Quiz #2 (ECE3075 Summer 2004): 07/01/04

There are six problems, worth a total of 120 points. You should choose five of them to finish first. Then work towards extra credit by doing the remaining one.

Problem 1 (20%)

Let X be the outcome of rolling one fair dice (with a possible outcome from 1 to 6 with equal probability), and Y be the outcome of independently rolling a second fair dice [This is Problem 3-1.4 of Cooper and McGillem]. [Hint: X and Y are discrete random variables]

(a) Find the probability of the event joint event $X \le 3$ and Y>3;

(b) Find $E[XY]$;

(c) Find $E[X/Y]$.

Problem 2 (20%)

A random variable X can be observed in the presence of independent additive noise, N. The observed random variable is $Y=X+N$. The joint pdf of X and Y is known as: $f(x,y) = K\exp[-(x^2+xy+y^2)]$ for all x and y [Part of this is Problem 3-2.4 of Cooper and McGillem].

(a) Find the value of K to make $f(x, y)$ a valid pdf;

(b) Find the conditional pdf of $f(x|y)$;

(c) Find the conditional mean of X given $Y=y$;

(d) Find the most likely estimate of X as a function of the observed $Y=y$.

Problem 3 (20%)

A random variable X has a pdf: $f_X(x) = e^{-x}u(x)$ [$u(x)$: step function at $x=0$], and an independent random variable Y with a pdf: $f_Y(y) = 3e^{-3y}u(y)$ [This is Problem 3-7.1 of Cooper and McGillem].

(a) Find the characteristic functions of X and Y, $\phi_X(u) = E[e^{juX}]$, $\phi_Y(u) = E[e^{juY}]$;

(b) Find the characteristic function of $Z=X+Y$;

(c) Find the probability density function of Z, using results from parts (a) and (b), and the inverse transformation $f_Z(z) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\phi_Z(u)e^{-juz}dz$ [Hint: this can be obtained with the results in part (a)].

Problem 4 (20%)

The sample mean is a random variable defined as: $\hat{\bar{X}} = \frac{1}{n}\sum_{i=1}^{n} X_i$, with X_i's being independent of each other and having an identical distribution, with a mean μ and a variance σ^2. The sample variance is an r. v. defined as: $S_2^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \hat{\hat{X}})^2$

(a) Find Var[$\hat{\hat{X}}$];

(b) Find $E[S_2^2]$.

Problem 5 (20%)

The City of XYZ is interested in knowing if the sale price of residential properties had changed significantly in the last five years. You were asked by the City Council to conduct a study. You collected 50 independent random samples, and found out the sample mean and sample standard deviation of the ratio of property values between 2000 and 1995 are 1.534 and 0.91, respectively.

(a) Formulate a statistical test, and determine the null and alternative hypotheses of interest. Is it a one-sided or two-sided test?

(b) What is the test statistic you will use? Can we assume that it is a Gaussian random variable?

(c) Based on the Gaussian assumption, suppose you want to perform a two-sided test at a confidence level of 99%, or a level of significance of $\alpha = 0.01$, do you reject the hypothesis that the property price does not change from 1995 to 2000? Why? [Hint: the critical value $z_{\alpha/2} = 2.33$ in this case]

(d) What can you report to the City Council about the results of your study and the probability of incorrect decision?

Problem 6 (20%)

Consider a wide sense stationary random process *X(t)*, and form a new process $Y(t) = X(t) + bX(t - \Delta t)$, *b* is a constant and Δt is usually a sampling interval.

(a) Express $E[Y(t)]$ in terms of $E[X(t)]$;

(b) Express $E[Y(t_1)Y(t_2)]$ in terms of $E[X(t_1)X(t_2)]$;

(c) Based on the results from parts (a) and (b), is *Y(t)* wide sense stationary?