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On a Class of Low-Reflection Transmission-Line Quasi-Gaussian Low-Pass Filters and Their Lumped-Element Approximations

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Abstract — **Gaussian-like filters are frequently used in digital signal transmission. Usually, these filters are made of lumped inductors and capacitors. In the stopband, these filters exhibit a high reflection, which can create unwanted signal interference. To prevent that, a new, low-reflection ladder network is introduced that consist of resistors, inductors, and capacitors. The network models fictitious transmission lines with Gaussian-like amplitude characteristics. Starting from the analysis of this network, a procedure is developed for synthesis of a new class of lumped-element RLC filters. These filters have transmission coefficients similar to the classical Bessel filters. In contrast to the Bessel filters, the new filters exhibit a low reflection both in the stopband and passband, they have a small span of element parameters, and they are easy for manufacturing and tuning.**

Indexing terms — **Linear phase filters, low-pass filters, distributed parameter filters, impedance matching.**

I. INTRODUCTION

In digital signal transmission, Gaussian-like frequency-domain transfer functions are usually desirable because they do not yield overshoots and ringing in the time domain. For practical filter design, the leading representatives for this kind of low-pass filters are the Bessel (Bessel-Thompson) filters, which have a maximally flat group delay [1]. Lumped element realizations of such filters and their implementations in the microwave range have been well developed and known, e.g., [2], [3].

These filters are, theoretically, lossless. For a low-pass filter of this kind, at low frequencies, the magnitude of the transfer function ($|s_{21}|=|s_{12}|$) is close to 1 (0 dB) and the magnitude of the reflection coefficient ($|s_{11}|=|s_{22}|$) is close to 0. The filters have a mild transition to the stopband, where the magnitude of the transfer function becomes close to 0, but the magnitude of the reflection coefficient becomes close to 1. Hence, the classical filters exhibit a high reflection (except near the zero frequency), which is undesirable in many digital-circuit applications as it can create signal interference.

A filter that exhibits little or no reflection both in the passband and in the stopband can be named a matched filter. However, the term "matched filter" has a different meaning in communications. Hence, we shall refer to such filters as low-reflection filters.

To achieve a good matching, the network must be lossy[1]. Matching at one port of the filter can be achieved by making a diplexer with a dummy complementary filter or using hybrids [4]. This procedure can yield a good selectivity, but has the disadvantage of increased complexity. To make a symmetrical low-reflection filter, the complexity is even further increased.

Losses in filter elements have been reported in [5] to improve matching in the passband. However, this concept has not been further elaborated, and, as far as the authors could search, there is no published theoretical

[1] For a lossless network, $|s_{11}|^2+|s_{21}|^2=|s_{22}|^2+|s_{12}|^2=1$, so matching cannot be achieved in the stopband.

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work that covers synthesis of low-reflection filters. Recently, Gaussian-like filters with lossy elements have been reported (e.g., [6]) where the magnitude of the reflection coefficient is kept reasonably low both in the passband and in the stopband, at both filter ports, but details of the design have not been published.

The present paper has several goals. Section II starts from the theory of the classical lossy transmission lines and reveals a derivation of a new class of distributed-parameter (transmission-line) low-reflection filters that have Gaussian-like amplitude characteristics. Section III presents a design procedure for lumped-element networks that approximate transmission-line Gaussian filters. A particular case is emphasized that yields a flat group delay. Section IV presents experimental results for a microstrip implementation and compares them with theoretical data.

II. Lossy Transmission Lines with Gaussian Characteristic

We start from the classical lossy transmission line, whose primary parameters are the per-unit-length inductance (l), capacitance (c), resistance (r), and conductance (g). The telegrapher's equations for this line are derived starting from the lumped-element model shown in Figure 1, where the elements of a half-cell[2] are

$$L = l\Delta x\ , \tag{1}$$

$$C = c\Delta x\ , \tag{2}$$

$$R = r\Delta x\ , \tag{3}$$

$$G = g\Delta x\ , \tag{4}$$

and Δx is the length of the line approximated by the half-cell. Note that the inductor L and the resistor R (which models conductor losses) are connected in series, whereas the capacitor C and the resistor G (which models dielectric losses) are connected in parallel. By letting $\Delta x \to 0$, a network with distributed parameters is obtained, and the Kirchhoff voltage and current laws are substituted by differential (telegraphers') equations.

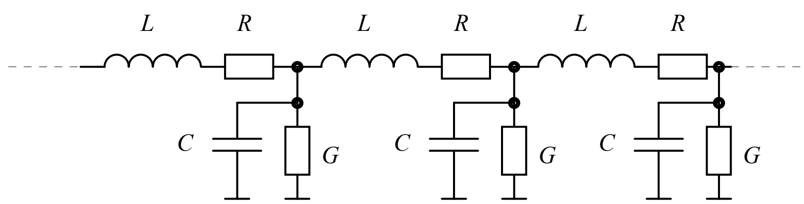

Fig. 1. Lumped-element ladder-network approximation of a lossy transmission line.

We assume the parameters r, l, c, and g to be frequency independent and to satisfy Heavyside's condition,

$$\frac{r}{l} = \frac{g}{c}\ . \tag{5}$$

The per-unit length impedance and admittance of the line are $z = r + \mathrm{j}\omega l$ and $y = g + \mathrm{j}\omega c$, respectively, where ω is the angular frequency. The characteristic impedance of the line is

$$Z_\mathrm{c} = \sqrt{\frac{z}{y}}\ . \tag{6}$$

Under the condition (5), equation (6) yields $Z_\mathrm{c} = \sqrt{\frac{l}{c}} = \sqrt{\frac{r}{g}}$, so the characteristic impedance is purely resistive and frequency-invariant. The propagation coefficient is

$$\gamma = \sqrt{zy} = \alpha + \mathrm{j}\beta\ . \tag{7}$$

[2] A half-cell consists of one series branch and one adjacent shunt branch.

Under the condition (5), the attenuation coefficient, $\alpha = \sqrt{rg}$, and the wave velocity, $v = \frac{\omega}{\beta} = \frac{1}{\sqrt{lc}}$, are also independent of frequency. The phase coefficient, β, is a linear function of frequency.

If we consider this line as a two-port network and if both port nominal impedances are equal to Z_c, the network is perfectly matched, i.e., the intrinsic reflection coefficients are $s_{11} = s_{22} = 0$. If the line length is D, the transfer function of the network is

$$s_{12} = s_{21} = \exp(-\gamma D) . \tag{8}$$

The magnitude of the transfer function is $|s_{12}|=|s_{21}|= \exp(-\alpha D)$. It can be expressed in decibels as

$$|s_{12}|_{\mathrm{dB}}=|s_{21}|_{\mathrm{dB}}= -8.686\ \alpha D\ \mathrm{dB} = -a , \tag{9}$$

where a is the attenuation (insertion loss) of the network. If (5) is fulfilled, then the attenuation is $a = 8.686\, D\sqrt{rg}$ dB and it is independent of the frequency. Hence, the lossy transmission line represents a broadband (allpass) attenuator, with a frequency-invariant attenuation. The phase (in radians) of the transfer function is

$$\arg(s_{12}) = \arg(s_{21}) = \varphi = -\beta D . \tag{10}$$

If (5) is fulfilled, the phase is $\varphi = -\omega\sqrt{lc}D$ and it is a linear function of frequency. The group delay, $\tau = -\frac{\mathrm{d}\varphi}{\mathrm{d}\omega}$, is constant, and the network is perfectly dispersionless.

A lumped-element approximation to this transmission line can be made in the form of a ladder network. This is the same network as shown in Figure 1, with a finite number of half-cells. The lumped-element network is a good approximation of the transmission line at lower frequencies. As the frequency increases, the lumped-element network behaves like a low-pass filter with the cutoff angular frequency

$$\omega_c = \frac{2}{\sqrt{LC}} . \tag{11}$$

Hence, for a given transmission line, the approximation will hold up to higher frequencies if the lumped-element network contains more half-cells, i.e., if Δx is smaller.

By a simple rearrangement of elements of the network of Figure 1, we obtain a low-pass filter, shown in Figure 2, whose transfer characteristic is similar to the ideal Gaussian filter, and, hence, akin to the Bessel filters and finite-order Gaussian filters.

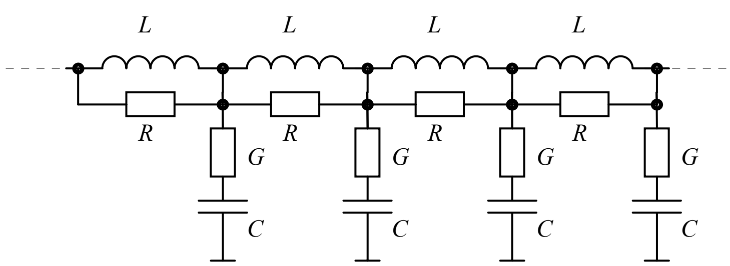

Fig. 2. Lumped-element ladder-network approximation of the complete Gaussian transmission line.

We can formally apply the same analysis as for the classical transmission line. The per-unit-length impedance of the line is $z = \frac{\mathrm{j}\omega rl}{r + \mathrm{j}\omega l}$ and the per-unit-length admittance is $y = \frac{\mathrm{j}\omega gc}{g + \mathrm{j}\omega c}$. Assuming (5) to be fulfilled, the characteristic impedance of the line is again independent of frequency,

$$Z_c = \sqrt{\frac{l}{c}} = \sqrt{\frac{r}{g}} . \tag{12}$$

We introduce the break-point angular frequency,

$$\omega_b = \frac{r}{l} = \frac{g}{c}. \tag{13}$$

Below this frequency, the real parts in the denominators of the per-unit-length impedance and admittance dominate. Beyond this frequency, the imaginary parts dominate. The per-unit length impedance and admittance are now $z = \frac{j\frac{\omega}{\omega_b} r}{1 + j\frac{\omega}{\omega_b}}$ and $y = \frac{j\frac{\omega}{\omega_b} g}{1 + j\frac{\omega}{\omega_b}}$, respectively. The propagation coefficient can be written in the form

$$\gamma = \frac{j\frac{\omega}{\omega_b}\sqrt{rg}}{1 + j\frac{\omega}{\omega_b}} = \frac{j\omega\sqrt{lc}}{1 + j\frac{\omega}{\omega_b}} \approx \omega_b\sqrt{lc}\left(\left(\frac{\omega}{\omega_b}\right)^2 - \left(\frac{\omega}{\omega_b}\right)^4\right) + j\omega_b\sqrt{lc}\left(\frac{\omega}{\omega_b} - \left(\frac{\omega}{\omega_b}\right)^3\right) \tag{14}$$

because $rg = \omega_b{}^2 lc$. The approximation is valid when $\omega << \omega_b$. The real part of the propagation coefficient, i.e., the attenuation coefficient of the line, is

$$\alpha = \frac{\left(\frac{\omega}{\omega_b}\right)^2 \sqrt{rg}}{1 + \left(\frac{\omega}{\omega_b}\right)^2} \approx \omega_b\sqrt{lc}\left(\left(\frac{\omega}{\omega_b}\right)^2 - \left(\frac{\omega}{\omega_b}\right)^4\right). \tag{15}$$

When $\omega << \omega_b$, the attenuation of the line follows the Gaussian form, i.e., it is proportional to frequency squared. When the frequency approaches ω_b, the amplitude characteristic flattens out, tending to a constant (i.e., $\alpha \to \sqrt{rg}$). The imaginary part of the propagation coefficient, i.e., the phase coefficient, is

$$\beta = \frac{\omega\sqrt{lc}}{1 + \left(\frac{\omega}{\omega_b}\right)^2} \approx \omega_b\sqrt{lc}\left(\frac{\omega}{\omega_b} - \left(\frac{\omega}{\omega_b}\right)^3\right). \tag{16}$$

When $\omega << \omega_b$, the phase characteristic is linear, $\beta \approx \omega\sqrt{lc}$, the phase velocity is approximately constant and equal to $1/\sqrt{lc}$, and the group delay is flat.

The amplitude and phase characteristics of the line show that the line can be used as a low-pass filter that approximates Gaussian and Bessel filters.

The transmission line derived from Figure 2 (when $\Delta x \to 0$) will be referred to as the complete Gaussian line, because losses are introduced both in series and parallel branches.

We introduce the incomplete Gaussian line, which is derived from the lumped-element network shown in Figure 3. In this figure, resistors exist only in shunt branches, so that the total number of resistors is halved when compared with Figure 2. (By duality, resistors can be located only in series branches.)

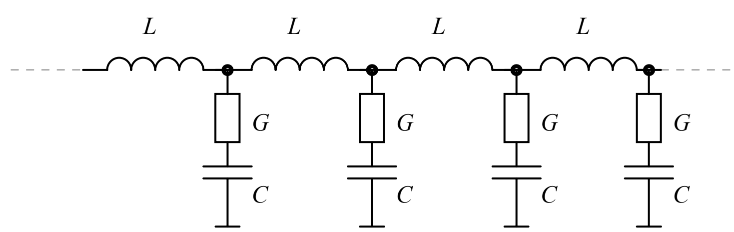

Fig. 3. Lumped-element ladder-network approximation of the incomplete Gaussian transmission line.

The per-unit-length impedance and admittance of the incomplete Gaussian line are $z = \mathrm{j}\omega l$ and $y = \dfrac{\mathrm{j}\omega g c}{g + \mathrm{j}\omega c} = \dfrac{\mathrm{j}\dfrac{\omega}{\omega_\mathrm{b}} g}{1 + \mathrm{j}\dfrac{\omega}{\omega_\mathrm{b}}}$, respectively, where, following (13), $\omega_\mathrm{b} = \dfrac{g}{c}$. The characteristic impedance of the incomplete Gaussian line depends on frequency,

$$Z_\mathrm{c} = \sqrt{\frac{l}{c}}\sqrt{\frac{g + \mathrm{j}\omega c}{g}} = \sqrt{\frac{l}{c}}\sqrt{1 + \mathrm{j}\frac{\omega}{\omega_\mathrm{b}}} \approx \sqrt{\frac{l}{c}}\left(1 + \mathrm{j}\frac{\omega}{2\omega_\mathrm{b}}\right), \tag{17}$$

where the approximate expression is valid at lower frequencies, when $\omega << \omega_\mathrm{b}$. The propagation coefficient is

$$\gamma = \mathrm{j}\omega\sqrt{lc}\sqrt{\frac{1}{1 + \mathrm{j}\dfrac{\omega}{\omega_\mathrm{b}}}} \approx \frac{1}{2}\omega_\mathrm{b}\sqrt{lc}\left(\left(\frac{\omega}{\omega_\mathrm{b}}\right)^2 - \frac{5}{8}\left(\frac{\omega}{\omega_\mathrm{b}}\right)^4\right) + \mathrm{j}\omega_\mathrm{b}\sqrt{lc}\left(\frac{\omega}{\omega_\mathrm{b}} - \frac{3}{8}\left(\frac{\omega}{\omega_\mathrm{b}}\right)^3\right). \tag{18}$$

We compare equations (14) and (18). The real part of the propagation coefficient has in both cases a Gaussian behavior at lower frequencies. The deviation from the ideal characteristic as $\omega \to \omega_\mathrm{b}$ is somewhat larger for (14) than for (18), because the coefficients with the $\left(\dfrac{\omega}{\omega_\mathrm{b}}\right)^4$ term are 1 and 0.625, respectively. The phase coefficient in both (14) and (18) is linear at lower frequencies, and it starts deviating when $\omega \to \omega_\mathrm{b}$. The deviation from the linear phase in (14) is larger than in (18), because the coefficients with the $\left(\dfrac{\omega}{\omega_\mathrm{b}}\right)^3$ term are 1 and 0.375, respectively.

Figure 4 shows magnitudes of transmission coefficients of the ideal Gaussian filter, the eighth-order Bessel and Gaussian lumped-element filters [1] (labeled Bessel 8 and Gauss 8, respectively), complete and incomplete Gaussian transmission lines for $\omega_\mathrm{b} = 3\omega_{3\mathrm{dB}}$ (labeled Complete GTL and Incomplete GTL, respectively), and their lumped-element approximations of the eighth-order (labeled Complete 8 and Incomplete 8, respectively). At the normalizing angular frequency, $\omega_{3\mathrm{dB}}$, the transmission coefficient is -3 dB. The order of a filter is twice the number of half-cells (N) in Figures 2 and 3.

Note that if ω_b is increased, the transmission coefficients of the complete and incomplete Gaussian line (as well as of their lumped-element approximations) follow much closer the ideal Gaussian filter or the Bessel filter than shown in Figure 4. For a given value of ω_b, the incomplete Gaussian line has a steeper transfer function than the corresponding complete line. The incomplete line does not have an asymptotic value for the insertion loss, i.e., it behaves like a low-pass filter with an infinitely decaying skirt.

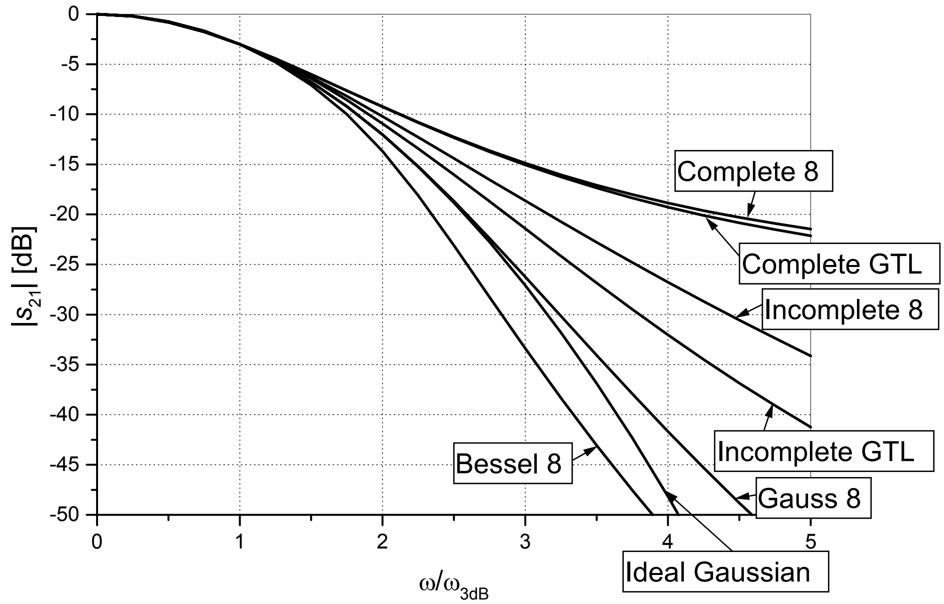

Fig. 4. Magnitude of the transmission coefficient.

III. Design Procedure for Lumped-Element Filters

We present two design cases. The first one is the design of a lumped-element network whose characteristics closely follow the Gaussian line, up to a certain frequency. The second design is an optimized filter, whose group-delay characteristic is superior to the Gaussian line. In both cases, we consider complete and incomplete Gaussian lines.

III.1. Approximation of Complete Gaussian Line

As the first step, we design a complete Gaussian line given the nominal impedance (equal to Z_c), the 3 dB attenuation frequency (f_{3dB}, viz. $\omega_{3dB} = 2\pi f_{3dB}$), and the break-point frequency (f_b, viz. $\omega_b = 2\pi f_b$). We have to evaluate the parameters *l*, *r*, *c*, *g*, and *D*. There is a total of four conditions, i.e., the three requests plus the condition (5), and a total of five parameters. To simplify the design, we reduce the number of parameters to four by defining the total parameters of the line, $L_t = lD$, $R_t = rD$, $C_t = cD$, and $G_t = gD$.

To evaluate the total parameters of the line, we start from the attenuation of the two-port network. According to (9) and (15),

$$a = 8.686 \frac{\left(\frac{\omega}{\omega_b}\right)^2 \sqrt{R_t G_t}}{1 + \left(\frac{\omega}{\omega_b}\right)^2} \text{ dB}. \tag{19}$$

Since $a = 3$ dB at ω_{3dB}, using equations (12), (13), and (19), we obtain

$$L_t = \frac{Z_c \ln 2}{2\omega_b} \frac{1 + \left(\frac{\omega_{3dB}}{\omega_b}\right)^2}{\left(\frac{\omega_{3dB}}{\omega_b}\right)^2}, \tag{20}$$

$$C_{\mathrm{t}} = \frac{L_{\mathrm{t}}}{Z_{\mathrm{c}}^{2}}, \tag{21}$$

$$R_{\mathrm{t}} = \omega_{\mathrm{b}} L_{\mathrm{t}}, \tag{22}$$

$$G_{\mathrm{t}} = \frac{R_{\mathrm{t}}}{Z_{\mathrm{c}}^{2}}. \tag{23}$$

As the second step, we find a lumped-element approximation to this line according to the scheme in Figure 2. We assume that the ladder network approximation consists of N identical half-cells. The values of the lumped elements are $L = L_{\mathrm{t}} / N$, $R = R_{\mathrm{t}} / N$, $C = C_{\mathrm{t}} / N$, and $G = G_{\mathrm{t}} / N$. All the elements in this design have identical values, which may be advantageous for manufacturing. However, to make a symmetrical network, it is possible to have a symmetrical topology, as in Figure 5. A dual alternative is also possible, starting and ending with a series branch.

To have a good approximation of the transmission-line behavior, the number N should be selected to keep the cutoff frequency of the LC filter, ω_{c}, high enough (e.g., close to ω_{b}). Hence, the low-pass behavior of the LC portion does not mask the quasi-Gaussian behavior of the network. Using (11), we obtain $N = \frac{\omega_{\mathrm{c}}}{2} \sqrt{L_{\mathrm{t}} C_{\mathrm{t}}}$.

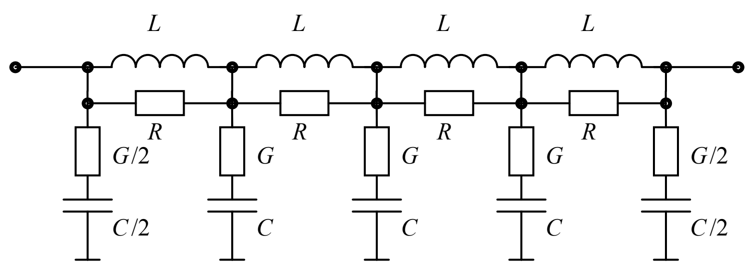

Fig. 5. An example of the topology of lumped-element filters derived from the complete Gaussian line ($N = 4$).

III.2. Optimized Design Based on Complete Gaussian Line

Lowering the frequency ω_{c}, the filter attenuation becomes steeper than for the Gaussian line within a certain frequency band, and it also flattens-out the group delay. Optimization of filters of various orders shows that the key factor for improving the filter performance is the proper choice of the attenuation introduced by a half-cell. To that purpose, we express the conductance in shunt branches as

$$G = \frac{1}{q Z_{\mathrm{c}}}, \tag{24}$$

so that

$$R = G Z_{\mathrm{c}}^{2} = \frac{Z_{\mathrm{c}}}{q}, \tag{25}$$

where q is a parameter. To find the optimal value of q that yields the most linear phase characteristic, we consider one half-cell in an infinite array of identical half-cells (Figure 6).

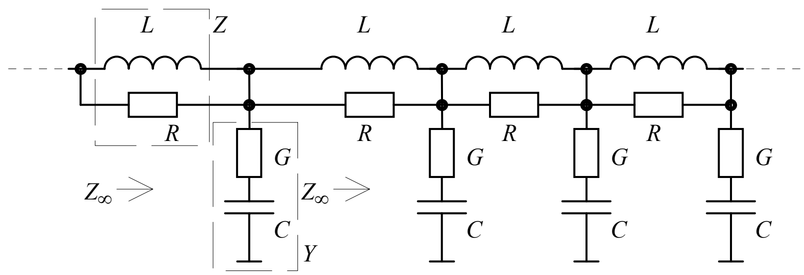

Fig. 6. A half-cell of the complete Gaussian line in an infinite array.

The half-cell consists of a series impedance, $Z = \dfrac{\mathrm{j}\omega LR}{R + \mathrm{j}\omega L}$, and a parallel admittance, $Y = \dfrac{\mathrm{j}\omega \mathrm{CG}}{G + \mathrm{j}\omega \mathrm{C}}$. On the right, the cell is backed by the infinite array. Let Z_∞ be the input impedance looking into the array. The impedance looking into the cell is also Z_∞, because the network is infinite. Hence,

$$Z_\infty = Z + \frac{Z_\infty}{Z_\infty Y + 1}. \tag{26}$$

Solving this equation results in

$$Z_\infty = \frac{ZY \pm \sqrt{ZY(ZY+4)}}{2Y}. \tag{27}$$

The sign (+ or −) should be selected to obtain a positive real part of Z_∞. The current transfer function of the half-cell in Figure 6 is

$$T = \frac{1}{Z_\infty Y + 1}. \tag{28}$$

This function can be represented as $T = A\mathrm{e}^{\mathrm{j}\varphi}$, where A is the amplitude and φ the phase of transfer function. We extract the linear frequency term from the phase, consider the residual phase, and vary the parameter q to obtain the flattest response. The optimum is numerically found to be

$$q = \frac{1}{6}, \tag{29}$$

when the first four derivatives of residual phase are zero at $\omega = 0$. Now, the remaining elements of the half-cell are

$$L = \frac{R}{\omega_\mathrm{b}}, \tag{30}$$

$$C = \frac{G}{\omega_\mathrm{b}}. \tag{31}$$

The corresponding cutoff frequency is

$$\omega_\mathrm{c} = \frac{2}{\sqrt{LC}} = \frac{2Z_\mathrm{c}}{R}\omega_\mathrm{b} = 2q\omega_\mathrm{b}. \tag{32}$$

Going back to the actual design, the number of half-cells is finally evaluated as

$$N = \frac{\omega_\mathrm{c}}{2}\sqrt{L_\mathrm{t} C_\mathrm{t}} = q\omega_\mathrm{b}\sqrt{L_\mathrm{t} C_\mathrm{t}} \approx 0.058\left(\frac{\omega_\mathrm{b}}{\omega_{3\mathrm{dB}}}\right)^2 = 0.52\left(\frac{\omega_\mathrm{c}}{\omega_{3\mathrm{dB}}}\right)^2. \tag{33}$$

Equation (33) is the starting point for the design. Once we have selected an integer value for N, we have to recompute

$$\omega_{cN} = 1.4\,\omega_{3dB}\sqrt{N} \tag{34}$$

to account for the discretization. From (12) and (32), we obtain the remaining equations needed for the design:

$$L = \frac{2Z_c}{\omega_{cN}}, \tag{35}$$

$$C = \frac{L}{Z_c^2}. \tag{36}$$

The lumped-element network obtained using this procedure can thereafter be further optimized, using a circuit simulator. By analyzing various lumped-element filters of the topology shown in Figure 5, it is found out that the optimum value of the conductance in the first and the last shunt branches is about $1/Z_c$. This choice provides a low reflection at very high frequencies.

III.3. Approximation of Incomplete Gaussian Line

In a similar way as in III.1, one can design a lumped-element filter (Figure 7) that approximates an incomplete Gaussian transmission line. We start from given Z_c, ω_{3dB}, and ω_b. Following the same procedure as before, we evaluate the total parameters of the incomplete Gaussian line as

$$L_t = \frac{Z_c \ln 2}{\omega_b} \frac{1 + \frac{5}{8}\left(\frac{\omega_{3dB}}{\omega_b}\right)^2}{\left(\frac{\omega_{3dB}}{\omega_b}\right)^2}, \tag{37}$$

$$C_t = \frac{L_t}{Z_c^2}, \tag{38}$$

$$G_t = \omega_b C_t. \tag{39}$$

As the second step, we find a lumped-element approximation to the incomplete Gaussian line. We assume that the ladder network approximation consists of N identical half-cells. The values of the elements are $L = L_t / N$, $C = C_t / N$, and $G = G_t / N$, where $N = \frac{\omega_c}{2}\sqrt{L_t C_t}$.

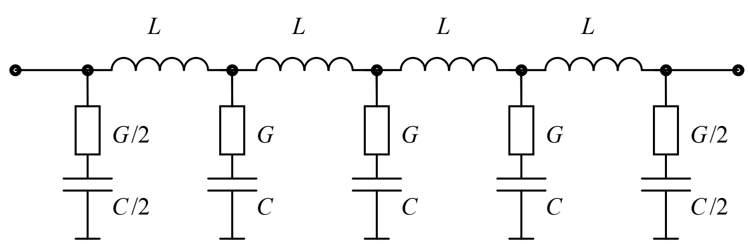

Fig. 7. An example of the topology of lumped-element filters derived from the incomplete Gaussian line ($N = 4$).

III.4. Optimized Design Based on Incomplete Gaussian Line

To find the optimal value for the elements in the ladder network from III.3, we represent the conductance in shunt branches by (24). The impedance of the series branch of one half-cell (Figure 8) is $Z = j\omega L$ and the admittance of the parallel branch is $Y = \frac{j\omega CG}{G + j\omega C}$. Following the same approach as before, we analyze one half-cell in an infinite array of identical half-cells. The optimization yields the optimal value for the parameter q,

$$q = \frac{1}{3}\,. \tag{40}$$

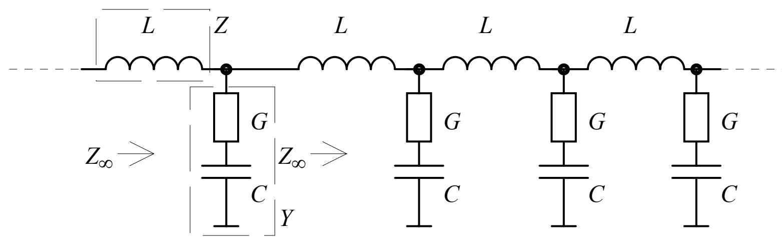

Fig. 8. A half-cell of the incomplete Gaussian line in an infinite array.

The final result of (32) is still valid, so that the number of half-cells for the actual design is given by

$$N \approx 0.23\left(\frac{\omega_b}{\omega_{3dB}}\right)^2 = 0.52\left(\frac{\omega_c}{\omega_{3dB}}\right)^2. \tag{41}$$

The remaining design procedure is the same as in Section III.2.

IV. Example

We present design results for a lumped-element filter based on the incomplete Gaussian line (Figure 7). Requests are $Z_c = 50\ \Omega$, $f_{3dB} = 4.2$ GHz, and $f_c = 11$ GHz. Using equations (37-41), we calculate the number of half-cells, $N = 4$ (filter of order 8), and the element values $L = 1.435$ nH, $C = 0.574$ pF, and $1/G = 16.7\ \Omega$. As mentioned in Section III.2, the resistances of the resistors in the first and the last shunt branches (denoted by $G/2$) should be Z_c.

A prototype of the filter was developed in the microstrip technique. The printed pattern and resistances are shown in Figure 9. The filter was made on a substrate of relative permittivity 2.33 and thickness 0.254 mm (10 mil). The inductors were made as narrow microstrips (trace width 0.15 mm) and capacitors as wide microstrips (trace width 2.5 mm). High-frequency SMD resistors (size 0603) were used. The printed pattern and resistances were optimized to include the parasitic effects of traces, junctions, and SMD components. These effects significantly increase the resistances, make sharper the slope of the transfer function magnitude, and create parasitic passbands at very high frequencies.

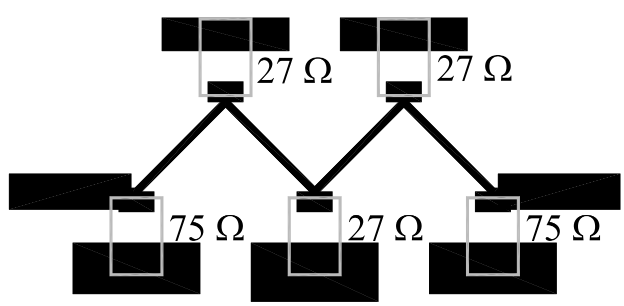

Fig. 9. A microstrip implementation of the filter from Figure 7.

Figure 10 shows the measured magnitude of the transmission coefficient compared with the results of computer simulation [7] and theoretical results for the classical Bessel filter of the eighth order. Figure 11 shows the phase of the transmission coefficient with the linear term extracted and Figure 12 shows the magnitude of the reflection coefficient. Excellent agreement between the theoretical and experimental results is obtained, although the theoretical model does not include the effect of SMA connectors. The experiments have shown the filters fairly insensitive to production tolerances and easy for tuning.

We do not compare our results with other data for low-reflection filters (e.g., [6]), as all other filters are of very low orders. Their transfer functions are inferior to those of the Bessel filters of order 3 or 4. In contrast to this, using the present approach, one can readily design and produce filters of virtually any order.

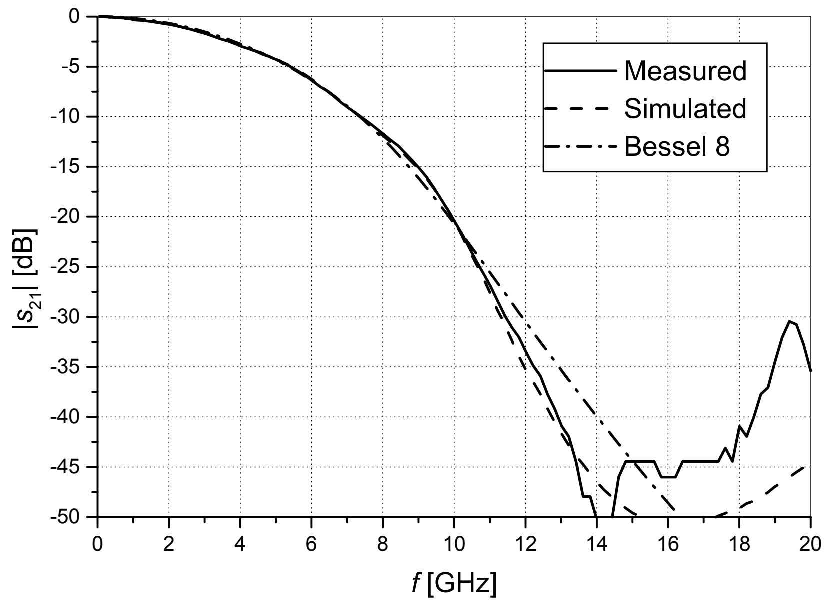

Fig. 10. Magnitude of the transmission coefficient.

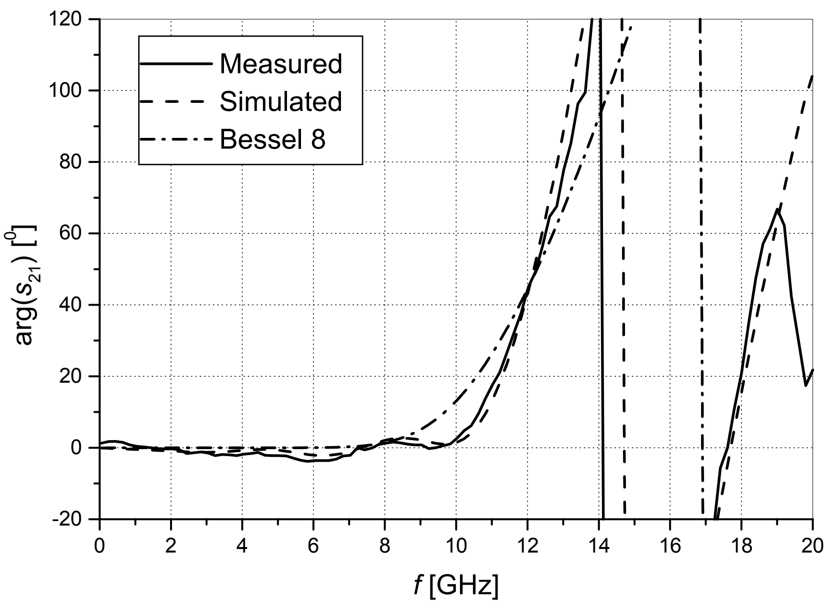

Fig. 11. Residual phase of the transmission coefficient.

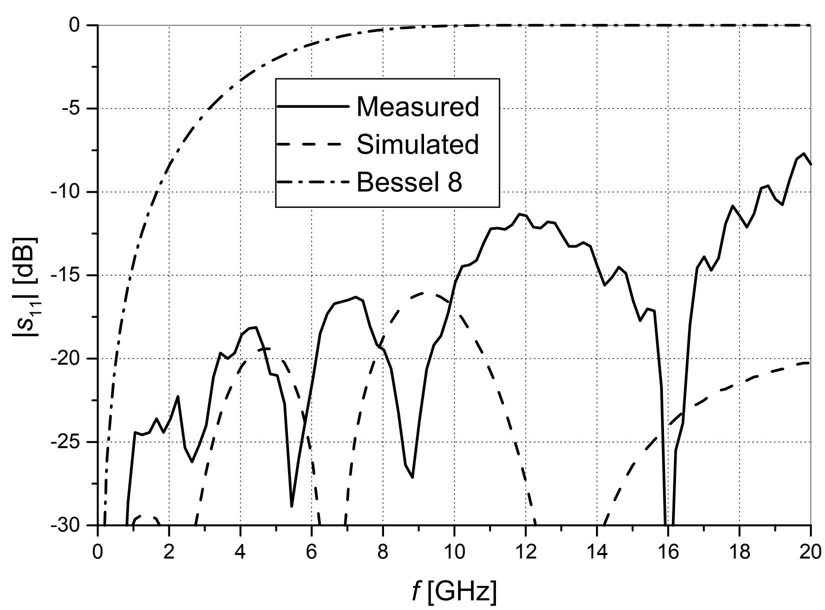

Fig. 12. Magnitude of the reflection coefficient.

V. Conclusion

This paper proposes a new, low-reflection ladder network that consists of resistors, inductors, and capacitors. The network models fictitious transmission lines with Gaussian-like amplitude characteristics. A theoretical analysis of these lines is presented, based on which a procedure for synthesis of low-reflection resistive filters is developed. These filters have transmission coefficients similar to the classical Bessel filters, but in contrast to them have low reflection, both in the stopband and passband. The new filters are convenient for manufacturing because the range of element parameters is small. In a typical design example given in the paper, all inductances are equal, capacitances are in the range 2:1 (as opposed to more than 10:1 for the classical Bessel and Gaussian filters), and resistances are in the range 3:1. The filters have been found fairly insensitive to element tolerances and easy for tuning. A patent application has been filed for the basic design. Further investigation is planned to extend this work to bandpass filters.

VI. References

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