Solution to HW#5 (ECE6255 Spring 2010) – Total = 4/100

Solution 1)

$$V(z) = \frac{1}{\prod_{k=1}^{q}\left(1-c_k z^{-1}\right)\left(1-c_k^* z^{-1}\right)}$$

$c_k = r_k e^{j\theta_k}$; $\quad |c_k| = r_k < 1$ for stability

$$\log V(z) = -\sum_{k=1}^{q}\log(1-c_k z^{-1}) - \sum_{k=1}^{q}\log(1-c_k^* z^{-1})$$

using the power series expansion:

$$\boxed{\log(1+x) = \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}x^n \qquad |x|<1}$$

$$\log[V(z)] = -\sum_{k=1}^{q}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}(-c_k)^n z^{-n} - \sum_{k=1}^{q}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}(-c_k^*)^n z^{-n}$$

$$= \sum_{k=1}^{q}\sum_{n=1}^{\infty}\left[\frac{(c_k)^n}{n} + \frac{(c_k^*)^n}{n}\right]z^{-n} = \sum_{k=1}^{q}\sum_{n=1}^{\infty}\left[\frac{(r_k e^{j\theta_k})^n}{n} + \frac{(r_k e^{-j\theta_k})^n}{n}\right]z^{-n}$$

$$= \sum_{n=1}^{\infty}\left[\sum_{k=1}^{q}\frac{(r_k)^n}{n}(e^{jn\theta_k} + e^{-jn\theta_k})\right]z^{-n} = \sum_{n=1}^{\infty}\hat{v}(n)z^{-n}$$

Therefore $\hat{v}(n) = 0 \qquad\qquad n \le 0$

$$= 2\sum_{k=1}^{q}\frac{(r_k)^n}{n}\cos(\theta_k n) \qquad n > 0$$

Solution 2)

$$\hat{x}(n) \leftrightarrow \hat{X}(e^{j\omega}) = \log\left|X(e^{j\omega})\right| + j\arg\left[X(e^{j\omega})\right]$$

$$c(n) \leftrightarrow \log\left|X(e^{j\omega})\right|$$

 show $c(n) = \dfrac{\hat{x}(n) + \hat{x}(-n)}{2}$

$$c(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi}\log\left|X(e^{j\omega})\right| e^{j\omega n}d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi}\log\left|X(e^{j\omega})\right|(\cos(\omega n) + j\sin(\omega n))d\omega$$

 recall that for $x(n)$ real, $\left|X(e^{j\omega})\right|$ is an even function; therefore

$$\int_{-\pi}^{\pi}\log\left|X(e^{j\omega})\right|(j\sin(\omega n))d\omega = 0$$

with the result

$$c(n)=\frac{1}{2\pi}\int_{-\pi}^{\pi}\log\left|X(e^{j\omega})\right|\cos(\omega n)d\omega$$

now inverse transforming $\hat{X}(e^{j\omega})$ we obtain

$$\hat{x}(n)=\frac{1}{2\pi}\int_{-\pi}^{\pi}\left[\log\left|X(e^{j\omega})\right|+j\arg\left\{X(e^{j\omega})\right\}\right]\left[\cos(\omega n)+j\sin(\omega n)\right]d\omega$$

for $x(n)$ real, $\arg\left\{X(e^{j\omega})\right\}$ is an odd function, therefore

$$\int_{-\pi}^{\pi} j\arg\left\{X(e^{j\omega})\right\}\cos(\omega n)d\omega=0$$

thus

$$\hat{x}(n)=\frac{1}{2\pi}\int_{-\pi}^{\pi}\left[\log\left|X(e^{j\omega})\right|\right]\left[\cos(\omega n)\right]d\omega-\frac{1}{2\pi}\int_{-\pi}^{\pi}\arg\left[X(e^{j\omega})\right]\left[\sin(\omega n)\right]d\omega$$

$$\hat{x}(-n)=\frac{1}{2\pi}\int_{-\pi}^{\pi}\left[\log\left|X(e^{j\omega})\right|\right]\left[\cos(\omega n)\right]d\omega+\frac{1}{2\pi}\int_{-\pi}^{\pi}\arg\left[X(e^{j\omega})\right]\left[\sin(\omega n)\right]d\omega$$

$$\frac{\hat{x}(n)+\hat{x}(-n)}{2}=\frac{1}{2\pi}\int_{-\pi}^{\pi}\left[\log\left|X(e^{j\omega})\right|\right]\left[\cos(\omega n)\right]d\omega=c(n)$$

Solution 3)

$a)$ $\qquad c_l(n)=c(n)\cdot l(n)$

 multiplication in the cepstral domain results in convolution of the Fourier transforms:

$$\tilde{X}(e^{j\omega})=\frac{1}{2\pi j}\int_{-\pi}^{\pi}\log\left|X(e^{j\theta})\right|L(e^{j(\omega-\theta)})d\theta$$

$b)$ A low-time window is required to smooth $\log\left|X(e^{j\omega})\right|$

$c)$ The Fourier transform of the cepstral window is convolved with $\log\left|X(e^{j\omega})\right|$.

The Fourier transform of the rectangular window has the narrowest main lobe for a given window length and will produce less blurring or smoothing of adjacent points. Since the first side lobe is only about 13 dB below the main lobe peak, oscillations in $\log\left|X(e^{j\omega})\right|$ will be introduced at discontinuities of $\log\left|X(e^{j\omega})\right|$.

The Hamming window has a wider main lobe, resulting in greater smoothing of adjacent points. The largest side lobe is approximately 43 dB below the main lobe peak.

$d)$ The cepstral window should be shorter than the expect pitch period. The objective of the smoothing is to eliminate the excitation information and recover an approximation to the overall vocal tract transfer function.

Solution 4)

$x(n)$ is a finite length sequence with complex cepstrum $\hat{x}(n)$

$y(n) = \alpha^n x(n)$ with complex cepstrum $\hat{y}(n)$

$a)$ $0 < \alpha < 1$; assume length N for $x(n)$, then

$$Y(z) = \sum_{n=0}^{N-1} \alpha^n x(n) z^{-n} = \sum_{n=0}^{N-1} x(n)(\alpha z^{-1})^n$$

$$Y(z) = X(z/\alpha)$$

□ If the zeros of $X(z)$ occur at $z_0, z_1, ..., z_{N-1}$, then the zeros of $Y(z)$ occur at $\alpha z_0, \alpha z_1, ..., \alpha z_{N-1}$; therefore $\hat{y}(n) = \alpha^n \hat{x}(n)$

$b)$ Let $\max_i |z_i|$ denote the magnitude of the maximum magnitude zero of X(z).

$y(n)$ is not minimum phase if any zero of $Y(z)$ lies on or outside the unit circle.

Therefore choose α so that

$$|\alpha| \max_i |z_i| \geq 1 \Rightarrow |\alpha| \geq \frac{1}{\max_i |z_i|}$$

$c)$ Let $\min_i |z_i|$ denote the magnitude of the minimum magnitude zero of X(z).

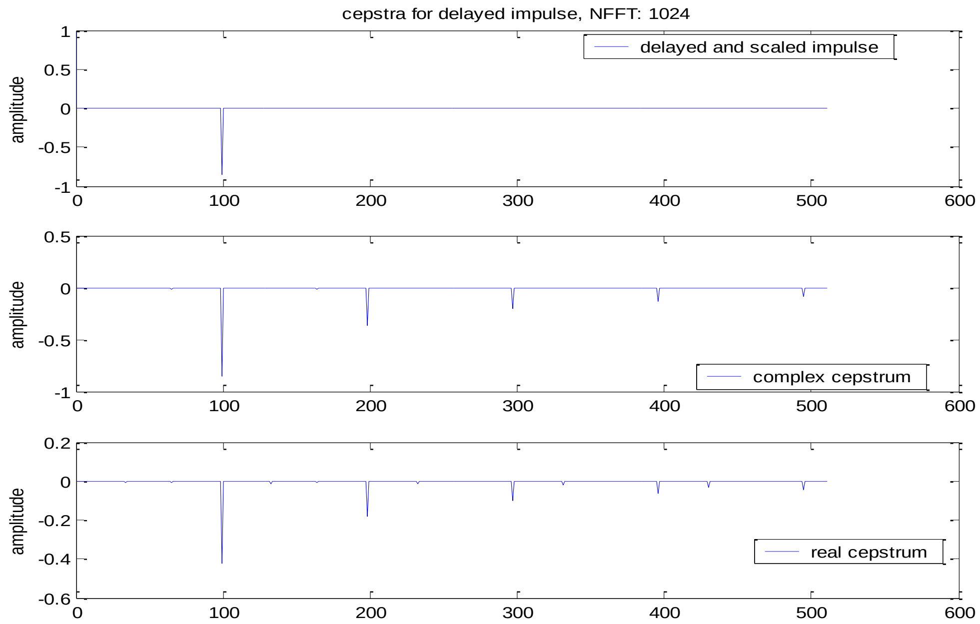
$y(n)$ is maximum phase if all zeros of $Y(z)$ lies outside the unit circle.

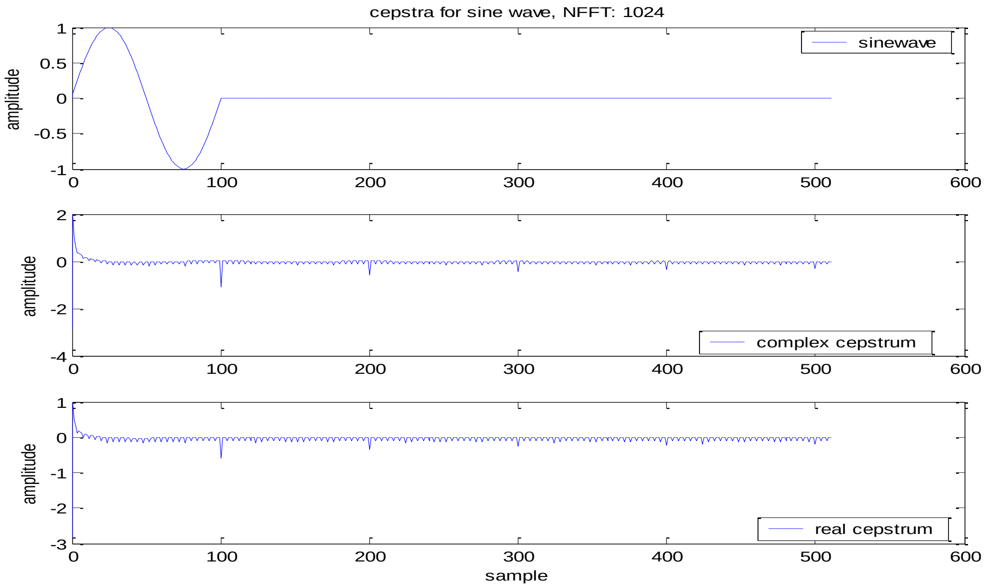
Therefore choose α so that

$$|\alpha| \min_i |z_i| > 1 \Rightarrow |\alpha| > \frac{1}{\min_i |z_i|}$$

Solution 5)

The following plots result:

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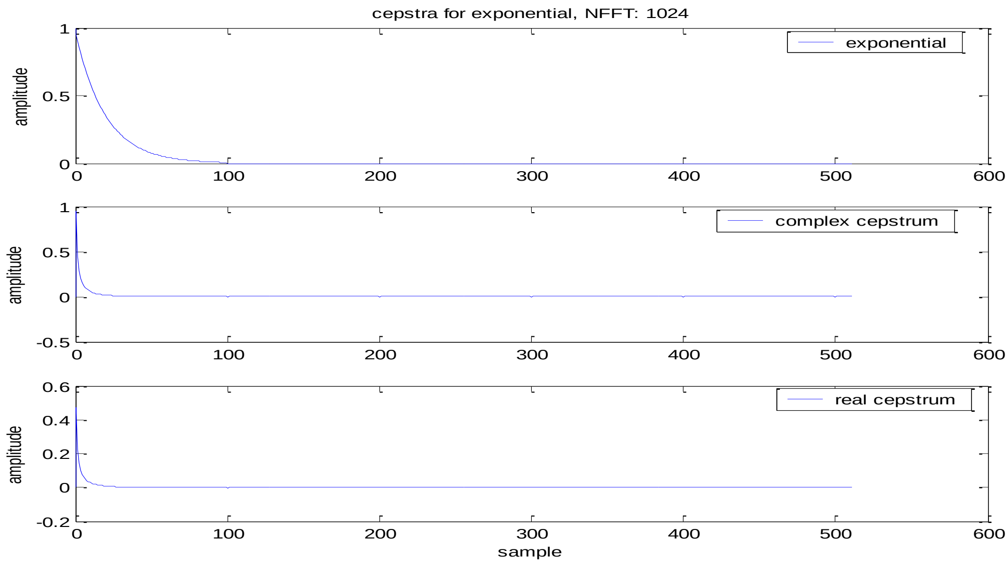
sinewave
complex cepstrum
real cepstrum
amplitude
sample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exponential
complex cepstrum
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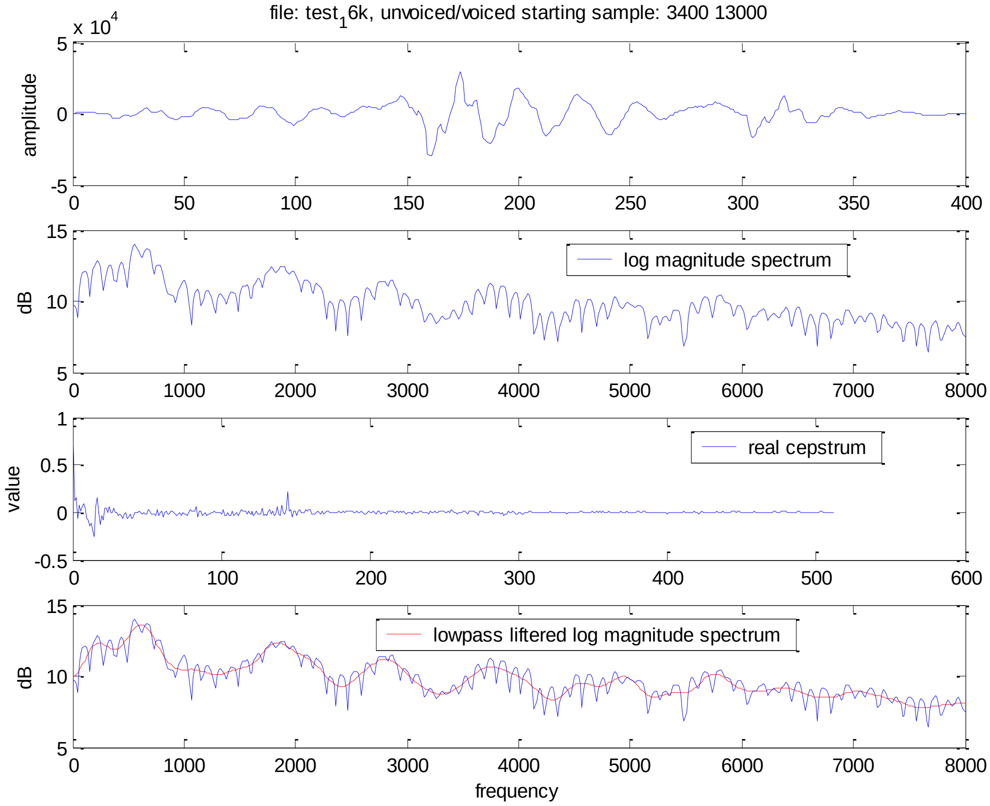
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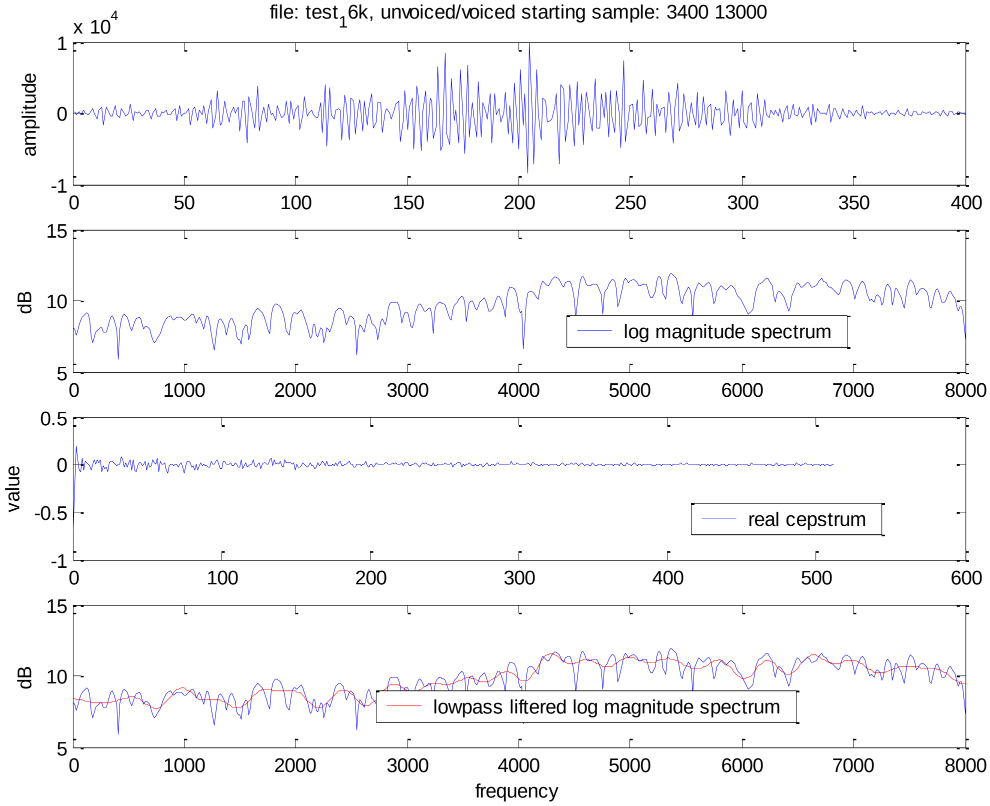
log magnitude spectrum
dB
real cepstrum
value
lowpass liftered log magnitude spectrum
frequency


file: test_16k, unvoiced/voiced starting sample: 3400 13000
x 10^4
amplitude
log magnitude spectrum
dB
real cepstrum
value
lowpass liftered log magnitude spectrum
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