Solution for HW#6 (ECE6255 Spring 2010) – Total = 4/100

Solution 1)

(a) The autocorrelation function *R(0)* and *R(1)* can be computed as

$$R(0)=\sum_{m=0}^{N-1}s(m)s(m)=\sum_{m=0}^{N-1}a^{2m}=\frac{1-a^{2N}}{1-a^{2}}$$

$$R(1)=\sum_{m=0}^{N-2}s(m)s(m=1)=\sum_{m=0}^{N-2}a^{2m+1}=a\frac{1-a^{2N-2}}{1-a^{2}}$$

So the estimated first order prediction coefficient is an "unbiased" estimate of *a* as:

$$\alpha_1=R(1)/R(0)=a\frac{1-a^{2N-2}}{1-a^{2N}}\to a(N\to\infty)$$

(b) $E_0=R(0)-\alpha_1 R(1)=R(0)(1-\alpha_1^2)\to\frac{1}{(1-a^2)}(1-a^2)=1(N\to\infty)$

Solution 2)

$$e(n)=\hat{s}(n)-\sum_{i=1}^{p}\alpha_i\hat{s}(n-i)$$

$$\alpha_0=1$$

$$e(n)=-\sum_{i=0}^{p}\alpha_i\hat{s}(n-i)$$

$$\hat{s}(n)=s(n)w(n)\qquad 0\le n\le N-1$$
$$=0\qquad otherwise$$

$$R_e(m)=\sum_{n=-\infty}^{\infty}e(n)e(n+m)=\sum_{n=-\infty}^{\infty}\sum_{i=0}^{p}\alpha_i\hat{s}(n-i)\sum_{j=0}^{p}\alpha_j\hat{s}(n+m-j)$$

let $n'=n-i$

$$R_e(m)=\sum_{n'=-\infty}^{\infty}\sum_{i=0}^{p}\sum_{j=0}^{p}\alpha_i\alpha_j\hat{s}(n')\hat{s}(n'+m+i-j)$$

let $j=l+i$

$$R_e(m)=\sum_{i=0}^{p}\sum_{l=-i}^{p-i}\alpha_i\alpha_{i+l}\sum_{n'=-\infty}^{\infty}\hat{s}(n')\hat{s}(n'+m-l)$$

since $\alpha_{i+l}=0$ for $i+l<0\Rightarrow l<-i$

$=0$ for $i+l>p\Rightarrow l>p-i$

$$R_e(m)=\sum_{l=-\infty}^{\infty}\left[\sum_{i=0}^{p}\alpha_i\alpha_{i+l}\right]\left[\sum_{n'=-\infty}^{\infty}\hat{s}(n')\hat{s}(n'+m-l)\right]=\sum_{l=-\infty}^{\infty}R_a(l)R_{\hat{s}}(m-l).$$

Solution 3)

$$h(n)=\sum_{k=1}^{p}\alpha_k h(n-k)+G\delta(n)$$

$$\tilde{R}(m)=\sum_{n=0}^{\infty}h(n)h(n+m)$$

$a)$ $$\tilde{R}(-m)=\sum_{n=0}^{\infty}h(n)h(n-m)$$

let $m'=n-m$

$$\tilde{R}(-m)=\sum_{m'=-n}^{\infty}h(m+m')h(m')$$

since $h(m')$ is causal, $h(m')=0,\ m'<0\Rightarrow$

$$\tilde{R}(-m)=\sum_{m'=0}^{\infty}h(m+m')h(m')=\tilde{R}(m)$$

$b)$ $$\tilde{R}(-m)=\sum_{n=0}^{\infty}h(n)h(n-m)=\tilde{R}(m)$$

$$=\sum_{n=0}^{\infty}\left[\sum_{k=1}^{p}\alpha_k h(n-k)+G\delta(n)\right]\left[\sum_{k=1}^{p}\alpha_l h(n-m-l)+G\delta(n-m)\right]$$

$$=\sum_{n=0}^{\infty}\left\{\sum_{k=1}^{p}\sum_{l=1}^{p}\alpha_k\alpha_l h(n-k)h(n-m-l)+G\sum_{k=1}^{p}\alpha_k h(n-k)\delta(n-m)+G\sum_{l=1}^{p}\alpha_l h(n-m-l)\delta(n)+G^2\delta(n)\delta(n-m)\right\}$$

assume $m\geq 0$

$$\tilde{R}(m)=\sum_{n=0}^{\infty}\sum_{k=1}^{p}\sum_{l=1}^{p}\alpha_k\alpha_l h(n-k)h(n-m-l)+G\sum_{k=1}^{p}\alpha_k h(m-k)+G\sum_{l=1}^{p}\alpha_l h(-m-l)$$

since for $m\geq 0,\ h(-m-l)=0$ for $l=1,2,\ldots,p$ then

$$\tilde{R}(m)=\sum_{n=0}^{\infty}\sum_{k=1}^{p}\alpha_k h(n-k)\left[\sum_{l=1}^{p}\alpha_l h(n-m-l)+G\delta(n-m)\right]$$

but $\left[\sum_{l=1}^{p}\alpha_l h(n-m-l)+G\delta(n-m)\right]=h(n-m)$

thus $$\tilde{R}(m)=\sum_{k=1}^{p}\alpha_k\sum_{n=0}^{\infty}h(n-k)h(n-m)$$

Solution 4)

$$x(n) \quad 0 \le n \le N-1 \Rightarrow R(k) = \sum_{n=0}^{N-1+k} x(n)x(n+k), \;\; 0 \le k \le p$$

$$\hat{x}(n) \quad 0 \le n \le N-1 \Rightarrow \hat{R}(k) = \sum_{n=0}^{N-1+k} \hat{x}(n)\hat{x}(n+k), \;\; 0 \le k \le p$$

$$x(n) \Rightarrow \alpha = \left[\alpha_0, \alpha_1, ..., \alpha_p\right]^t, \;\; \alpha_0 = -1$$

$$\hat{x}(n) \Rightarrow \hat{\alpha} = \left[\hat{\alpha}_0, \hat{\alpha}_1, ..., \hat{\alpha}_p\right]^t, \;\; \hat{\alpha}_0 = -1$$

(a)
$$E^{(p)} = \sum_{n=0}^{N-1+p} e^2(n) = \sum_{n=0}^{N-1+p} \left[-\sum_{i=0}^{p} \alpha_i x(n-i)\right]^2$$

$$= \sum_{i=0}^{p} \alpha_i \sum_{j=0}^{p} \alpha_j \sum_{n=0}^{N-1+p} x(n-i)x(n-j)$$

$$= \sum_{i=0}^{p} \alpha_i \sum_{j=0}^{p} \alpha_j R(i-j) = \alpha^t R_\alpha \alpha$$

$$R_\alpha = \begin{bmatrix} R(0) & R(1) & . & . & R(p) \\ R(1) & R(0) & . & . & R(p-1) \\ . & . & . & . & . \\ . & . & . & . & . \\ R(p) & R(p-1) & . & . & R(0) \end{bmatrix}$$

(b) $\tilde{e}(n) = \hat{x}(n) - \sum_{i=0}^{p} \alpha_i \hat{x}(n-i)$, following part (a) we have

$$\tilde{E}^{(p)} = \sum_{i=0}^{p} \alpha_i \sum_{j=0}^{p} \alpha_j \hat{R}(i-j) = \alpha^t \hat{R}_\alpha \alpha$$

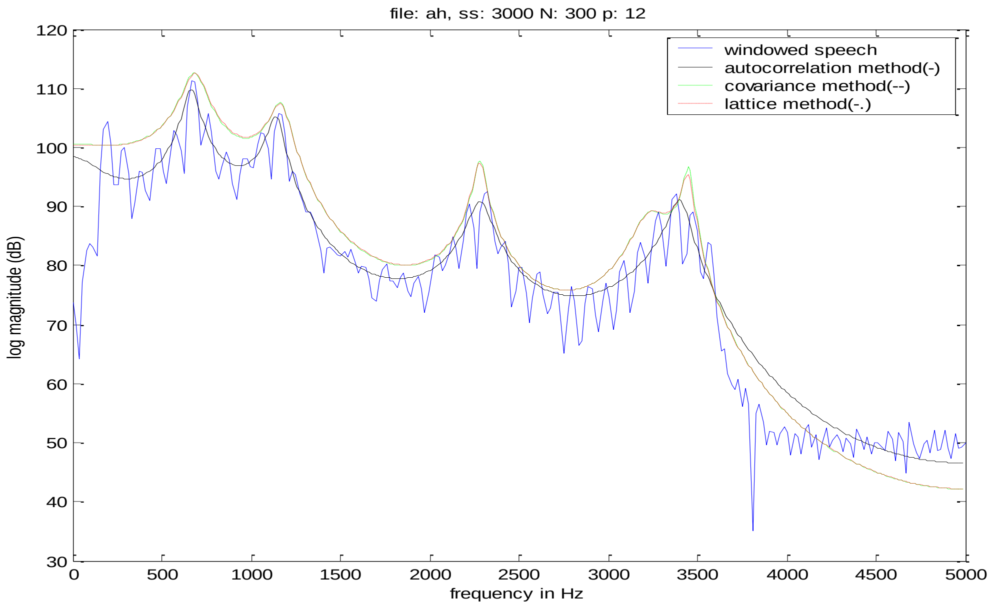
$$\hat{R}_\alpha = \begin{bmatrix} \hat{R}(0) & \hat{R}(1) & . & . & \hat{R}(p) \\ \hat{R}(1) & \hat{R}(0) & . & . & \hat{R}(p-1) \\ . & . & . & . & . \\ . & . & . & . & . \\ \hat{R}(p) & \hat{R}(p-1) & . & . & \hat{R}(0) \end{bmatrix}$$

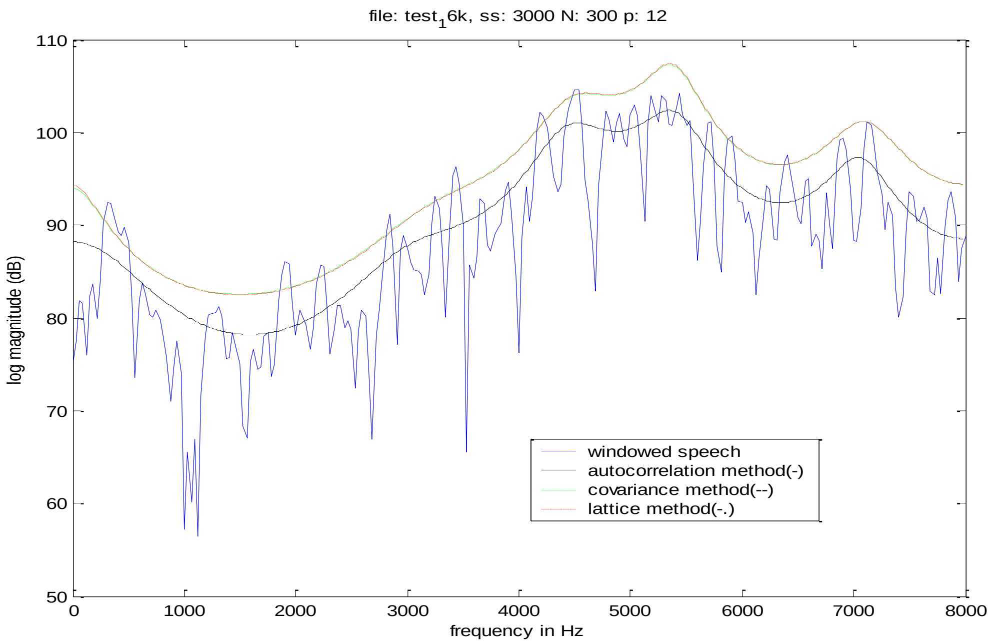
(c) Clearly, $\tilde{\tilde{E}}^{(p)} = \sum_{i=0}^{p} \hat{\alpha}_i \sum_{j=0}^{p} \hat{\alpha}_j R(i-j) = \hat{\alpha}^t R_\alpha \hat{\alpha}$

(d) $\tilde{D} = \dfrac{\tilde{E}^{(p)}}{E^{(p)}} = \dfrac{\alpha^t \hat{R}_\alpha \alpha}{\alpha^t R_\alpha \alpha}$ and $\tilde{\tilde{D}} = \dfrac{\tilde{\tilde{E}}^{(p)}}{E^{(p)}} = \dfrac{\hat{\alpha}^t R_\alpha \hat{\alpha}}{\alpha^t R_\alpha \alpha}$

 since $E^{(p)}$ is the minimum prediction residual, then $\tilde{\tilde{E}}^{(p)} \ge E^{(p)} \Rightarrow \tilde{\tilde{D}} \ge 1$
but we cannot say much about $\tilde{D}$.

Solution 5)


file: ah, ss: 3000 N: 300 p: 12
windowed speech
autocorrelation method(-)
covariance method(--)
lattice method(-.)
log magnitude (dB)
frequency in Hz
120
110
100
90
80
70
60
50
40
30
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000


file: test$_1$6k, ss: 3000 N: 300 p: 12
windowed speech
autocorrelation method(-)
covariance method(--)
lattice method(-.)
log magnitude (dB)
frequency in Hz
110
100
90
80
70
60
50
0
1000
2000
3000
4000
5000
6000
7000
8000