Finite Antenna Arrays
and FSS
Finite Antenna Arrays and FSS

Ben A. Munk
To the increasing number of my friends who realize that computer power is a supplement, not a substitute, for brain power.

The constant support of the Electroscience Laboratory and my family—in particular, my wife Aase—is deeply appreciated.

B. A. M.
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It has often been said that a good teacher must have a number of attributes, among which are true expertise in the subject to be taught, and, just as important, the ability to put the subject across to the students, regardless of its complexity. We have all suffered under instructors who got straight A’s as students, but who never understood how their own students did not do as well because the material was presented as if it should be obvious. Professor Ben Munk has no problems in either regard.

In this book, Ben treats a number of subjects related to antennas and both their intended usage as transmission or reception devices, as well as the important (these days) radar cross section (RCS) that they can contribute. A constant theme behind the presented results is how often investigators approach the problem with no apparent understanding of the real-world factors that bear heavily on the practicality and/or quality of the result. He takes issue with those who have become so enchanted with high-powered computers that they simply feed the machine some wonderful equations and sit back while it massages these and “optimizes” a result. Sad to say, Ben has been able to document all too many examples to prove his point.

All this is not intended in any way to say that powerful computers are useless. Far from it. Without the use of such machines, much of the work described herein could not have been done in a lifetime, but the approach has to be controlled by investigators who understand the physics and electromagnetic realities that make a solution truly optimal and practical.

Throughout this book, Ben makes excellent use of the work he described in his first book, *Frequency Selective Surfaces*, in which he demonstrated how what he called the “Periodic Moment Method” could be used to obtain excellent results for problems previously hampered by “micro” calculation methods. His array
theory approach, combined appropriately with the detailed “method of moments,” produced successful solutions to a number of critical problems.

Here he further applies this approach and gives many examples of problems solved by himself and his graduate associates, with the goal of teaching by practical example. This is done by walking the reader, case by case, through the basic technology that applies, then to a logical solution. He then gives hard results to validate what was done, and then, to quickly bring the reader up to speed, he provides a problem or two for solution without further guidance.

Throughout all this, Ben uses his wonderful sense of humor to make various points, which goes a long way in making this book anything but tedious. Saying that about a book on heavy electromagnetic theory and design is certainly a far cry from the usual. His sections on “Common Misconceptions” are his way of highlighting how often “results” are developed and publicized without the necessary understanding of the basic rules of the game. He calls a spade a spade, for sure, and there may well be some who, though unnamed, might feel a twinge after reading these sections. All in all, this is an excellent book that will certainly benefit any serious investigator in the technology areas it discusses. Highly recommended!

WILLIAM F. BAHRET

Mr. W. Bahret was with the United States Air Force but is now retired. From the early 1950s he sponsored numerous projects concerning radar cross section of airborne platforms—in particular, antennas and absorbers. Under his leadership grew many of the concepts used extensively today—for example, the metallic radome. In fact he is considered by many to be the father of stealth technology.

BEN MUNK

Wow!! The former student (now a professor emeritus) has succeeded in advancing the former teacher’s (an even older professor emeritus) knowledge of array design tremendously.

The information contained in this book is going to change the way that large, broadbanded arrays are designed. This also leads to new insights in the area of antenna scattering. I strongly recommend it to the designers of such arrays. The concept of starting a finite array design from an infinite array is a remarkable one.

A simple example of why I make this comment comes to mind. I was reading the papers in the December 2002 IEEE Magazine which discuss the transmission of power to earth from space. Several problems with interference created by reradiation of energy at harmonic frequencies were discussed. I could see potential cures simply from scanning the initial chapters. I would also be interested in applying these concepts to my current research namely, time-domain ground-penetrating radar (GPR). Some neat antennas may become practical.

Those who have read Ben’s first book, Frequency Selective Surfaces, Theory and Design, will recall that I also wrote the Foreword to it. I was his teacher,
project supervisor, and later co-worker on much of that material. In reading it, I would turn pages and simply agree with many of the concepts.

At this time I have only scanned some of the chapters of the present book. For what I have seen thus far, I would scan a part of it and simply say, “Wow.” The reader should understand that there were points where I would have said, “Bet a Coke” (Ben and I used to bet a Coke every time we disagreed. Neither of us ever paid up.). These points are provocative to those readers with an interest in antenna scattering and should make those readers think carefully about them but most of them are resolved when one recalls that the emphasis of this book is on arrays.

This book is a must for anyone involved in the design of large arrays. I fully intend to read it very carefully after it is published.

Finally, I would observe that Ben’s comments about the review of journal papers are borne out of frustration. While Ben has worked in these areas throughout his career, most of his work was at that time classified. Thus when a paper in these general areas was published, he saw various flaws because of his experience but he could not comment. Neither the paper’s author nor the reviewers, not having Ben’s unique background, would see these flaws. The problem is in reality created by the necessity of security. This same factor has led to the very interesting sections he has titled “Common Misconceptions.”

Columbus, Ohio

Leon Peters, Jr.

Leon Peters, Jr., was a professor at the Ohio State University but is now retired. From the early 1960s he worked on, among many other things, RCS problems involving antennas and absorbers. In fact, he became my supervisor when I joined the group in the mid-1960s.

Ben Munk
Why did I write this book?

The approach to engineering design has changed considerably over the last decades.

Earlier, it was of utmost importance to first gain insight into the physics of the problem. You would then try to express the problem in mathematical form. The beauty here was, of course, that it then often was quite simple to determine the location of the extreme values such as the maxima and minima as well as nulls and asymptotic behavior. You would then, in many cases, be able to observe which parameters were pertinent to your problem and in particular which were not. It was then followed by actual calculations and eventually by a meaningful parametric study that took into account what was already observed earlier.

The problem with this approach was, of course, that it required engineers and scientists with considerable insight and extensive training (I deliberately did not say experience, although it helps). However, not everyone that started down this road would finish and not without a liberal dose of humiliation.

It is therefore quite understandable that when the purely numerical approaches appeared on the scene, they soon became quite popular. Most importantly, only a minimum of physical insight was required (or so it was thought). The computers would be so fast that they would be able to calculate all the pertinent cases. These would then be sorted out by using a more or less sophisticated optimization scheme, and the results would be presented on a silver platter completely untouched by the human mind.

It would be incorrect to state that the numerical approach has failed. It has in many cases produced remarkable results. However, the author is keenly aware of several cases that have been the subject of intense investigation for years and still have not produced a satisfactory solution, although some do exist—most often
because the computer has been directed to incorporate all kinds of parameters that are alien to this particular problem. Or lack of physical insight has prevented the operator from obtaining a meaningful parametric study—for example, in cases where a solution does not exist in the parametric space considered.

The author has watched this development with considerable concern for several years. One of his colleagues stated recently that a numerical solution to a somewhat complex problem of his could only be used to check out specific designs. An actual optimization was not possible because of the excessive computer time involved.

That almost sounds like an echo of other similar statements coming from the numerical camp.

A partial remedy for this calamity would be, of course, to give the students a better physical understanding. However, a fundamental problem here is that many professors today are themselves lacking in that discipline. The emphasis in the education of the younger generation is simply to write a computer program, run it, and call themselves engineers! The result is that many educators and students today simply are unaware of the most basic fundamentals in electromagnetics. Many of these shortcomings have been exposed at the end of each chapter of this book, in a section titled “Common Misconceptions.” Others are so blatantly naive that I am embarrassed to even discuss them. What is particularly disturbing is the fact that many pursue these erroneous ideas and tales for no other reason than when “all the others do it, it must be OK!”

Neither this book nor my earlier one, *Frequency Selective Surfaces, Theory and Design*, make any claims to having the answers to all problems. However, there are strong signals from the readers out there that they more and more appreciate the analytic approach based on physical understanding followed up by a mathematical analysis. It is hoped that this second book will be appreciated as well.

The author shared this preface with some of his friends in the computational camp. All basically agreed with his philosophy, although one of them found the language a bit harsh!

However, another informed him before reading this preface that design by optimization has lately taken a back seat as far as he was concerned. Today, he said, there is a trend toward understanding the underlying mathematics and physics of the problem.

Welcome to the camp of real engineering. As they say, “there is greater joy in Heaven over one sinner who makes penance than over ninety-nine just ones.”

Columbus, Ohio

Ben Munk
As in my first book, *Frequency Selective Surfaces, Theory and Design*, three of my many mentors stand out: Mr. William Bahret, Professor Leon Peters, Jr., and Professor Robert Kouyoumjian. They were always ready with consultation and advice. That will not be forgotten.

Further support and interest in my work was shown by Dr. Brian Kent, Dr. Stephen Schneider, and Mr. Ed Utt from the U.S. Air Force. After completion of the development of the Periodic Method of Moments, the PMM code, the Hybrid radome, low RCS antennas, and more, the funding from the Air Force shifted into more hardware-oriented programs. Fortunately, the U.S. Navy needed our help in designing very broadbanded bandstop panels. Ultimately, this work resulted in the discovery of surface waves unique to finite periodic structures, which are treated in great detail in this book. The help and advice from Mr. Jim Logan, Dr. John Meloling, and Dr. John Rockway is deeply appreciated.

However, the most discussed subject was the Broadband Array Concept. It was set in motion by two of the author’s oldest friends, namely Mr. William Croswell and Mr. Robert Taylor from the Harris Corporation. This relationship resulted in many innovative ideas as well as support. So did my cooperation with Mission Research (home of many of the author’s old students). My deep-felt thanks goes to all who participated in particular Errol English who wrote Section 9.6 about Tapered Periodic Surfaces, and Peter Munk who supplied Section 3.7 investigating Periodic Surfaces with arbitrary oriented elements.

My good friend and mentor, Professor John Kraus, once stated that students really are at the university to “straighten” the professors out, not the other way around. I whole-heartedly agree. In fact, had it not been for my last two students, Dr. Dan Janning and Jonothan Pryor, this book would not have been written. I am particularly indebted to Jonothan, who tirelessly ran computer programs and
curves for numerous cases in this book. He is currently interviewing. Lucky is the company that “secures” him.

Deep-felt thanks also go to my many friends and colleagues at the OSU ElectroScience Lab who supported me—in particular to Prof. Robert Garbacz, who graciously reviewed Chapter 2 concerning the RCS of antennas.

Finally, I was very lucky to secure my old editorial team, namely, Mrs. Ann Dominek, who did the typing, and Mr. Jim Gibson, who did a great deal of the drawings. In spite of their leaving the laboratory, they both agreed to help me out. And a fine job they did. Thank you.

BEN MUNK
Symbols and Definitions

\(a\)  
horizontal distance between column \(q\) and point of observation \(\mathbf{R}\)

\(a, a_1\)  
wire radius of elements

\(a\)  
side length of square elements

\(d\mathbf{A}\)  
vector potential for double infinite array of Hertzian elements

\(d\mathbf{A}_q\)  
vector potential of Hertzian elements located in column \(q\)

\(d\mathbf{A}_{qm}\)  
vector potential of a single Hertzian element located in column \(q\) and row \(m\)

\(b_{m-1}\)  
location of the front face of dielectric slab \(m\) in dipole case

\(b_m\)  
location of the back face of dielectric slab \(m\) in dipole case

\(C_p\)  
equivalent shunt capacitance from the orthogonal elements in a CA absorber

\(d\)  
diameter of circular plate element

\(d_m\)  
thickness of dielectric slab in dipole case

\(D_N\)  
determinant of admittance matrix for \(N\) slot arrays

\(D_x\)  
terelement spacings in the \(X\) direction

\(D_z\)  
terelement spacings in the \(Z\) direction

\(\mathbf{e} = [\hat{p} \times \hat{r}] \times \hat{r}\)  
field vector for infinite array of Hertzian elements

\(\mathbf{e} = \hat{r}_e + \hat{n}_e\)  
electric field at \(\mathbf{R}\) in medium \(m\)
\( \overline{E}_m(\overline{R}) \)  
incident electric field at \( \overline{R} \) in medium \( m \)

\( \overline{E}_m^{(r)} \)  
reflected electric field of \( \overline{R} \) in medium \( m \)

\( f \)  
frequency

\( f_g \)  
onset frequency of grating lobe

\( F(w) \)  
Fourier transform of \( f(t) \), not necessarily a function of time

\( \overline{H}_m(\overline{R}) \)  
magnetic field of \( \overline{R} \) in medium \( m \)

\( \overline{H}_m^{(i)}(\overline{R}) \)  
incident magnetic field at \( \overline{R} \) in medium \( m \)

\( \overline{H}_m^{(r)}(\overline{R}) \)  
reflected magnetic field at \( \overline{R} \) in medium \( m \)

\( H_{n}^{(2)}(x) \)  
Hankels function of the second kind, order \( n \) and argument \( x \)

\( I_{qm}(l) \)  
current along element in column \( q \) and row \( m \)

\( k, n \)  
indices for the spectrum of plane, inhomogeneous waves from an infinite array

\( l \)  
distance from a reference point to an arbitrary point on the element

\( 2l_1 \)  
total element length

\( dl \)  
inﬁnitesimal element length

\( \Delta l \)  
element length of Hertzian dipoles

\( \overline{\mathbf{m}}_{\pm} = \overline{\mathbf{E}} \times \hat{n}_D_{\pm} \)  
magnetic current density

\( \overline{M}_{\pm} \)  
total magnetic current in slots

\( \hat{n}_D \)  
unit vector orthogonal to dielectric interface pointing into the dielectric medium in question

\( \hat{n}_n \)  
unit vector(s) orthogonal to the planes of incidence or reradiation in medium \( m \)

\( \|\hat{n}_m = \bot \hat{n}_m \times \overline{r} \)  
unit vector(s) parallel to the planes of incidence or reradiation in medium \( m \)

\( n, n_0, n_1, n_2, \ldots \)  
integers

\( \hat{p} \)  
orientation vector for elements

\( \hat{p}^{(p)} \)  
orientation vector for element section \( p \)

\( \hat{p}^{p,n} \)  
orientation vector for element section \( p \) in array \( n \)

\( P^{(p)} \)  
scattering pattern function associated with element section \( p \)

\( P^{(p)\uparrow} \)  
transmitting pattern function associated with element section \( p \)

\( P_m^{(p)} \)  
scattering pattern function associated with element section \( p \) in medium \( m \)

\( \perp P_{m\pm}^{(p)} = \hat{p}^{(p)} \cdot \bot \hat{n}_{m\pm} P_{m\pm}^{(p)} \)  
orthogonal and parallel pattern components of scattering pattern in medium \( m \)

\( P_m^{(p)\uparrow} \)  
transmitting pattern function associated with element section \( p \) in medium \( m \)
SYMBOLS AND DEFINITIONS

\( F_{m}^{(p)t} \)
\( \mathcal{P}_{n} \)
\( q, m \)
\( \hat{r}_{\pm} = \hat{x}r_x \pm \hat{y}r_y + \hat{z}r_z \)
\( \hat{r}_{m\pm} = \hat{x}r_{mx} \pm \hat{y}r_{my} + \hat{z}r_{mz} \)
\( r_{\rho} = \sqrt{1 - \left( s_{z} + n \frac{\lambda}{D_{z}} \right)^{2}} \)
\( \hat{s} = \hat{x}s_{x} + \hat{y}s_{y} + \hat{z}s_{z} \)
\( \hat{s}_m = \hat{x}s_{mx} + \hat{y}s_{my} + \hat{z}s_{mz} \)
\( t \)
\( \perp \)
\( T_{m} \)
\( E_{\mp} T_{m} \)
\( H_{\perp} T_{m} \)
\( \perp T_{m-m'} \)
\( T.C._{\pm1} V_{1,1}^{1,1} \)
\( V_{D_{1\pm}}^{(1)} \)
\( V_{D_{1\pm}}^{(1)} \)
\( V_{S_{\pm}}^{(1)} \)
\( w \)

Orthogonal and parallel pattern components of transmitting pattern in medium \( m \)

Polynomial for a bandpass filter comprised of \( n \) slot arrays

The position of a single element in column \( q \) and row \( m \)

Direction vectors of the plane wave spectrum from an infinite array

Direction vectors in medium \( m \) of the plane wave spectrum from an infinite array

The \( \rho \) component of \( \hat{r}_{\pm} \)

Direction of incident field

Direction of incident field in medium \( m \)

Variable used in Poisson’s sum formula

Orthogonal and parallel transformation functions for single dielectric slab of thickness \( d_m \)

Orthogonal and parallel transformation function for the \( E \) field in a single dielectric slab of thickness \( d_m \)

Orthogonal and parallel transformation function for the \( H \) field in a single dielectric slab of thickness \( d_m \)

Orthogonal and parallel generalized transformation function when going from one dielectric slab of thickness \( d_m \) to another of thickness \( d_{m'} \), both of which are located in a general stratified medium

Transmission coefficient at the roots \( Y_{1\pm} \), etc.

Induced voltage in an external element with reference point \( \overline{R}_{(1)}^{1} \) caused by all the currents from an array with reference element at \( \overline{R}_{(1)}^{1} \)

Induced voltage in an external element with reference point \( \overline{R}_{(1)}^{1} \) caused by a direct wave only from the entire array

Induced voltage in an external element with reference point \( \overline{R}_{(1)}^{1} \) caused by double bounded modes ending in the \( \pm \) direction

Induced voltage in an external element with reference point \( \overline{R}_{(1)}^{1} \) caused by a single bounded mode ending up in the \( \pm \) direction

Dipole or slot width
SYMBOLS AND DEFINITIONS

\( \perp W_m \) \( W^e_m \) orthogonal and parallel components for the Wronskian for a single dielectric slab of thickness \( d_m \) orthogonal and parallel components for the effective Wronskian for a single dielectric slab of thickness \( d_m \) and located in a general stratified medium

\( Y \) \( Y_{1\pm}, Y_{2\pm}, \ldots \) intrinsic admittance roots of polynomial for bandpass filter

\( Y_A \) scan admittance as seen at the terminals of an element in the array

\( Y_L \) load admittance at the terminals of the elements

\( Y_0 = \frac{1}{Z_0} \) intrinsic admittance of free space

\( Y_m = 1/Z_m \) intrinsic admittance of medium \( m \)

\( Y^{1,2} \) array mutual admittance between array 1 and 2

\( Z \) the dependent variable as a function of the independent variable \( z \) in a bilinear transformation

\( Z = \frac{a + b z}{c + d z} \) intrinsic impedance of free space

\( Z_0 = 1/Y_0 \) scan impedance as seen at the terminals of an element in the array

\( Z_A = R_A + j X_A \) load impedance at the terminals of the elements

\( Z_m = 1/Y_m \) intrinsic impedance of medium \( m \)

\( Z^{n,n'} \) array mutual impedance between a reference element in array \( n \) and double infinite array \( n' \)

\( Z^{q,q'} \) column mutual impedance between a reference element in column \( q \) and an infinite line array at \( q' \)

\( Z_{q,q'm} \) mutual impedance between reference element in column \( q \) and element \( m \) in column \( q' \)

\( \alpha \) angle between plane of incidence and the \( xy \) plane

\( \beta_m = \frac{2\pi}{\lambda_m} \) propagation constant in medium \( m \)

\( \Delta l \) total element length of Hertzian dipole

\( \varepsilon \) dielectric constant

\( \varepsilon_{eff} \) effective dielectric constant of a thin dielectric slab as it affects the resonant frequency

\( \varepsilon_m \) dielectric constant in medium \( m \)

\( \varepsilon_{rm} \) relative dielectric constant in medium \( m \)

\( \eta \) angle of incidence from broadside

\( \eta_g \) angle of grating lobe direction from broadside

\( \theta_m \) angle of incidence from broadside in medium \( m \)
orthogonal and parallel Fresnel reflection coefficient for the \( E \) field when incidence is from media \( m \) to \( m+1 \)

orthogonal and parallel Fresnel reflection coefficient for the \( H \) field when incidence is from media \( m \) to \( m+1 \)

orthogonal and parallel effective reflection coefficient for the \( E \) field when incidence is from media \( m \) to \( m+1 \)

orthogonal and parallel effective reflection coefficient for the \( H \) field when incidence is from media \( m \) to \( m+1 \)

wavelength in medium \( m \)

permeability in medium \( m \)

relative permeability in medium \( m \)

orthogonal and parallel Fresnel transmission coefficient for the \( E \) field when incidence is from media \( m \) to \( m+1 \)

orthogonal and parallel Fresnel transmission coefficient for the \( H \) field when incidence is from media \( m \) to \( m+1 \)

orthogonal and parallel effective transmission coefficient for the \( E \) field when incidence is from media \( m \) to \( m+1 \)

orthogonal and parallel effective transmission coefficient for the \( H \) field when incidence is from media \( m \) to \( m+1 \)

angular frequency

variables used in Poisson’s sum formula (not angular frequencies)