Solution for HW#2 (ECE6255 Spring 2010) – Total = 3/100

Solution 1)

Summing geometric series (refer to Examples 2.2 and 2.4 on p. 19).

$$\frac{1}{P}\sum_{m=0}^{P-1} e^{j\frac{2\pi}{P}mn} = \frac{1}{P} \bullet \frac{1-e^{j\frac{2\pi}{P}nP}}{1-e^{j\frac{2\pi}{P}n}} = \begin{cases} 1 & \text{if } n = kP \\ 0 & \text{otherwise} \end{cases} = \sum_{k=-\infty}^{\infty} \delta(n-kP) = x(n).$$

Now, the inverse Fourier transfor of $Y(\omega) = \sum_{k=-\infty}^{\infty} \frac{2\pi}{P}\delta(\omega - \frac{2\pi}{P}k)$ is

$y(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} Y(\omega)e^{j\omega n}d\omega$ with $Y(\omega)$ a periodic function, so

$$y(n) = \frac{1}{2\pi}\sum_{k=0}^{P-1}\frac{2\pi}{P}e^{j\frac{2\pi}{P}kn} = x(n).$$

Therefore, $X(\omega) = Y(\omega) = \sum_{k=-\infty}^{\infty} \frac{2\pi}{P}\delta(\omega - \frac{2\pi}{P}k).$

Solution 2)

a) time duration =1024/20,000=51.1 msec
b) 1024 point DFT => frequency resolution=20,000/1024=19.531 Hz
c) If the time duration is 512 samples, it corresponds to 25.55 msec, but the frequency resolution remains the same since it is the sampling frequency divided by the FFT size

Solution 5)

We can use a process of elimination to identify the digits (knowing that each digit occurs exactly once in the list). We begin by immediately identifying the easiest digits, namely /6/ (as seen by strong noise at the beginning and end), /7/ (as seen by strong noise at the beginning and a clear nasal at the end), and /oh/ (as seen by the almost steady vocalic sound). Thus /6/ is the only digit in the sixth row, /7/ is the second digit in the first row, and /oh/ is the first digit in the fifth row.

Next we find the digit /1/ which is identified by the strong nasal at the end, and the strong low frequency convergence of the first and second formant at the beginning (due to the /w/ sound). Thus we identify /1/ as the first digit in the third row.

Next we identify the digit /2/ by the strong frication at the release of the /t/ and the strong low frequency convergence of the first two formants at the end of the digits. Thus we find the first digit in the fourth row is /2/.

Next we identify the digit /8/ by the initial /ey/ diphthong (as seen by the low first formant and the high second formant), followed by a stop gap and a strong release of the /t/ at the end of the digit. Thus we find the second digit in the fourth row is /8/.

Next we identify the digit /9/ which is cued by nasal sounds at the beginning and end of the utterance, and the /ay/ diphthong in the middle of the digit. Thus we find the second digit in the second row is /9/.

Next we identify the digit /zero/ by the frication at the beginning (due to the /z/ sound), the high initial second formant of the /ih/ sound, and the low second formant of the /ow/ sound at the end of the utterance. Thus we find the first digit in the second row is /zero/.

Finally we make choices for the final three digits, namely /3/, /4/, and /5/. All three digits begin with weak fricatives, but we identify /3/ by the strong /iy/ sound at the end, having a high second formant. Although there are two good choices, it seems clear that the first digit

in the first row is the better fit to the characteristics of /3/ than the second digit in the third row, which we then identify as /5/, since the second digit in the fifth row is the best remaining match to /4/ with the characteristic /r/ sound at the end having the second and third resonances come together.

i) By way of summary, we have the following results. The digits of the top row are 3 and 7:
   a) The digit 3 is cued by the distinctive brief initial fricative (/θ/), followed by the semivowel /r/, where the second and third formants both get very low in frequency, followed by the /i/ where F2 and F3 both become very high in frequency
   b) The digit 7 is cued by the strong /s/ frication at the beginning, the distinctive /eh/, followed by the voiced fricative /v/, a short vowel /ax/, and the strong nasal /n/ at end.

ii) The digits in the second row are 0 and 9:
   a) The initial /z/ is cued by the strong frication with the presence of voicing at low frequencies; the following /I/ is seen by the high F2 and F3, the /r/ is signaled by the low F2 and F3 and the diphthong /o/ is signaled by the gliding motion of F2 and F3 toward an /u/-like sound.
   b) The digit 9 is cued by the distinct initial and final nasals /n/ and by the /a^y/ glide between the nasals.

iii) The digits in the third row are 1 and 5:
   a) The digit1 is cued by the strong initial semivowel /w/ with very low F2 and by the strong final nasal /n/.
   b) The digit 5 is cued by the weak initial frication of /f/, followed by the strong diphthong /a^y/ and ending in the very weak fricative /v/.

iv) The digits in the fourth row are 2 and 8:
   a) The digit 2 is cued by the strong /t/ burst and release followed by the glide to /u/.
   b) The digit 8 is cued by the initial weak diphthong /e^y/ followed by a clear stop gap of the /t/ and then the /t/ release.

v) The digits in the fifth row are “oh” and 4:
   a) The digit “oh” is virtually steady with a slight gliding tendency toward /u/ at the end
   b) The digit 4 is cued by the weak initial fricative /f/, followed by the strong /o/vowel and ending with a classic /r/ where F2 and F3 merge together.

vi) The digit in the last row is 6:
   a) The digit 6 is cued by the strong /s/ frication at the beginning and end, and by the steady vowel /I/ followed by the stop gap and release of the /k/.