peeta and Mahmassani, 1995b), our \textit{a priori} solution procedure suggests that an appropriate combination of the optimal (MUCTDTA) solutions of the L realizations can be robust vis-à-vis on-line applications.

While the shape of the O-D demand distribution influences the \textit{a priori} solution, the associated solution procedure is independent of properties of the distribution. Hence, the \textit{a priori} solution procedure can be generically applied under any observed O-D demand distribution. Also, the lack of well-behaved properties for general DTA problems precludes the guarantee of a
globally optimal solution under the MUCTDTA algorithm. Hence, the \textit{a priori} solution for some realizations may be slightly better than the corresponding MUCTDTA solutions because of the larger path set and favorable proportioning of path assignments.

Several on-line issues arise in the context of the off-line \textit{a priori} solution. A key issue is the mechanism by which the \textit{a priori} solution can be efficiently updated on-line to incorporate unfolding demand and/or supply (incidents) conditions. Peeta and Zhou (1999) propose an efficient heuristic approach that uses available on-line information on O-D demand and incident characteristics. The critical on-line issues relate to the multi-dimensional facets of the user class fractions. These fractions are random variables in reality and can vary over time across the planning horizon for a given day, as well as across days. In our experiments, only their mean values are used in obtaining the \textit{a priori} solution. Hence, substantial deviations from these mean values under any actual O-D demand pattern unfolding on-line can degrade the robustness of the off-line \textit{a priori} solution. This possibility is primarily viewed in our framework as an on-line issue, to be addressed through the on-line update strategy or within the context of the on-line consistency checking component (see Peeta and Bulusu, 1999). Similarly, the users’ response characteristics to supplied information (for SO and UE classes) are assumed to be known while computing the \textit{a priori} solution. However, this issue as well can be addressed on-line through consistency checking. The above issues are common to on-line DTA problems in general.

The four user classes (PS, SO, UE, and BR) are considered for algorithmic completeness, as being \textit{representative} of the various user classes in terms of information availability, information supply strategies, and user response behavior under ATIS/ATMS (Peeta and Mahmassani, 1995a). Their implications for on-line DTA deployment under ATIS/ATMS are in terms of: (i) the interpretation of these classes, and (ii) the determination of their actual fractions (or even their mean fractions) in an on-line scenario. These issues are also relevant for the consistent conceptual interpretation and realism of the \textit{a priori} solution for on-line operations.

The path assignment proportions for the SO and UE classes under the \textit{a priori} solution do not necessarily satisfy the SO and UE principles, respectively. This is because equations (1) do not explicitly address individual user class constraints, though the MUCTDTA solutions for the individual realizations satisfy them. To address this issue, a consistent interpretation is provided for the SO and UE classes which has critical implications for the information dissemination system architecture under ATIS/ATMS. This interpretation, discussed hereafter, is consistent from the perspective of the operators (traffic control center, private for-profit route guidance information provider) and the equipped users (SO, UE, and BR class users) in the ATIS market. Upon request, the individual SO class users are provided paths by the traffic controller (system operator) based on the \textit{a priori} solution SO user class proportions. To the extent that these paths satisfy the controller’s objectives, this information may be provided cost-free unless cost penalties are needed to limit the SO class percentage (demand). Similarly, upon request, private information providers sell individual users paths that almost satisfy UE objectives (shortest equilibrated travel time paths) using the \textit{a priori} solution UE class path assignment proportions or the M5 solution proportions. These providers may buy historical information on the traffic system from the controller or use public domain information to determine the UE path proportions. Such a framework is consistent with market economics, equitable, and reinforced by the experimental results which highlight the superior performance of the UE and SO classes (in that order) even under the \textit{a priori} solution approach.

The mean fractions of SO and UE class users can be projected from the time-dependent frequency of requests to the controller and private providers, respectively. The average fraction
of BR class users can be projected through market surveys that determine the number of users equipped with in-vehicle navigation systems in an ATIS market. These can be used to estimate the mean fraction of PS class users. The above discussion represents a plausible scenario for an ATIS market in which user class fractions can be estimated and the \textit{a priori} solution can be consistently interpreted.

**CONCLUDING COMMENTS**

A hybrid framework that combines off-line and on-line strategies was proposed to address the on-line DTA problem by explicitly considering the stochastic elements. It involves the determination of an off-line solution that serves as a robust initial on-line solution, thereby enabling computationally efficient updates on-line (Peeta and Zhou, 1999) in response to unfolding traffic conditions. It also envisages addressing the computationally intensive components off-line, thereby ensuring that the on-line heuristic is computationally feasible. In this context, an \textit{a priori} stochastic DTA solution was developed through a computationally intensive off-line mechanism that is realistic and exploits available historical data on O-D desires. It also circumvents the need for good on-line O-D demand predictions. Four scenarios were evaluated to assess the robustness of the off-line \textit{a priori} solution. The results indicate the superiority of the \textit{a priori} solution over the mean O-D demand based DTA solution for on-line ATMS/ATIS operations. The robustness is conserved when the actual O-D demand distributions are different from the assumed ones, and strongly emphasized under incidents and across congestion levels.

Incidents are addressed as on-line events. The corresponding solution methodology (Peeta and Zhou, 1999) is consistent with the hybrid framework discussed here. It consists of a computationally intensive off-line component that exploits historical data on incidents and an on-line dynamic update strategy. Hence, the fully integrated hybrid framework addresses incidents as well as randomness in O-D demands. Current efforts are focused on evaluating various heuristics for the on-line dynamic update strategy under incidents. In addition, the robustness of the off-line \textit{a priori} solution under moderate variations to the user class fractions is being evaluated.

**ACKNOWLEDGMENTS**

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**REFERENCES**


## Table 1  Comparison of average travel times (minutes)

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Table 2: Comparison of average travel time (minutes) for another set of O-D demand realizations.
Table 3  Comparison of average travel times under incidents (minutes)

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Table 4  Comparison of average travel times for positive skewed distributions (minutes)

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Table 6  Comparison of average travel times under higher congestion levels (minutes)

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<td>(100)</td>
<td>(104.2)</td>
<td>(115.0)</td>
</tr>
<tr>
<td>UE class</td>
<td>5.15</td>
<td>5.38</td>
<td>6.07</td>
</tr>
<tr>
<td></td>
<td>(100)</td>
<td>(104.5)</td>
<td>(117.9)</td>
</tr>
<tr>
<td>BR class</td>
<td>7.83</td>
<td>8.07</td>
<td>8.79</td>
</tr>
<tr>
<td></td>
<td>(100)</td>
<td>(103.1)</td>
<td>(112.3)</td>
</tr>
</tbody>
</table>
Table 7  Comparison of average travel times under lower congestion levels (minutes)

<table>
<thead>
<tr>
<th></th>
<th>MUCTDTA</th>
<th>M1</th>
<th>M4</th>
</tr>
</thead>
<tbody>
<tr>
<td>All classes</td>
<td>3.66 (100)</td>
<td>3.73 (101.9)</td>
<td>3.76 (102.7)</td>
</tr>
<tr>
<td>PS class</td>
<td>4.56 (100)</td>
<td>4.54 (99.6)</td>
<td>4.58 (100.4)</td>
</tr>
<tr>
<td>SO class</td>
<td>3.15 (100)</td>
<td>3.39 (107.6)</td>
<td>3.40 (107.9)</td>
</tr>
<tr>
<td>UE class</td>
<td>2.97 (100)</td>
<td>3.09 (104.0)</td>
<td>3.12 (105.1)</td>
</tr>
<tr>
<td>BR class</td>
<td>3.95 (100)</td>
<td>3.91 (99.0)</td>
<td>3.95 (100)</td>
</tr>
</tbody>
</table>
Fig. 1 The Hybrid Solution Framework for the On-line DTA Problem
Fig. 2. Solution framework for the off-line \textit{a priori} stochastic DTA problem.

\begin{itemize}
  \item \textbf{Network and system data}
  \item \textbf{Realization counter }\(\ell = 1\)
  \item \textbf{Historical O-D demand distributions}

  \begin{itemize}
    \item \textbf{Generate a realization of O-D demands for the entire planning horizon}
    \item \textbf{Multiple User Classes Time-dependent Traffic Assignment Algorithm (MU TDTA)}
    \item \(r^\tau_{ijk(\ell)} \forall i, j, k, u, \tau\)
    \item \(\ell < L?\)
    \item \textbf{Yes}
    \item \(\ell = \ell + 1\)
    \item \textbf{No}
  \end{itemize}

  \begin{itemize}
    \item Use \(r^\tau_{ijk(\ell)}\) for \(\ell = 1, \ldots, L\), to compute the \textit{a priori} solution as the proportion of the expected number of vehicles to be assigned to each path, \(\forall i, j, k, u, \text{ and } \tau\)
  \end{itemize}
\end{itemize}
Fig. 3. *A priori* stochastic DTA solution algorithm

- Set realization counter $\ell = 1$
- Historical time-dependent O-D distributions $R_{ij}^\tau(\mu, \sigma)$
- Generate $r_{ij}^{\tau,l}$ from $R_{ij}^\tau(\mu, \sigma)$, set iteration counter $\alpha = 0$

**DYNASMART**

- Time-dependent link average travel times
- Time-dependent link marginal travel times
- Time-dependent shortest path algorithm
- Time-dependent least cost path algorithm
- All-or-nothing assignment of O-D desires to auxiliary paths
- Update of paths and number of vehicles assigned to those paths (UE) using MSA

**Paths for BR class users**

- Time-dependent link average travel times
- Time-dependent link marginal travel times
- Time-dependent shortest path algorithm
- Time-dependent least cost path algorithm
- All-or-nothing assignment of O-D desires to auxiliary paths
- Update of paths and number of vehicles assigned to those paths (SO) using MSA

**Paths for PS class users**

- Paths of vehicles of equipped user classes
- Convergence criteria satisfied?
- Update iteration counter $\alpha = \alpha + 1$
- Update realization counter $\ell = \ell + 1$

Using $r_{ijk(u)}^{nl}$ for $\ell = 1, \ldots, L$, compute the *a priori* solution as the fraction of the expected number of vehicles to be assigned to each path $\forall i, j, k, u,$ and $\tau$
Fig. 4. Time-dependent distribution for O-D desires for a given i, j, and τ
Fig. 5. Network structure
(a) Positive skewed distribution
(b) Negative skewed distribution

Fig. 6. Skewed O-D demand distributions